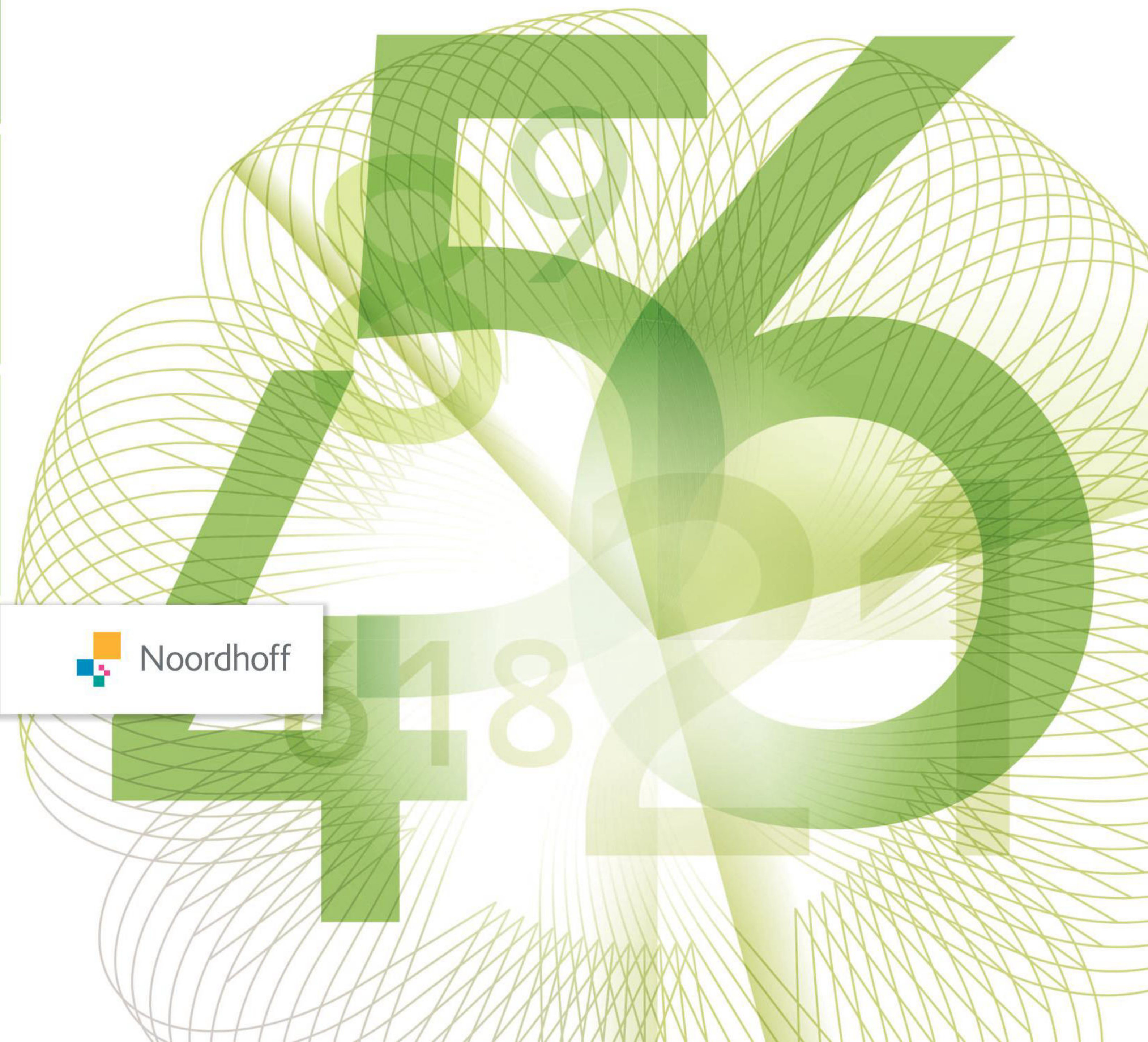
UITWERKINGEN | WISKUNDE B | 12E EDITIE | VWO DEEL 2

GETAL & RUIMTE

B



Noordhoff

Getal & Ruimte

Uitwerkingen
vwo B deel 2

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Inhoud

5 Machten, exponenten en logaritmen

Voorkennis Herleiden van machten

Bladzijde 10

1
a $x^2 \cdot x^3 = x^5$
b $2p^3 \cdot 3p^2 = 6p^5$
c $4a^2b \cdot 5a^3b^2 = 20a^5b^3$
d $-2p^4q^3 \cdot -3pq = 6p^5q^4$
e $5x^2y \cdot 2x - 3x^3y = 10x^3y - 3x^3y = 7x^3y$
f $12a^4b \cdot \frac{1}{4}ab - 8ab = 3a^5b^2 - 8ab$

2
a $(p^2q)^3 = p^6q^3$
b $(3x^2)^3 = 27x^6$
c $(-5x^2y^3)^2 = 25x^4y^6$
d $(-4ab^4)^2 = 16a^2b^8$
e $(3a)^2 \cdot (2a^2)^3 = 9a^2 \cdot 8a^6 = 72a^8$
f $(3a^3)^2 + (2a^2)^3 = 9a^6 + 8a^6 = 17a^6$

3
a $\frac{12x^6}{4x^2} = 3x^4$
b $\frac{5x^{10}}{15x^5} = \frac{x^5}{3} = \frac{1}{3}x^5$
c $\frac{24a^4b^2}{6ab} = 4a^3b$
d $\frac{-15p^6q}{5p^2q} = -3p^4$
e $\frac{10x^3y^2}{5x^2y} = 2xy$
f $\frac{(2ab)^3}{(3ab)^2} = \frac{8a^3b^3}{9a^2b^2} = \frac{8}{9}ab$

4
a $\frac{-28a^6}{7a} = -4a^5$
b $-(3a^4)^2 = -9a^8$
c $(-2a^2)^5 = -32a^{10}$
d $(-a^3)^3 = -a^9$
e $(5a)^3 \cdot -3a = 125a^3 \cdot -3a = -375a^4$
f $\left(\frac{9a^4}{a}\right)^2 = (9a^3)^2 = 81a^6$

5
a $(ab)^4 \cdot a = a^4b^4 \cdot a = a^5b^4$
b $(-2ab)^3 \cdot b = -8a^3b^3 \cdot b = -8a^3b^4$
c $(3a)^2 + (2b)^2 = 9a^2 + 4b^2$
d $(3a)^3 - 8a^3 = 27a^3 - 8a^3 = 19a^3$
e $(\frac{1}{2}a)^2 + (-a)^2 = \frac{1}{4}a^2 + a^2 = 1\frac{1}{4}a^2$
f $(5a^4)^2 + (-a^2)^4 = 25a^8 + a^8 = 26a^8$

6
a $a^{2n} \cdot a^{n-1} = a^{2n+n-1} = a^{3n-1}$
b $a^{n^2-1} \cdot a^{n-1} = a^{n^2-1+n-1} = a^{n^2+n-2}$
c $\frac{a^{n^2-n}}{a^{n-1}} = a^{n^2-n-(n-1)} = a^{n^2-n-n+1} = a^{n^2-2n+1}$

5.1 Machten met negatieve en gebroken exponenten

Bladzijde 11

1
a De exponenten worden telkens 1 minder en de getallen worden steeds door 2 gedeeld,

dus $2^1 = \frac{4}{2} = 2$, $2^0 = \frac{2}{2} = 1$, $2^{-1} = \frac{1}{2}$ en $2^{-2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$.

b $2^{-3} = \frac{1}{2^3}$

$2^{-4} = \frac{1}{2^4}$

c $x^0 = 1$

$x^{-1} = \frac{1}{x}$

$x^{-3} = \frac{1}{x^3}$

Bladzijde 12

2

a $\frac{1}{a^2} = a^{-2}$

b $a^4 \cdot \frac{1}{a^6} = a^4 \cdot a^{-6} = a^{-2}$

c $\frac{a^n}{\left(\frac{1}{a^4}\right)} = \frac{a^n}{a^{-4}} = a^{n+4}$

d $\frac{a^8}{a^0} = a^{8-0} = a^8$

e $(a^3)^{-2} = a^{-6}$

f $\frac{\left(\frac{1}{a^5}\right)}{a} = \frac{a^{-5}}{a^1} = a^{-5-1} = a^{-6}$

g $\frac{a}{a^{12}} = \frac{a^1}{a^{12}} = a^{1-12} = a^{-11}$

h $\frac{1}{a^8} \cdot (a^3)^n = a^{-8} \cdot a^{3n} = a^{-8+3n}$

i $\frac{\left(\frac{1}{a^n}\right)}{a^{-3}} = \frac{a^{-n}}{a^{-3}} = a^{-n--3} = a^{-n+3}$

Bladzijde 13

3

a $6a^{-5}b^3 = 6 \cdot \frac{1}{a^5} \cdot b^3 = \frac{6b^3}{a^5}$

b $\frac{1}{3}a^{-3} = \frac{1}{3} \cdot \frac{1}{a^3} = \frac{1}{3a^3}$

c $(5a^{-4}b^2)^{-1} = 5^{-1} \cdot (a^{-4})^{-1} \cdot (b^2)^{-1} = \frac{1}{5}a^4b^{-2} = \frac{a^4}{5b^2}$

d $\frac{3}{5}a^{-4} = \frac{3}{5} \cdot \frac{1}{a^4} = \frac{3}{5a^4}$

e $(\frac{1}{2}a)^{-3} = (2^{-1} \cdot a)^{-3} = 2^3 \cdot a^{-3} = 8 \cdot \frac{1}{a^3} = \frac{8}{a^3}$

f $\frac{1}{6}a^{-2}b^4 = \frac{1}{6} \cdot \frac{1}{a^2} \cdot b^4 = \frac{b^4}{6a^2}$

g $-4 \cdot (3a)^{-2} = -4 \cdot \frac{1}{(3a)^2} = -4 \cdot \frac{1}{9a^2} = -\frac{4}{9a^2}$

h $(3a)^{-2}b^{-3} = \frac{1}{(3a)^2} \cdot \frac{1}{b^3} = \frac{1}{9a^2b^3}$

i $(\frac{3}{8}a^{-1}b)^{-2} = (\frac{3}{8})^{-2} \cdot (a^{-1})^{-2} \cdot b^{-2} = \frac{1}{(\frac{3}{8})^2} \cdot a^2 \cdot \frac{1}{b^2} = \frac{64}{9} \cdot a^2 \cdot \frac{1}{b^2} = \frac{64a^2}{9b^2}$

4 Uit $(\sqrt[3]{x})^3 = x$ en $(x^{\frac{1}{3}})^3 = x$ volgt $x^{\frac{1}{3}} = \sqrt[3]{x}$.

Bladzijde 14

5

a $a \cdot \sqrt[3]{a} = a^1 \cdot a^{\frac{1}{3}} = a^{1\frac{1}{3}}$

b $\frac{1}{\sqrt{a}} = \frac{1}{a^{\frac{1}{2}}} = a^{-\frac{1}{2}}$

c $\frac{1}{a} = \frac{1}{a^1} = a^{-1}$

d $\frac{1}{a^3} = a^{-3}$

e $a^2 \cdot \sqrt{a} = a^2 \cdot a^{\frac{1}{2}} = a^{2\frac{1}{2}}$

f $\sqrt[3]{\frac{1}{a^2}} = \sqrt[3]{a^{-2}} = a^{-\frac{2}{3}}$

g $\sqrt[3]{a^{12}} = a^{\frac{12}{3}} = a^4$

h $\frac{1}{a^4} \cdot \sqrt[3]{a} = a^{-4} \cdot a^{\frac{1}{3}} = a^{-4+\frac{1}{3}} = a^{-3\frac{2}{3}}$

i $\frac{a^3}{\sqrt[3]{a}} = \frac{a^3}{a^{\frac{1}{3}}} = a^{3-\frac{1}{3}} = a^{2\frac{2}{3}}$

6

a $8\sqrt{2} = 2^3 \cdot 2^{\frac{1}{2}} = 2^{3\frac{1}{2}}$

b $\frac{1}{3}\sqrt{3} = 3^{-1} \cdot 3^{\frac{1}{2}} = 3^{-1+\frac{1}{2}} = 3^{-\frac{1}{2}}$

c $\frac{4\sqrt{2}}{\sqrt[3]{2}} = \frac{2^2 \cdot 2^{\frac{1}{2}}}{2^{\frac{1}{3}}} = 2^{2+\frac{1}{2}-\frac{1}{3}} = 2^{2\frac{1}{6}}$

d $\frac{1}{100}\sqrt{10} = 10^{-2} \cdot 10^{\frac{1}{2}} = 10^{-2+\frac{1}{2}} = 10^{-1\frac{1}{2}}$

e $\frac{1}{8} \cdot \sqrt[3]{\frac{1}{4}} = \frac{1}{2^3} \cdot \sqrt[3]{2^{-2}} = 2^{-3} \cdot 2^{-\frac{2}{3}} = 2^{-3-\frac{2}{3}} = 2^{-3\frac{2}{3}}$

f $10 \cdot \sqrt[3]{0{,}1} = 10^1 \cdot \sqrt[3]{10^{-1}} = 10^1 \cdot 10^{-\frac{1}{3}} = 10^{1-\frac{1}{3}} = 10^{\frac{2}{3}}$

7 **a** $5a^{3\frac{1}{3}} = 5a^3 \cdot a^{\frac{1}{3}} = 5a^3 \cdot \sqrt[3]{a}$

b $\frac{1}{2}a^{-\frac{1}{4}}b = \dfrac{b}{2a^{\frac{1}{4}}} = \dfrac{b}{2 \cdot \sqrt[4]{a}}$

c $3a^{-\frac{2}{3}} = \dfrac{3}{a^{\frac{2}{3}}} = \dfrac{3}{\sqrt[3]{a^2}}$

d $\frac{2}{3}a^{-3} \cdot b^{1\frac{1}{2}} = \dfrac{2b^{1\frac{1}{2}}}{3a^3} = \dfrac{2b\sqrt{b}}{3a^3}$

e $\frac{1}{5}a^{-\frac{1}{2}} \cdot b^{\frac{1}{3}} = \dfrac{b^{\frac{1}{3}}}{5a^{\frac{1}{2}}} = \dfrac{\sqrt[3]{b}}{5\sqrt{a}}$

f $(5a)^{-\frac{1}{2}} = \dfrac{1}{(5a)^{\frac{1}{2}}} = \dfrac{1}{\sqrt{5a}}$

8 **a** $\dfrac{x^6}{x^2 \cdot \sqrt{x}} = \dfrac{x^6}{x^2 \cdot x^{\frac{1}{2}}} = \dfrac{x^6}{x^{2\frac{1}{2}}} = x^{6 - 2\frac{1}{2}} = x^{3\frac{1}{2}}$

b $x \cdot \sqrt[7]{x^3} = x^1 \cdot x^{\frac{3}{7}} = x^{1\frac{3}{7}}$

c $\dfrac{x}{\sqrt[5]{x}} = \dfrac{x^1}{x^{\frac{1}{5}}} = x^{1 - \frac{1}{5}} = x^{\frac{4}{5}}$

d $x^4 \cdot \sqrt{x} = x^4 \cdot x^{\frac{1}{2}} = x^{4\frac{1}{2}}$

e $\dfrac{\sqrt[3]{x}}{\sqrt{x}} = \dfrac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{\frac{1}{3} - \frac{1}{2}} = x^{-\frac{1}{6}}$

f $\dfrac{x^4 \cdot \sqrt[5]{x}}{x^5 \cdot \sqrt[4]{x}} = \dfrac{x^4 \cdot x^{\frac{1}{5}}}{x^5 \cdot x^{\frac{1}{4}}} = \dfrac{x^{4\frac{1}{5}}}{x^{5\frac{1}{4}}} = x^{4\frac{1}{5} - 5\frac{1}{4}} = x^{-1\frac{1}{20}}$

9 De getallen als macht van 2 geschreven zijn: $2^0, 2^1, 2^{-1}, 2^2, 2^{-3}, 2^5, 2^{-8}, 2^{13}, 2^{-21}, 2^{34}, \ldots$
Het tiende getal is dus 2^{34}.

10 **a** $y = (2x^2)^3 \cdot \dfrac{2}{x^{10}} = 8x^6 \cdot 2x^{-10} = 16x^{-4}$
Dus $y = 16x^{-4}$.

b $y = \dfrac{3}{x} \cdot \sqrt[4]{x^3} = 3x^{-1} \cdot x^{\frac{3}{4}} = 3x^{-1 + \frac{3}{4}} = 3x^{-\frac{1}{4}}$
Dus $y = 3x^{-\frac{1}{4}}$.

c $y = 3(\frac{1}{3}x^2)^{-2} \cdot 6x^2 = 3 \cdot (\frac{1}{3})^{-2} \cdot (x^2)^{-2} \cdot 6x^2 = 3 \cdot 9 \cdot x^{-4} \cdot 6x^2 = 162x^{-2}$
Dus $y = 162x^{-2}$.

d $y = \dfrac{5}{3x\sqrt{x}} = \frac{5}{3} \cdot \dfrac{1}{x \cdot x^{\frac{1}{2}}} = 1\frac{2}{3} \cdot \dfrac{1}{x^{1\frac{1}{2}}} = 1\frac{2}{3}x^{-1\frac{1}{2}}$
Dus $y = 1\frac{2}{3}x^{-1\frac{1}{2}}$.

11 **a** $y = \dfrac{5}{x\sqrt{x}} = \dfrac{5}{x^{1\frac{1}{2}}} = 5x^{-1\frac{1}{2}}$
Dus $y = 5x^{-1\frac{1}{2}}$.

b $y = 5x\sqrt{x^3} = 5x^1 \cdot x^{\frac{3}{2}} = 5x^{1 + 1\frac{1}{2}} = 5x^{2\frac{1}{2}}$
Dus $y = 5x^{2\frac{1}{2}}$.

c $y = \dfrac{5}{x^3} \cdot 2\sqrt{x} = 5 \cdot x^{-3} \cdot 2x^{\frac{1}{2}} = 10x^{-3 + \frac{1}{2}} = 10x^{-2\frac{1}{2}}$
Dus $y = 10x^{-2\frac{1}{2}}$.

d $y = 72x(\frac{1}{4}x\sqrt{x})^3 = 72x \cdot (\frac{1}{4})^3 \cdot (x^{1\frac{1}{2}})^3 = 72x \cdot \frac{1}{64} \cdot x^{4\frac{1}{2}} = 1\frac{1}{8}x^{5\frac{1}{2}}$
Dus $y = 1\frac{1}{8}x^{5\frac{1}{2}}$.

Bladzijde 15

12 **a** $(x^{\frac{2}{3}})^{\frac{3}{2}} = 10^{\frac{3}{2}}$

b $(x^{\frac{2}{3}})^{\frac{3}{2}} = 10^{\frac{3}{2}}$
$x^1 = 10^1 \cdot 10^{\frac{1}{2}}$
$x = 10\sqrt{10}$

13 **a** $x^{\frac{2}{3}} = 9$
$x = 9^{\frac{3}{2}}$
$x = 9\sqrt{9} = 27$

b $8x^{-1\frac{1}{2}} = 1$
$x^{-1\frac{1}{2}} = \frac{1}{8}$
$x = (\frac{1}{8})^{-\frac{2}{3}}$
$x = (2^{-3})^{-\frac{2}{3}}$
$x = 2^2 = 4$

c $5 - 2x^{-3} = 4$
$-2x^{-3} = -1$
$x^{-3} = \frac{1}{2}$
$x = (\frac{1}{2})^{-\frac{1}{3}}$
$x = (2^{-1})^{-\frac{1}{3}}$
$x = 2^{\frac{1}{3}} = \sqrt[3]{2}$

d $\sqrt{(2x)^3} = \frac{1}{8}$
$(2x)^3 = (\frac{1}{8})^2$
$8x^3 = \dfrac{1}{8^2}$
$x^3 = \dfrac{1}{8^3}$
$x = \frac{1}{8}$

Bladzijde 16

14 **a** $\frac{1}{3}x^{1\frac{1}{2}} = 2\frac{2}{3} - \frac{1}{3}x^{1\frac{1}{2}}$
$\frac{2}{3}x^{1\frac{1}{2}} = 2\frac{2}{3}$
$x^{\frac{3}{2}} = 4$
$x = 4^{\frac{2}{3}} = \sqrt[3]{16}$

b $3 \cdot \sqrt[4]{(2x)^{-1}} = 6$
$\sqrt[4]{(2x)^{-1}} = 2$
$(2x)^{-\frac{1}{4}} = 2$
$2x = 2^{-4}$
$2x = \frac{1}{16}$
$x = \frac{1}{32}$

c $(x + 9)^{-1\frac{1}{2}} = \frac{8}{27}$
$x + 9 = (\frac{8}{27})^{-\frac{2}{3}}$
$x + 9 = ((\frac{2}{3})^3)^{-\frac{2}{3}}$
$x + 9 = (\frac{2}{3})^{-2} = \dfrac{1}{(\frac{2}{3})^2} = \dfrac{1}{\frac{4}{9}} = \frac{9}{4}$
$x = -6\frac{3}{4}$

d $\frac{1}{3}(x^2 - 3)^{2\frac{1}{2}} = 81$
$(x^2 - 3)^{2\frac{1}{2}} = 243$
$x^2 - 3 = 243^{\frac{2}{5}} = (3^5)^{\frac{2}{5}} = 3^2 = 9$
$x^2 = 12$
$x = \sqrt{12} \vee x = -\sqrt{12}$
$x = 2\sqrt{3} \vee x = -2\sqrt{3}$

15 $(x^2 + 10)^{1\frac{1}{2}} = 3x^2(x^2 + 10)^{\frac{1}{2}}$
$(x^2 + 10)^1 \cdot (x^2 + 10)^{\frac{1}{2}} = 3x^2(x^2 + 10)^{\frac{1}{2}}$
$(x^2 + 10)^{\frac{1}{2}} = 0 \vee x^2 + 10 = 3x^2$
$x^2 + 10 = 0 \vee 2x^2 = 10$
$x^2 = -10 \vee x^2 = 5$
$x = \sqrt{5} \vee x = -\sqrt{5}$

16 $y = 27x^3$
$27x^3 = y$
$x^3 = \frac{1}{27}y$
$x = (\frac{1}{27}y)^{\frac{1}{3}}$
$x = (\frac{1}{27})^{\frac{1}{3}} \cdot y^{\frac{1}{3}}$
$x = (3^{-3})^{\frac{1}{3}} \cdot y^{\frac{1}{3}}$
$x = 3^{-1} \cdot y^{\frac{1}{3}}$
$x = \frac{1}{3} \cdot y^{\frac{1}{3}}$
Dus $x = \frac{1}{3}y^{\frac{1}{3}}$.

Bladzijde 17

17 **a** $y = 5x^{1\frac{1}{2}}$

$5x^{1\frac{1}{2}} = y$

$x^{\frac{3}{2}} = 0{,}2y$

$x = (0{,}2y)^{\frac{2}{3}}$

$x = 0{,}2^{\frac{2}{3}} \cdot y^{\frac{2}{3}}$

$x \approx 0{,}34 \cdot y^{0{,}67}$

Dus $x = 0{,}34y^{0{,}67}$.

b $y = 0{,}1x^{-1\frac{2}{3}}$

$0{,}1x^{-1\frac{2}{3}} = y$

$x^{-\frac{5}{3}} = 10y$

$x = (10y)^{-\frac{3}{5}}$

$x = 10^{-\frac{3}{5}} \cdot y^{-\frac{3}{5}}$

$x \approx 0{,}25 \cdot y^{-0{,}6}$

Dus $x = 0{,}25y^{-0{,}6}$.

c $y = 125x^{-2\frac{1}{2}}$

$125x^{-2\frac{1}{2}} = y$

$x^{-\frac{5}{2}} = 0{,}008y$

$x = (0{,}008y)^{-\frac{2}{5}}$

$x = 0{,}008^{-\frac{2}{5}} \cdot y^{-\frac{2}{5}}$

$x \approx 6{,}90 \cdot y^{-0{,}4}$

Dus $x = 6{,}90y^{-0{,}4}$.

18 **a** $y = 15x \cdot \sqrt[3]{x} = 15x \cdot x^{\frac{1}{3}} = 15x^{1\frac{1}{3}}$

$y = 15x^{1\frac{1}{3}}$ geeft $15x^{1\frac{1}{3}} = y$

$x^{\frac{4}{3}} = \frac{1}{15}y$

$x = (\frac{1}{15}y)^{\frac{3}{4}}$

$x = (\frac{1}{15})^{\frac{3}{4}} \cdot y^{\frac{3}{4}}$

$x \approx 0{,}13 \cdot y^{0{,}75}$

Dus $x = 0{,}13y^{0{,}75}$.

b $y = \dfrac{12}{x \cdot \sqrt[4]{x}} = \dfrac{12}{x \cdot x^{\frac{1}{4}}} = \dfrac{12}{x^{1\frac{1}{4}}} = 12x^{-1\frac{1}{4}}$

$y = 12x^{-1\frac{1}{4}}$ geeft $12x^{-1\frac{1}{4}} = y$

$x^{-\frac{5}{4}} = \frac{1}{12}y$

$x = (\frac{1}{12}y)^{-\frac{4}{5}}$

$x = (\frac{1}{12})^{-\frac{4}{5}} \cdot y^{-\frac{4}{5}}$

$x \approx 7{,}30 \cdot y^{-0{,}8}$

Dus $x = 7{,}30y^{-0{,}8}$.

c $y = \frac{6}{x^2 \cdot \sqrt[5]{x^3}} = \frac{6}{x^2 \cdot x^{\frac{3}{5}}} = \frac{6}{x^{2\frac{3}{5}}} = 6x^{-2\frac{3}{5}}$

$y = 6x^{-2\frac{3}{5}}$ geeft $6x^{-2\frac{3}{5}} = y$

$x^{-\frac{13}{5}} = \frac{1}{6}y$

$x = (\frac{1}{6}y)^{-\frac{5}{13}}$

$x = (\frac{1}{6})^{-\frac{5}{13}} \cdot y^{-\frac{5}{13}}$

$x \approx 1{,}99 \cdot y^{-0{,}38}$

Dus $x = 1{,}99y^{-0{,}38}$.

19 **a** $K = 15q^{-1{,}6}$ geeft $15q^{-1{,}6} = K$

$q^{-\frac{8}{5}} = \frac{1}{15}K$

$q = (\frac{1}{15}K)^{-\frac{5}{8}}$

$q = (\frac{1}{15})^{-\frac{5}{8}} \cdot K^{-\frac{5}{8}}$

$q \approx 5{,}43 \cdot K^{-0{,}625}$

Dus $q = 5{,}43K^{-0{,}625}$.

b $v = 25t\sqrt{t} = 25t^{1\frac{1}{2}}$

$v = 25t^{1\frac{1}{2}}$ geeft $25t^{1\frac{1}{2}} = v$

$t^{\frac{3}{2}} = 0{,}04v$

$t = (0{,}04v)^{\frac{2}{3}}$

$t = (0{,}04)^{\frac{2}{3}} \cdot v^{\frac{2}{3}}$

$t \approx 0{,}12 \cdot v^{0{,}67}$

Dus $t = 0{,}12v^{0{,}67}$.

20 $F = \frac{m\sqrt{m}}{m\sqrt{m} - 1}$

$F(m\sqrt{m} - 1) = m\sqrt{m}$

$m\sqrt{m} \cdot F - F = m\sqrt{m}$

$m\sqrt{m} \cdot F - m\sqrt{m} = F$

$m\sqrt{m}(F - 1) = F$

$m\sqrt{m} = \frac{F}{F - 1}$

$m^{\frac{3}{2}} = \frac{F}{F - 1}$

$m = \left(\frac{F}{F - 1}\right)^{\frac{2}{3}}$

Bladzijde 18

21 **a** $h_0 = 0{,}6$ en $d_0 = 1000$ geeft

$h = 0{,}6 \cdot \left(\frac{1000}{d}\right)^{0{,}25} = 0{,}6 \cdot (1000d^{-1})^{0{,}25} = 0{,}6 \cdot 1000^{0{,}25} \cdot d^{-0{,}25} \approx 3{,}37 \cdot d^{-0{,}25}$

Dus $h = 3{,}37d^{-0{,}25}$.

b $d = 300$ geeft $h = 3{,}37 \cdot 300^{-0{,}25} \approx 0{,}8$

De waterhoogte bij een waterdiepte van 300 meter is 8 dm.

c $h = 3{,}37d^{-0{,}25}$ geeft $3{,}37d^{-0{,}25} = h$

$$d^{-\frac{1}{4}} = \frac{1}{3{,}37}h$$

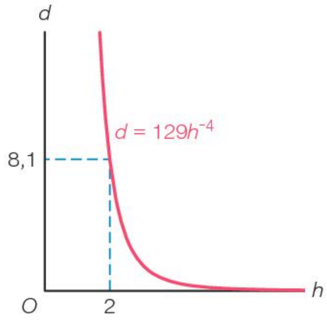
$$d = \left(\frac{1}{3{,}37}h\right)^{-4}$$

$$d = \left(\frac{1}{3{,}37}\right)^{-4} \cdot h^{-4}$$

$$d \approx 129 \cdot h^{-4}$$

Dus $d = 129h^{-4}$.

d $h = 2$ geeft $d = 129 \cdot 2^{-4} \approx 8{,}1$



$h > 2$ geeft $d < 8{,}1$

Dus bij waterdieptes minder dan 8,1 meter is de golf meer dan 2 meter hoog.

22 **a** $h = 1{,}5$ geeft $D = 1014(1 - 0{,}0226 \cdot 1{,}5)^{5{,}26} \approx 846$

Dus op een hoogte van 1,5 km is de luchtdruk 846 mbar.

b $D = 1014(1 - 0{,}0226h)^{5{,}26}$ geeft $1014(1 - 0{,}0226h)^{5{,}26} = D$

$$(1 - 0{,}0226h)^{5{,}26} = \frac{1}{1014}D$$

$$1 - 0{,}0226h = \left(\frac{1}{1014}D\right)^{\frac{1}{5{,}26}}$$

$$-0{,}0226h = \left(\frac{1}{1014}\right)^{\frac{1}{5{,}26}} \cdot D^{\frac{1}{5{,}26}} - 1$$

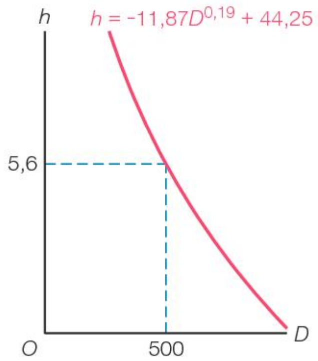
$$h = \frac{\left(\frac{1}{1014}\right)^{\frac{1}{5{,}26}}}{-0{,}0226} \cdot D^{\frac{1}{5{,}26}} - \frac{1}{-0{,}0226}$$

$$h \approx -11{,}87 \cdot D^{0{,}19} + 44{,}25$$

Dus $h = -11{,}87D^{0{,}19} + 44{,}25$.

c 0,5 bar = 500 mbar

$D = 500$ geeft $h = -11{,}87 \cdot 500^{0{,}19} + 44{,}25 \approx 5{,}6$



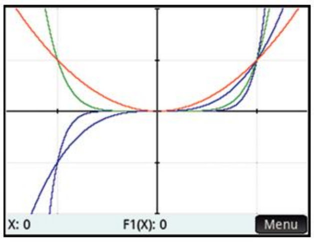
$D < 500$ geeft $h > 5{,}6$

Bij hoogten hoger dan 5,6 km is de luchtdruk minder dan 500 mbar = 0,5 bar.

5.2 Machtsfuncties en wortelfuncties

Bladzijde 20

23 **a**



b (0, 0) en (1, 1)

c Van de grafieken van f en h ligt geen enkel punt onder de x-as.

Bladzijde 22

24 **a** $y = -\frac{1}{2}x^3$

$\downarrow$ translatie (−3, −5)

$y = -\frac{1}{2}(x + 3)^3 - 5$

$\downarrow$ verm. x-as, −3

$y = -3(-\frac{1}{2}(x + 3)^3 - 5)$

oftewel $y = 1\frac{1}{2}(x + 3)^3 + 15$

b Het punt van symmetrie is (−3, 15).

25 **a** $y = 0{,}3x^4$

$\downarrow$ translatie (−5, 6)

$y = 0{,}3(x + 5)^4 + 6$

$\downarrow$ verm. x-as, −4

$y = -4(0{,}3(x + 5)^4 + 6)$

oftewel $y = -1{,}2(x + 5)^4 - 24$

De top is (−5, −24).

b $y = 0{,}3x^4$

$\downarrow$ verm. x-as, −4

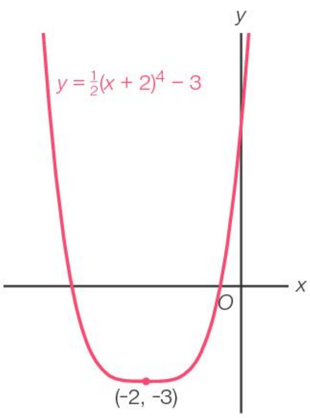
$y = -1{,}2x^4$

$\downarrow$ translatie (−5, 6)

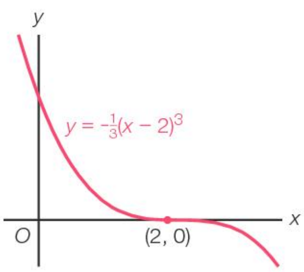
$y = -1{,}2(x + 5)^4 + 6$

De top is (−5, 6).

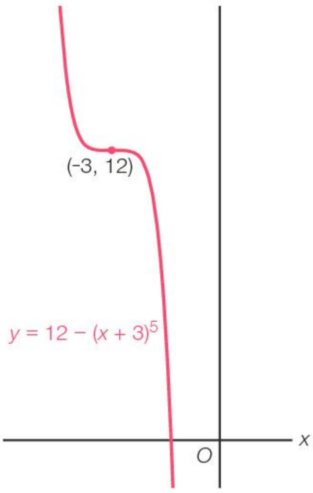
26 **a**



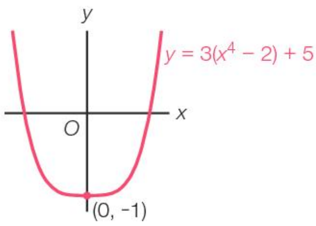
b



c



d $y = 3(x^4 - 2) + 5 = 3x^4 - 6 + 5 = 3x^4 - 1$

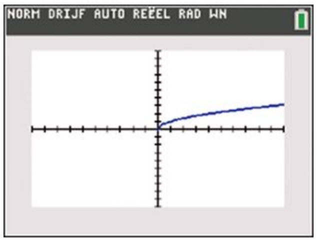


27 **a** $f(x) = \frac{1}{2}(x - 3)^5 + 7$
$\downarrow$ translatie (1, 2)
$y = \frac{1}{2}(x - 1 - 3)^5 + 7 + 2$
oftewel $y = \frac{1}{2}(x - 4)^5 + 9$
$\downarrow$ verm. x-as, -1
$y = -\frac{1}{2}(x - 4)^5 - 9$
Het punt van symmetrie is (4, -9).

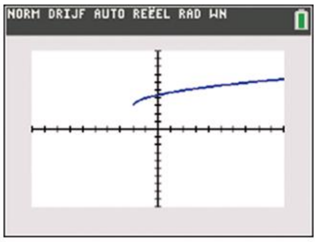
b $g(x) = -2\frac{1}{2}(x + 4)^6 - 1$
$\downarrow$ verm. x-as, -4
$y = 10(x + 4)^6 + 4$
$\downarrow$ translatie (-5, -4)
$h(x) = 10(x + 5 + 4)^6 + 4 - 4$
oftewel $h(x) = 10(x + 9)^6$
min. is $h(-9) = 0$

28 **a** Voor f geldt $y = (x - 5)^3 + 5$, dus voor f^{inv} geldt $x = (y - 5)^3 + 5$.
$x = (y - 5)^3 + 5$ geeft $(y - 5)^3 + 5 = x$
$(y - 5)^3 = x - 5$
$y - 5 = \sqrt[3]{x - 5}$
$y = \sqrt[3]{x - 5} + 5$
Dus $g(x) = \sqrt[3]{x - 5} + 5$ is de inverse van f.

b $f(x) = g(x)$ geeft $(x - 5)^3 + 5 = \sqrt[3]{x - 5} + 5$
$(x - 5)^3 = \sqrt[3]{x - 5}$
$(x - 5)^9 = x - 5$
$(x - 5)^8 \cdot (x - 5) = x - 5$
$x - 5 = 0 \vee (x - 5)^8 = 1$
$x = 5 \vee x - 5 = 1 \vee x - 5 = -1$
$x = 5 \vee x = 6 \vee x = 4$
De snijpunten zijn (4, 4), (5, 5) en (6, 6).

29 **a** 

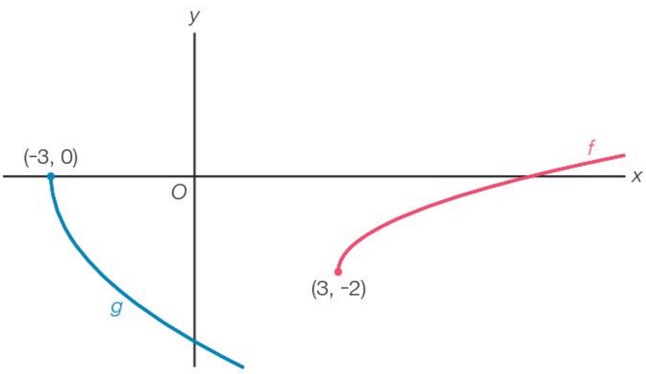
Het domein is $[0, \rightarrow\rangle$ en het bereik is $[0, \rightarrow\rangle$.

b 

De grafiek van $y = \sqrt{x+2} + 3$ ontstaat uit de grafiek van $y = \sqrt{x}$ bij de translatie $(-2, 3)$.

Bladzijde 23

30 **a** De grafiek van $f(x) = \sqrt{x-3} - 2$ ontstaat uit die van $y = \sqrt{x}$ bij de translatie $(3, -2)$.
De grafiek van $g(x) = -2\sqrt{x+3}$ ontstaat uit die van $y = \sqrt{x}$ bij de vermenigvuldiging ten opzichte van de x-as met -2 en de translatie $(-3, 0)$.

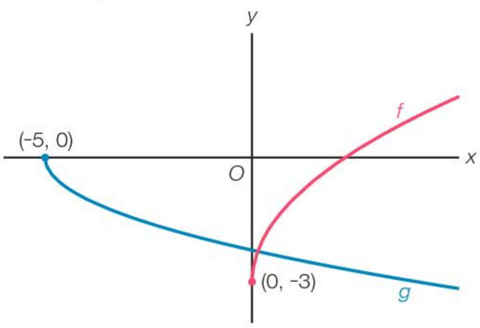
b 

c $D_f = [3, \rightarrow\rangle$, $B_f = [-2, \rightarrow\rangle$, $D_g = [-3, \rightarrow\rangle$ en $B_g = \langle\leftarrow, 0]$

Bladzijde 24

31 **a**

$y = \sqrt{x}$	$y = \sqrt{x}$
$\downarrow$ verm. x-as, 2	$\downarrow$ verm. x-as, -1
$y = 2\sqrt{x}$	$y = -\sqrt{x}$
$\downarrow$ translatie $(0, -3)$	$\downarrow$ translatie $(-5, 0)$
$f(x) = 2\sqrt{x} - 3$	$g(x) = -\sqrt{x+5}$

b 

c $D_f = [0, \rightarrow\rangle$, $B_f = [-3, \rightarrow\rangle$, $D_g = [-5, \rightarrow\rangle$ en $B_g = \langle\leftarrow, 0]$

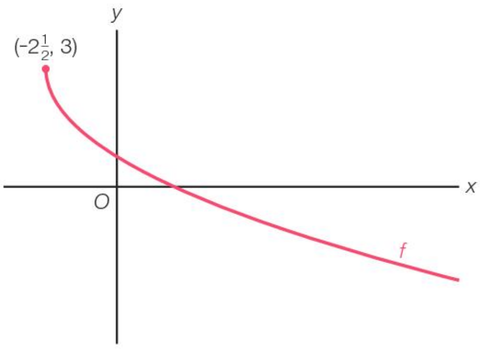
32 **a** randpunt (−5, 3), $D_f = [-5, \rightarrow\rangle$ en $B_f = [3, \rightarrow\rangle$
b randpunt (−3, −7), $D_g = [-3, \rightarrow\rangle$ en $B_g = [-7, \rightarrow\rangle$
c randpunt (−1, 0), $D_h = [-1, \rightarrow\rangle$ en $B_h = \langle\leftarrow, 0]$
d randpunt (0, 1), $D_k = [0, \rightarrow\rangle$ en $B_k = [1, \rightarrow\rangle$
e randpunt (1, −1), $D_l = [1, \rightarrow\rangle$ en $B_l = \langle\leftarrow, -1]$
f randpunt (0, −3), $D_m = [0, \rightarrow\rangle$ en $B_m = [-3, \rightarrow\rangle$

33 **a** Voor het domein van $f(x) = 5 - \sqrt{2x - 6}$ moet gelden $2x - 6 \geq 0$
$2x \geq 6$
$x \geq 3$
Dus het domein is $D_f = [3, \rightarrow\rangle$.
b De uitkomst van een wortel is minstens 0, dus de uitkomst van $f(x)$ is hoogstens 5.
Dus het bereik is $B_f = \langle\leftarrow, 5]$.

Bladzijde 26

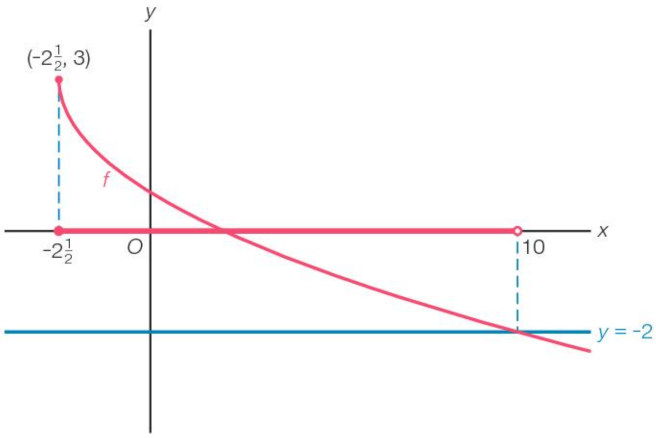
34 **a** $8 - 4x \geq 0$
$-4x \geq -8$
$x \leq 2$
Dus $D_f = \langle\leftarrow, 2]$, $B_f = [3, \rightarrow\rangle$ en randpunt (2, 3).
b $4x - 8 \geq 0$
$4x \geq 8$
$x \geq 2$
Dus $D_g = [2, \rightarrow\rangle$, $B_g = [3, \rightarrow\rangle$ en randpunt (2, 3).
c $2x + 6 \geq 0$
$2x \geq -6$
$x \geq -3$
Dus $D_h = [-3, \rightarrow\rangle$, $B_h = \langle\leftarrow, 5]$ en randpunt (−3, 5).
d $x \geq 0$
Dus $D_k = [0, \rightarrow\rangle$, $B_k = \langle\leftarrow, 3]$ en randpunt (0, 3).

35 **a** $2x + 5 \geq 0$
$2x \geq -5$
$x \geq -2\frac{1}{2}$
$D_f = [-2\frac{1}{2}, \rightarrow\rangle$
randpunt $(-2\frac{1}{2}, 3)$



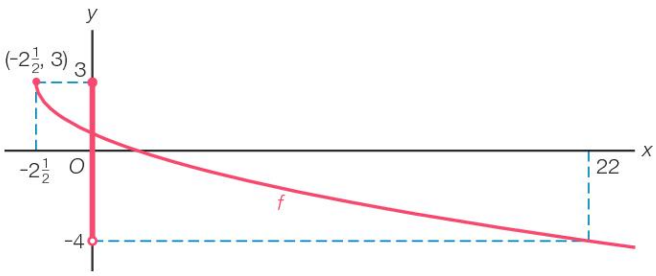
$B_f = \langle\leftarrow, 3]$

b $f(x) = -2$ geeft $3 - \sqrt{2x+5} = -2$
$\sqrt{2x+5} = 5$
$2x + 5 = 25$
$2x = 20$
$x = 10$



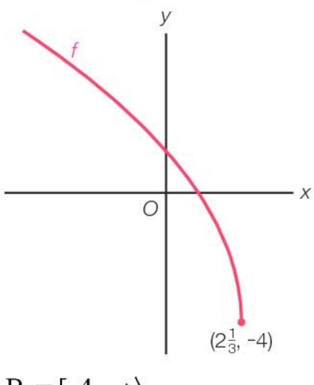
$f(x) > -2$ geeft $-2\frac{1}{2} \leq x < 10$

c $f(22) = -4$



$x < 22$ geeft $-4 < f(x) \leq 3$

36 **a** $7 - 3x \geq 0$
$-3x \geq -7$
$x \leq 2\frac{1}{3}$ dus $D_f = \langle \leftarrow, 2\frac{1}{3}]$
randpunt $(2\frac{1}{3}, -4)$



$B_f = [-4, \rightarrow\rangle$

b $f(x) = -2$ geeft $2\sqrt{7-3x} - 4 = -2$

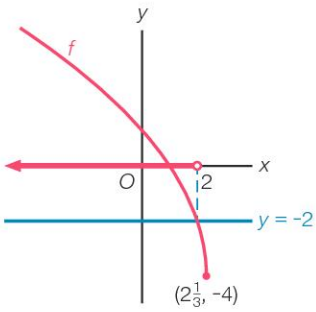
$2\sqrt{7-3x} = 2$

$\sqrt{7-3x} = 1$

$7 - 3x = 1$

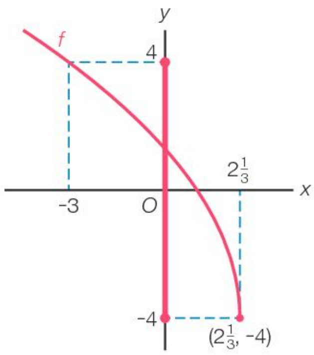
$-3x = -6$

$x = 2$



$f(x) > -2$ geeft $x < 2$

c $f(-3) = 4$



$x \geq -3$ geeft $-4 \leq f(x) \leq 4$

37 Stel $f(x) = 2 + \sqrt{7-2x}$ en $g(x) = x$.

$7 - 2x \geq 0$

$-2x \geq -7$

$x \leq 3\frac{1}{2}$

Dus $D_f = \langle \leftarrow, 3\frac{1}{2}]$ en het randpunt van de grafiek van f is $(3\frac{1}{2}, 2)$.

$f(x) = g(x)$ geeft $2 + \sqrt{7-2x} = x$

$\sqrt{7-2x} = x - 2$

kwadrateren geeft

$7 - 2x = x^2 - 4x + 4$

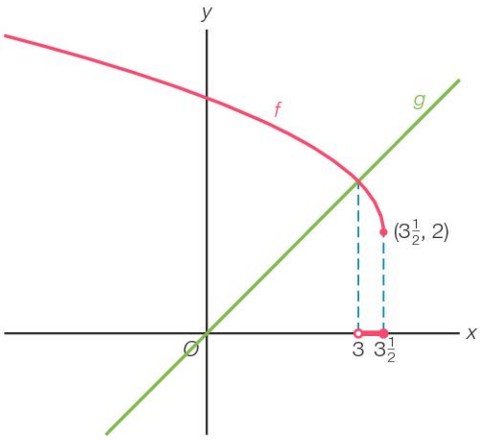
$-x^2 + 2x + 3 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1 \lor x = 3$

vold. niet vold.



$2 + \sqrt{7-2x} < x$ geeft $3 < x \leq 3\frac{1}{2}$

38 Er geldt $D_f = [-8, \rightarrow\rangle$, $B_f = \langle\leftarrow, 3]$ en $g = f^{\text{inv}}$, dus $D_g = B_f = \langle\leftarrow, 3]$ en $B_g = D_f = [-8, \rightarrow\rangle$.

39 $5 + ax = 0$

$ax = -5$

$x = -\frac{5}{a}$

Dus het randpunt van de grafiek van f is $\left(-\frac{5}{a}, 4\right)$.

$\left(-\frac{5}{a}, 4\right)$ op de lijn $y = 2x - 1$ geeft $2 \cdot -\frac{5}{a} - 1 = 4$

$-\frac{10}{a} = 5$

$a = -2$

40 $(5, 3)$ op de grafiek van $f(x) = a\sqrt{x + b}$ geeft $a\sqrt{5 + b} = 3$

$a = \frac{3}{\sqrt{b + 5}}$

$(13, 9)$ op de grafiek van $f(x) = a\sqrt{x + b}$ geeft $a\sqrt{13 + b} = 9$

$a = \frac{9}{\sqrt{b + 13}}$

$a = \frac{3}{\sqrt{b + 5}}$ en $a = \frac{9}{\sqrt{b + 13}}$ geeft $\frac{3}{\sqrt{b + 5}} = \frac{9}{\sqrt{b + 13}}$

$3\sqrt{b + 13} = 9\sqrt{b + 5}$

$\sqrt{b + 13} = 3\sqrt{b + 5}$

kwadrateren geeft

$b + 13 = 9(b + 5)$

$b + 13 = 9b + 45$

$-8b = 32$

$b = -4$ vold.

$b = -4$ en $a = \frac{3}{\sqrt{b + 5}}$ geeft $a = \frac{3}{\sqrt{-4 + 5}} = 3$

Dus $a = 3$ en $b = -4$.

41 **a** $y = 2\sqrt{x}$ geeft $2\sqrt{x} = y$

kwadrateren geeft

$4x = y^2$

$x = \frac{1}{4}y^2$

b $y = \sqrt{x - 2}$ geeft $\sqrt{x - 2} = y$

kwadrateren geeft

$x - 2 = y^2$

$x = y^2 + 2$

Uit $y = \sqrt{x - 2}$ volgt $x = y^2 + 2$.

c $y = 2\sqrt{x - 2}$ geeft $2\sqrt{x - 2} = y$

kwadrateren geeft

$4(x - 2) = y^2$

$x - 2 = \frac{1}{4}y^2$

$x = \frac{1}{4}y^2 + 2$

Uit $y = 2\sqrt{x - 2}$ volgt $x = \frac{1}{4}y^2 + 2$.

Bladzijde 27

42 **a** $F = 3\sqrt{2t-1}$ geeft $3\sqrt{2t-1} = F$

kwadrateren geeft

$9(2t-1) = F^2$

$2t - 1 = \frac{1}{9}F^2$

$2t = \frac{1}{9}F^2 + 1$

$t = \frac{1}{18}F^2 + \frac{1}{2}$

b $A = 5 + \sqrt{4-3B}$ geeft $5 + \sqrt{4-3B} = A$

$\sqrt{4-3B} = A - 5$

kwadrateren geeft

$4 - 3B = (A-5)^2$

$4 - 3B = A^2 - 10A + 25$

$-3B = A^2 - 10A + 21$

$B = -\frac{1}{3}A^2 + 3\frac{1}{3}A - 7$

c $2x\sqrt{y} - 5 = 0$ geeft $2x\sqrt{y} = 5$

kwadrateren geeft

$4x^2y = 25$

$y = \dfrac{25}{4x^2}$

d $R\sqrt{q} - \sqrt{R} = 6$ geeft $R\sqrt{q} = 6 + \sqrt{R}$

kwadrateren geeft

$R^2q = (6 + \sqrt{R})^2$

$R^2q = 36 + 12\sqrt{R} + R$

$q = \dfrac{36 + 12\sqrt{R} + R}{R^2}$

Bladzijde 28

43 **a** $T = -27{,}4$ geeft $v = 331\sqrt{1 - \dfrac{27{,}4}{273}} = 313{,}9...$

$T = 38{,}6$ geeft $v = 331\sqrt{1 + \dfrac{38{,}6}{273}} = 353{,}6...$

Het verschil is $353{,}6... - 313{,}9... \approx 40$ m/s.

b $v = 340$ geeft $331\sqrt{1 + \dfrac{T}{273}} = 340$

$\sqrt{1 + \dfrac{T}{273}} = \dfrac{340}{331}$

kwadrateren geeft

$1 + \dfrac{T}{273} = \left(\dfrac{340}{331}\right)^2$

$\dfrac{T}{273} = \left(\dfrac{340}{331}\right)^2 - 1$

$T = 273 \cdot \left(\dfrac{340}{331}\right)^2 - 273 \approx 15$

Bij een temperatuur van 15 °C is de geluidssnelheid 340 m/s.

c $T_0 = 25$ en $h = 2{,}5$ geeft $T = 25 - 6{,}5 \cdot 2{,}5 = 8{,}75$

$T = 8{,}75$ geeft $v = 331\sqrt{1 + \dfrac{8{,}75}{273}} = 336{,}2...$ m/s $= 1210{,}5...$ km/uur

De snelheid is 1211 km/uur.

d $T = -1$ en $h = 2$ geeft $T_0 - 6{,}5 \cdot 2 = -1$

$T_0 = 12$

Dus $T = 12 - 6{,}5h$.

Dit geeft $v = 331\sqrt{1 + \frac{12 - 6{,}5h}{273}} = 331\sqrt{1 + \frac{12}{273} - \frac{6{,}5}{273}h} \approx 331\sqrt{-0{,}0238h + 1{,}0440}$.

Dus $v = 331\sqrt{-0{,}0238h + 1{,}0440}$.

e $v = 331\sqrt{1 + \frac{T}{273}}$ geeft $331\sqrt{1 + \frac{T}{273}} = v$

$\sqrt{1 + \frac{T}{273}} = \frac{v}{331}$

kwadrateren geeft

$1 + \frac{T}{273} = \left(\frac{v}{331}\right)^2$

$\frac{T}{273} = \left(\frac{v}{331}\right)^2 - 1$

$T = 273 \cdot \left(\frac{v}{331}\right)^2 - 273$

$T = \frac{273}{109\,561}v^2 - 273$

$T = T_0 - 6{,}5h$ en $T = \frac{273}{109\,561}v^2 - 273$ geeft $T_0 - 6{,}5h = \frac{273}{109\,561}v^2 - 273$

$-6{,}5h = \frac{273}{109\,561}v^2 - T_0 - 273$

$h = -\frac{273}{109\,561 \cdot 6{,}5}v^2 + \frac{1}{6{,}5}T_0 + \frac{273}{6{,}5}$

$h \approx -0{,}0004v^2 + 0{,}1538T_0 + 42$

Dus $h = -0{,}0004v^2 + 0{,}1538T_0 + 42$.

44 Voor f geldt $y = a + \sqrt{bx + c}$, dus voor f^{inv} geldt $x = a + \sqrt{by + c}$.

$x = a + \sqrt{by + c}$ geeft $a + \sqrt{by + c} = x$

$\sqrt{by + c} = x - a$

kwadrateren geeft

$by + c = (x - a)^2$

$by + c = x^2 - 2ax + a^2$

$by = x^2 - 2ax + a^2 - c$

$y = \frac{1}{b}x^2 - \frac{2a}{b}x + \frac{a^2 - c}{b}$

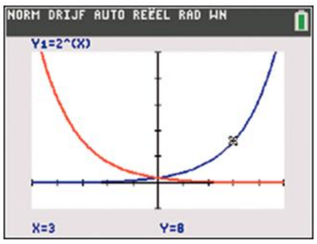
$f^{\text{inv}}(x) = \frac{1}{2}x^2 - 3x$ geeft $\frac{1}{b} = \frac{1}{2} \wedge -\frac{2a}{b} = -3 \wedge \frac{a^2 - c}{b} = 0$

Dus $b = 2 \wedge -a = -3 \wedge a^2 - c = 0$ oftewel $a = 3$, $b = 2$ en $c = 9$.

5.3 Exponentiële functies

Bladzijde 30

45 **a**



De grafieken zijn elkaars gespiegelde in de y-as.

b $f(-10) = 2^{-10} \approx 9{,}77 \cdot 10^{-4}$
$f(-20) = 2^{-20} \approx 9{,}54 \cdot 10^{-7}$
$f(-100) = 2^{-100} \approx 7{,}89 \cdot 10^{-31}$

c Voor elke x is $2^x > 0$, dus er is geen origineel te vinden waarvan het beeld 0 is.

d $B_f = B_g = \langle 0, \rightarrow \rangle$

Bladzijde 32

46 **a** $y = 3^x$
$\downarrow$ translatie $(-2, -1)$
$y = 3^{x+2} - 1$
De asymptoot is de lijn $y = -1$.

b $y = 0{,}5^x$
$\downarrow$ verm. x-as, 2
$y = 2 \cdot 0{,}5^x$
$\downarrow$ translatie $(0, 3)$
$y = 2 \cdot 0{,}5^x + 3$
De asymptoot is de lijn $y = 3$.

c $y = 2^x$
$\downarrow$ verm. y-as, $\frac{1}{3}$
$y = 2^{3x}$
$\downarrow$ verm. x-as, 3
$y = 3 \cdot 2^{3x}$
$\downarrow$ translatie $(0, 4)$
$y = 3 \cdot 2^{3x} + 4$
De asymptoot is de lijn $y = 4$.

d $y = 0{,}8^x$
$\downarrow$ verm. x-as, -1
$y = -0{,}8^x$
$\downarrow$ translatie $(-1, 10)$
$y = 10 - 0{,}8^{x+1}$
$\downarrow$ verm. y-as, 2,5
$y = 10 - 0{,}8^{0{,}4x+1}$
De asymptoot is de lijn $y = 10$.

47 **a** Het bereik is $\langle -6, \rightarrow \rangle$ en de asymptoot is de lijn $y = -6$.
b Het bereik is $\langle \leftarrow, 5 \rangle$ en de asymptoot is de lijn $y = 5$.
c Het bereik is $\langle \leftarrow, 1000 \rangle$ en de asymptoot is de lijn $y = 1000$.
d Het bereik is $\langle \leftarrow, 1000 \rangle$ en de asymptoot is de lijn $y = 1000$.

48 **a** $y = 3^x$
$\downarrow$ spiegelen in de x-as
$y = -3^x$
$\downarrow$ translatie $(0, -1)$
$y = -3^x - 1$

b $y = 3^x$
$\downarrow$ translatie $(2, 5)$
$y = 3^{x-2} + 5$
$\downarrow$verm. y-as, $\frac{1}{4}$
$y = 3^{4x-2} + 5$

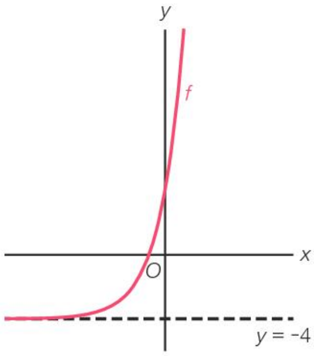
c $y = 3^x$
$\downarrow$ translatie $(4, -5)$
$y = 3^{x-4} - 5$
$\downarrow$ verm. x-as, 3
$y = 3(3^{x-4} - 5)$
oftewel $y = 3^{x-3} - 15$

d $y = 3^x$
$\downarrow$ verm. x-as, 3
$y = 3 \cdot 3^x$
$\downarrow$ translatie $(4, -5)$
$y = 3 \cdot 3^{x-4} - 5$
oftewel $y = 3^{x-3} - 5$

49 **a** $y = 2^x$
$\downarrow$ translatie $(-3, -4)$
$f(x) = 2^{x+3} - 4$

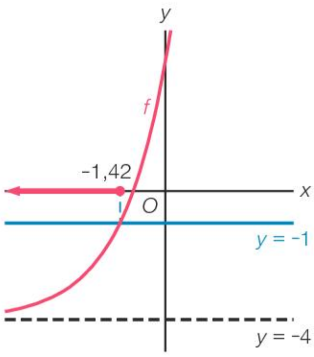
b $B_f = \langle -4, \rightarrow \rangle$
De horizontale asymptoot is de lijn $y = -4$.

c



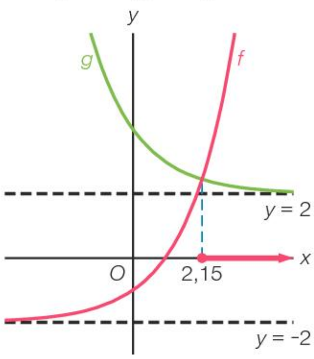
d $f(3) = 60$
$x \leq 3$ geeft $-4 < f(x) \leq 60$

e Voer in $y_2 = -1$.
De optie snijpunt geeft $x \approx -1{,}42$.



$f(x) \leq -1$ geeft $x \leq -1{,}42$

50 **a** Voer in $y_1 = 2^x - 2$ en $y_2 = (\frac{1}{2})^{x-1} + 2$.
De optie snijpunt geeft $x \approx 2{,}15$.



$f(x) \geq g(x)$ geeft $x \geq 2{,}15$

b $B_f = \langle -2, \rightarrow \rangle$, dus $f(x) = p$ heeft geen oplossingen voor $p \leq -2$.

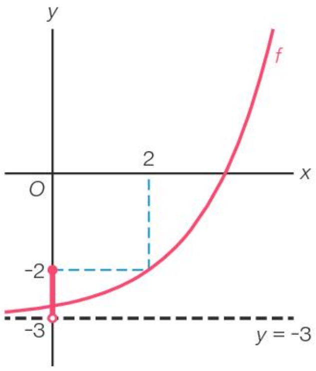
Bladzijde 33

51 **a** $y = 2^x$
$\downarrow$ translatie $(2, -3)$
$f(x) = 2^{x-2} - 3$
$B_f = \langle -3, \rightarrow \rangle$

$y = 0{,}5^x$
$\downarrow$ verm. x-as 4
$y = 4 \cdot 0{,}5^x$
$\downarrow$ translatie $(3, -1)$
$g(x) = 4 \cdot 0{,}5^{x-3} - 1$
$B_g = \langle -1, \rightarrow \rangle$

$B_f = \langle -3, \rightarrow \rangle$ en $B_g = \langle -1, \rightarrow \rangle$, dus $f(x) = p$ heeft één oplossing voor $p > -3$ en $g(x) = p$ heeft geen oplossingen voor $p \leq -1$.
Dus voor $-3 < p \leq -1$ heeft $f(x) = p$ één oplossing en $g(x) = p$ geen oplossingen.

b $f(2) = -2$



$x \leq 2$ geeft $-3 < f(x) \leq -2$

c $f(1) = -2\frac{1}{2}$ en $g(1) = 15$, dus $A(1, -2\frac{1}{2})$ en $B(1, 15)$.
$AB = y_B - y_A = 15 - -2\frac{1}{2} = 17\frac{1}{2}$

d Voer in $y_1 = 2^{x-2} - 3$, $y_2 = 4 \cdot 0{,}5^{x-3} - 1$ en $y_3 = 5$.
De optie snijpunt met y_1 en y_3 geeft $x = 5$, dus $P(5, 5)$.
De optie snijpunt met y_2 en y_3 geeft $x = 2{,}415...$, dus $Q(2{,}415...; 5)$.
$PQ = x_P - x_Q = 5 - 2{,}415... \approx 2{,}585$

52 **a** $8 \cdot 4^x = 2^3 \cdot (2^2)^x = 2^3 \cdot 2^{2x} = 2^{2x+3}$

b Bij de herleiding van $(2^2)^x$ tot 2^{2x} is de regel $(a^p)^q = a^{pq}$ gebruikt.
Bij de herleiding van $2^3 \cdot 2^{2x}$ tot 2^{2x+3} is de regel $a^p \cdot a^q = a^{p+q}$ gebruikt.

53 **a** $y = 15 \cdot 2^{3x+2} = 15 \cdot 2^{3x} \cdot 2^2 = 15 \cdot (2^3)^x \cdot 4 = 60 \cdot 8^x$
Dus $y = 60 \cdot 8^x$.

b $y = 50 \cdot 2^{3x-1} = 50 \cdot 2^{3x} \cdot 2^{-1} = 50 \cdot (2^3)^x \cdot \frac{1}{2} = 25 \cdot 8^x$
Dus $y = 25 \cdot 8^x$.

c $y = 260 \cdot 4^{1\frac{1}{2}x-1} = 260 \cdot (2^2)^{1\frac{1}{2}x-1} = 260 \cdot 2^{3x-2} = 260 \cdot (2^3)^x \cdot 2^{-2} = 260 \cdot 8^x \cdot \frac{1}{4} = 65 \cdot 8^x$
Dus $y = 65 \cdot 8^x$.

d $y = 8 \cdot 4^{-2x-1} = 8 \cdot 4^{-2x} \cdot 4^{-1} = 8 \cdot (4^{-2})^x \cdot \frac{1}{4} = 2 \cdot (\frac{1}{16})^x$
Dus $y = 2 \cdot (\frac{1}{16})^x$.

Bladzijde 34

54 **a** $y = 2^x \cdot 2^{2x-3} = 2^x \cdot 2^{2x} \cdot 2^{-3} = 2^{3x} \cdot \frac{1}{8} = \frac{1}{8} \cdot (2^3)^x = \frac{1}{8} \cdot 8^x$
Dus $y = \frac{1}{8} \cdot 8^x$.

b $y = 63 \cdot 3^{\frac{1}{2}x-2} = 63 \cdot 3^{\frac{1}{2}x} \cdot 3^{-2} = 63 \cdot (3^{\frac{1}{2}})^x \cdot \frac{1}{9} = 7 \cdot (\sqrt{3})^x$
Dus $y = 7 \cdot (\sqrt{3})^x$.

c $y = 50\,000 \cdot 100^{-x-2\frac{1}{2}} = 50\,000 \cdot 100^{-x} \cdot 100^{-2\frac{1}{2}} = 50\,000 \cdot (100^{-1})^x \cdot 10^{-5} = \frac{1}{2} \cdot (\frac{1}{100})^x$
Dus $y = \frac{1}{2} \cdot (\frac{1}{100})^x$.

d $y = \dfrac{3}{4^{2x-1}} = 3 \cdot 4^{-2x+1} = 3 \cdot 4^{-2x} \cdot 4^1 = 3 \cdot (4^{-2})^x \cdot 4 = 12 \cdot (\frac{1}{16})^x$
Dus $y = 12 \cdot (\frac{1}{16})^x$.

55 $f(x) = 5 - 2^{\frac{1}{2}x+4}$
$\downarrow$ translatie (12, −8)
$h(x) = 5 - 2^{\frac{1}{2}(x-12)+4} - 8 = -3 - 2^{\frac{1}{2}x-6+4} = -3 - 2^{\frac{1}{2}x} \cdot 2^{-2} = -3 - (2^{\frac{1}{2}})^x \cdot \frac{1}{4} = -3 - \frac{1}{4} \cdot (\sqrt{2})^x$
Dus $a = -3$, $b = -\frac{1}{4}$ en $g = \sqrt{2}$.

56 $f(x) = 2^{x-3}$
$\downarrow$ translatie (−5, 0)
$g(x) = 2^{x+5-3} = 2^{x-3} \cdot 2^5 = 32 \cdot 2^{x-3}$
De vermenigvuldiging ten opzichte van de x-as met 32 toepassen op de grafiek van f geeft de grafiek van g.
Dus $a = 32$.

57 $4\sqrt{2} = 2^2 \cdot 2^{\frac{1}{2}} = 2^{2\frac{1}{2}}$
$2^{x-1} = 4\sqrt{2}$
$2^{x-1} = 2^{2\frac{1}{2}}$
$x - 1 = 2\frac{1}{2}$
$x = 3\frac{1}{2}$

Bladzijde 35

58
a $2^{x+1} = 64$
$2^{x+1} = 2^6$
$x + 1 = 6$
$x = 5$
b $2^{x-3} = \frac{1}{8}$
$2^{x-3} = 2^{-3}$
$x - 3 = -3$
$x = 0$
c $3^{2x} = 3$
$3^{2x} = 3^1$
$2x = 1$
$x = \frac{1}{2}$
d $(\frac{1}{2})^x = 1$
$(\frac{1}{2})^x = (\frac{1}{2})^0$
$x = 0$
e $2^x = \frac{1}{4}\sqrt{2}$
$2^x = 2^{-2} \cdot 2^{\frac{1}{2}}$
$2^x = 2^{-1\frac{1}{2}}$
$x = -1\frac{1}{2}$
f $5^{x+6} = (\frac{1}{5})^x$
$5^{x+6} = (5^{-1})^x$
$5^{x+6} = 5^{-x}$
$x + 6 = -x$
$2x = -6$
$x = -3$

59
a $3^{2x+1} = 27\sqrt{3}$
$3^{2x+1} = 3^3 \cdot 3^{\frac{1}{2}}$
$3^{2x+1} = 3^{3\frac{1}{2}}$
$2x + 1 = 3\frac{1}{2}$
$2x = 2\frac{1}{2}$
$x = 1\frac{1}{4}$
b $10^{2x+1} = 0{,}01$
$10^{2x+1} = 10^{-2}$
$2x + 1 = -2$
$2x = -3$
$x = -1\frac{1}{2}$
c $3^x - 2 = 25$
$3^x = 27$
$3^x = 3^3$
$x = 3$
d $5 \cdot 2^x = 80$
$2^x = 16$
$2^x = 2^4$
$x = 4$
e $10 \cdot 3^x = 270$
$3^x = 27$
$3^x = 3^3$
$x = 3$
f $3 \cdot 8^{2-x} = 48$
$8^{2-x} = 16$
$(2^3)^{2-x} = 2^4$
$2^{6-3x} = 2^4$
$6 - 3x = 4$
$-3x = -2$
$x = \frac{2}{3}$

60 **a** $3 \cdot 2^x + 4 = 28$
$3 \cdot 2^x = 24$
$2^x = 8$
$2^x = 2^3$
$x = 3$

b $5^{2x-6} = 0{,}04$
$5^{2x-6} = \frac{1}{25}$
$5^{2x-6} = 5^{-2}$
$2x - 6 = -2$
$2x = 4$
$x = 2$

c $3 \cdot 7^{3x+1} = 147$
$7^{3x+1} = 49$
$7^{3x+1} = 7^2$
$3x + 1 = 2$
$3x = 1$
$x = \frac{1}{3}$

d $32^{x-2} = \frac{1}{16}$
$(2^5)^{x-2} = 2^{-4}$
$2^{5x-10} = 2^{-4}$
$5x - 10 = -4$
$5x = 6$
$x = 1\frac{1}{5}$

e $5 \cdot 4^{x-1} = 2\frac{1}{2}$
$4^{x-1} = \frac{1}{2}$
$(2^2)^{x-1} = 2^{-1}$
$2^{2x-2} = 2^{-1}$
$2x - 2 = -1$
$2x = 1$
$x = \frac{1}{2}$

f $8 \cdot 2^x = 4^{x+1}$
$2^3 \cdot 2^x = (2^2)^{x+1}$
$2^{3+x} = 2^{2x+2}$
$3 + x = 2x + 2$
$-x = -1$
$x = 1$

61 $f(x) = 3^{x+2} - 5$
$\downarrow$ translatie (3, 7)
$y = 3^{x-1} + 2$
$\downarrow$ verm. y-as, b
$g(x) = 3^{\frac{1}{b}x-1} + 2$

$\left.\begin{array}{l} g(x) = 3^{\frac{1}{b}x-1} + 2 \\ \text{door } A(-15, 83) \end{array}\right\} 3^{\frac{-15}{b}-1} + 2 = 83$

$3^{\frac{-15}{b}-1} = 81$
$3^{\frac{-15}{b}-1} = 3^4$
$\frac{-15}{b} - 1 = 4$
$\frac{-15}{b} = 5$
$b = -3$

62 **a** $2^{x+1} = 2^x \cdot 2^1 = 2 \cdot 2^x$, dus uit $2^{x+1} + 2^x = 48$ volgt $2 \cdot 2^x + 2^x = 48$.

b $2 \cdot 2^x + 2^x = 2 \cdot 2^x + 1 \cdot 2^x = 3 \cdot 2^x$, dus uit $2 \cdot 2^x + 2^x = 48$ volgt $3 \cdot 2^x = 48$.
$3 \cdot 2^x = 48$
$2^x = 16$
$2^x = 2^4$
$x = 4$

63 **a** $3^{x+2} + 3^x = 810$
$3^x \cdot 3^2 + 3^x = 810$
$9 \cdot 3^x + 3^x = 810$
$10 \cdot 3^x = 810$
$3^x = 81$
$3^x = 3^4$
$x = 4$

b $2^{x-1} + 2^{x+1} = 10$
$2^x \cdot 2^{-1} + 2^x \cdot 2^1 = 10$
$\frac{1}{2} \cdot 2^x + 2 \cdot 2^x = 10$
$2\frac{1}{2} \cdot 2^x = 10$
$2^x = 4$
$2^x = 2^2$
$x = 2$

c $2^{x+3} - 2^x = \frac{7}{8}$
$2^x \cdot 2^3 - 2^x = \frac{7}{8}$
$8 \cdot 2^x - 2^x = \frac{7}{8}$
$7 \cdot 2^x = \frac{7}{8}$
$2^x = \frac{1}{8}$
$2^x = 2^{-3}$
$x = -3$

d $3^{x+2} = 24 + 3^x$
$3^x \cdot 3^2 = 24 + 3^x$
$9 \cdot 3^x - 3^x = 24$
$8 \cdot 3^x = 24$
$3^x = 3$
$3^x = 3^1$
$x = 1$

e $3^x - 3^{x-1} = 2\sqrt{3}$
$3^x - 3^x \cdot 3^{-1} = 2\sqrt{3}$
$3^x - \frac{1}{3} \cdot 3^x = 2\sqrt{3}$
$\frac{2}{3} \cdot 3^x = 2\sqrt{3}$
$3^x = 3\sqrt{3}$
$3^x = 3^{1\frac{1}{2}}$
$x = 1\frac{1}{2}$

f $5^{x-1} + 5^{x-2} = 6\sqrt{5}$
$5^x \cdot 5^{-1} + 5^x \cdot 5^{-2} = 6\sqrt{5}$
$\frac{1}{5} \cdot 5^x + \frac{1}{25} \cdot 5^x = 6\sqrt{5}$
$\frac{6}{25} \cdot 5^x = 6\sqrt{5}$
$5^x = 25\sqrt{5}$
$5^x = 5^{2\frac{1}{2}}$
$x = 2\frac{1}{2}$

Bladzijde 36

64 **a** $3^{x+1} = 9^{x+2}$
$3^{x+1} = (3^2)^{x+2}$
$3^{x+1} = 3^{2x+4}$
$x + 1 = 2x + 4$
$-x = 3$
$x = -3$

b $3^{x+1} - 3^{x-1} = 8\sqrt{3}$
$3^x \cdot 3^1 - 3^x \cdot 3^{-1} = 8\sqrt{3}$
$3 \cdot 3^x - \frac{1}{3} \cdot 3^x = 8\sqrt{3}$
$2\frac{2}{3} \cdot 3^x = 8\sqrt{3}$
$3^x = 3\sqrt{3}$
$3^x = 3^{1\frac{1}{2}}$
$x = 1\frac{1}{2}$

c $3^{x^2} = (\frac{1}{3})^{x-6}$
$3^{x^2} = (3^{-1})^{x-6}$
$3^{x^2} = 3^{-x+6}$
$x^2 = -x + 6$
$x^2 + x - 6 = 0$
$(x-2)(x+3) = 0$
$x = 2 \vee x = -3$

d $5^{x^2+5} = 125^{x+1}$
$5^{x^2+5} = (5^3)^{x+1}$
$5^{x^2+5} = 5^{3x+3}$
$x^2 + 5 = 3x + 3$
$x^2 - 3x + 2 = 0$
$(x-1)(x-2) = 0$
$x = 1 \vee x = 2$

e $2^{x+2} - (\frac{1}{2})^{-x+1} = 28$
$2^x \cdot 2^2 - (2^{-1})^{-x+1} = 28$
$4 \cdot 2^x - 2^{x-1} = 28$
$4 \cdot 2^x - 2^x \cdot 2^{-1} = 28$
$4 \cdot 2^x - \frac{1}{2} \cdot 2^x = 28$
$3\frac{1}{2} \cdot 2^x = 28$
$2^x = 8$
$2^x = 2^3$
$x = 3$

f $4^{x^2+1} = 8^{x^2-1}$
$(2^2)^{x^2+1} = (2^3)^{x^2-1}$
$2^{2x^2+2} = 2^{3x^2-3}$
$2x^2 + 2 = 3x^2 - 3$
$-x^2 = -5$
$x^2 = 5$
$x = \sqrt{5} \vee x = -\sqrt{5}$

65 $f(x) = 2^x$
$\downarrow$ translatie (3, 8)
$y = 2^{x-3} + 8$
$\downarrow$verm. x-as, 4
$g(x) = 4 \cdot 2^{x-3} + 32$
$f(x) = g(x)$ geeft $2^x = 4 \cdot 2^{x-3} + 32$
$2^x = 4 \cdot 2^x \cdot 2^{-3} + 32$
$2^x = \frac{1}{2} \cdot 2^x + 32$
$\frac{1}{2} \cdot 2^x = 32$
$2^x = 64$
$2^x = 2^6$
$x = 6$

Het snijpunt van de grafieken van f en g is het punt (6, 64).

66 **a** $f(x) = g(x)$ geeft $3^{x+1} - 4 = 6 - 3^{x-1}$
$3^x \cdot 3 - 4 = 6 - 3^x \cdot 3^{-1}$
$3 \cdot 3^x - 4 = 6 - \frac{1}{3} \cdot 3^x$
$3\frac{1}{3} \cdot 3^x = 10$
$3^x = 3$
$x = 1$

$f(x) \leq g(x)$ geeft $x \leq 1$

b $f(2\frac{1}{2}) = 3^{3\frac{1}{2}} - 4 = 3^3 \cdot 3^{\frac{1}{2}} - 4 = 27\sqrt{3} - 4$, dus $A(2\frac{1}{2}, 27\sqrt{3} - 4)$
$g(2\frac{1}{2}) = 6 - 3^{1\frac{1}{2}} = 6 - 3 \cdot 3^{\frac{1}{2}} = 6 - 3\sqrt{3}$, dus $B(2\frac{1}{2}, 6 - 3\sqrt{3})$
$AB = y_A - y_B = 27\sqrt{3} - 4 - (6 - 3\sqrt{3}) = 30\sqrt{3} - 10$

c $f(x) - g(x) = 80$ geeft $3^{x+1} - 4 - (6 - 3^{x-1}) = 80$
$3^x \cdot 3 - 4 - 6 + 3^x \cdot 3^{-1} = 80$
$3 \cdot 3^x - 10 + \frac{1}{3} \cdot 3^x = 80$
$3\frac{1}{3} \cdot 3^x = 90$
$3^x = 27$
$3^x = 3^3$
$x = 3$

d $g(x) - f(x) = 6 - 3^{x-1} - (3^{x+1} - 4) = 6 - 3^x \cdot 3^{-1} - 3^x \cdot 3 + 4 = 6 - \frac{1}{3} \cdot 3^x - 3 \cdot 3^x + 4 = 10 - 3\frac{1}{3} \cdot 3^x$
Het bereik van $g(x) - f(x)$ is $\langle \leftarrow, 10 \rangle$.
Dus de vergelijking $g(x) - f(x) = p$ heeft geen oplossingen voor $p \geq 10$.

5.4 Logaritmen

Bladzijde 38

67
a $2^3 = 8$
b $2^{-2} = \frac{1}{4}$
c $2^{\frac{1}{2}} = \sqrt{2}$
d $3^2 = 9$
e $3^{-3} = \frac{1}{27}$
f $3^{\frac{1}{5}} = \sqrt[5]{3}$

68
a ${}^5\log(125) = {}^5\log(5^3) = 3$
b ${}^{10}\log(0{,}1) = {}^{10}\log(10^{-1}) = -1$
c ${}^2\log(4) = {}^2\log(2^2) = 2$
d ${}^7\log(49) = {}^7\log(7^2) = 2$
e ${}^2\log(\sqrt{2}) = {}^2\log(2^{\frac{1}{2}}) = \frac{1}{2}$
f ${}^2\log(0{,}5) = {}^2\log(2^{-1}) = -1$
g ${}^4\log(0{,}25) = {}^4\log(4^{-1}) = -1$
h ${}^4\log(4) = {}^4\log(4^1) = 1$
i ${}^4\log(1) = {}^4\log(4^0) = 0$

Bladzijde 39

69
a ${}^2\log(64\sqrt{2}) = {}^2\log(2^6 \cdot 2^{\frac{1}{2}}) = {}^2\log(2^{6\frac{1}{2}}) = 6\frac{1}{2}$
b ${}^3\log(\frac{1}{9}\sqrt{3}) = {}^3\log(3^{-2} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-1\frac{1}{2}}) = -1\frac{1}{2}$
c ${}^3\log(3^{21{,}5}) = 21{,}5$
d ${}^5\log(\frac{1}{125}) = {}^5\log(5^{-3}) = -3$
e ${}^{\frac{1}{3}}\log(\frac{1}{27}) = {}^{\frac{1}{3}}\log((\frac{1}{3})^3) = 3$
f ${}^{\frac{1}{2}}\log(\frac{1}{4}) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) = 2$
g ${}^2\log(\frac{1}{32} \cdot \sqrt[3]{2}) = {}^2\log(2^{-5} \cdot 2^{\frac{1}{3}}) = {}^2\log(2^{-4\frac{2}{3}}) = -4\frac{2}{3}$
h ${}^5\log(1) = {}^5\log(5^0) = 0$
i ${}^3\log(81 \cdot \sqrt[5]{27}) = {}^3\log(3^4 \cdot \sqrt[5]{3^3}) = {}^3\log(3^4 \cdot 3^{\frac{3}{5}}) = {}^3\log(3^{4\frac{3}{5}}) = 4\frac{3}{5}$

70
a ${}^3\log(9) = {}^3\log(3^2) = 2$
Dus ${}^3\log(x) = 2$ geeft $x = 9$.
b ${}^5\log(\frac{1}{25}) = {}^5\log(5^{-2}) = -2$
Dus ${}^5\log(x) = -2$ geeft $x = \frac{1}{25}$.

71
a $x = {}^5\log(0{,}2) = {}^5\log(\frac{1}{5}) = {}^5\log(5^{-1}) = -1$
b ${}^9\log(x) = \frac{1}{2}$
$x = 9^{\frac{1}{2}} = \sqrt{9} = 3$
c ${}^x\log(1000) = 3$
$x^3 = 1000$
$x = 10$

72 **a** $^{3}\log(x + 2) = 2$
$x + 2 = 3^2$
$x + 2 = 9$
$x = 7$

b $1 + {}^{\frac{1}{2}}\log(x) = 4$
$^{\frac{1}{2}}\log(x) = 3$
$x = (\frac{1}{2})^3$
$x = \frac{1}{8}$

c $^{3}\log(2x + 1) = 4$
$2x + 1 = 3^4$
$2x + 1 = 81$
$2x = 80$
$x = 40$

d $5 + {}^{4}\log(x) = 3$
$^{4}\log(x) = -2$
$x = 4^{-2}$
$x = \frac{1}{16}$

e $^{\frac{1}{2}}\log(x - 1) = 3$
$x - 1 = (\frac{1}{2})^3$
$x - 1 = \frac{1}{8}$
$x = 1\frac{1}{8}$

f $^{2}\log(x^2 - 4) = 5$
$x^2 - 4 = 2^5$
$x^2 - 4 = 32$
$x^2 = 36$
$x = 6 \vee x = -6$

73 **a** $4 \cdot {}^{3}\log(x) = 2$
$^{3}\log(x) = \frac{1}{2}$
$x = 3^{\frac{1}{2}}$
$x = \sqrt{3}$

b $^{3}\log(4x - 1) = -2$
$4x - 1 = 3^{-2}$
$4x - 1 = \frac{1}{9}$
$4x = 1\frac{1}{9}$
$x = \frac{5}{18}$

c $3 + {}^{2}\log(x) = -1$
$^{2}\log(x) = -4$
$x = 2^{-4}$
$x = \frac{1}{16}$

d $^{5}\log(3x + 2) = 1$
$3x + 2 = 5^1$
$3x + 2 = 5$
$3x = 3$
$x = 1$

e $^{3}\log(0{,}4x - 5) = 2$
$0{,}4x - 5 = 3^2$
$0{,}4x - 5 = 9$
$0{,}4x = 14$
$x = 35$

f $4 + 2 \cdot {}^{2}\log(x) = 7$
$2 \cdot {}^{2}\log(x) = 3$
$^{2}\log(x) = 1\frac{1}{2}$
$x = 2^{1\frac{1}{2}}$
$x = 2^1 \cdot 2^{\frac{1}{2}}$
$x = 2\sqrt{2}$

74 Voer in $y_1 = 2^x$ en $y_2 = 30$.
De optie snijpunt geeft $x \approx 4{,}91$.

Bladzijde 40

75 **a** $2^{x-1} = 15$
$x - 1 = {}^{2}\log(15)$
$x = 1 + {}^{2}\log(15)$

b $1 + 2^x = 15$
$2^x = 14$
$x = {}^{2}\log(14)$

c $4 + 3^{x+1} = 25$
$3^{x+1} = 21$
$x + 1 = {}^{3}\log(21)$
$x = -1 + {}^{3}\log(21)$

d $14 - 2^{x+3} = 2$
$-2^{x+3} = -12$
$2^{x+3} = 12$
$x + 3 = {}^{2}\log(12)$
$x = -3 + {}^{2}\log(12)$

e $7 + 4^{2x} = 12$
$4^{2x} = 5$
$2x = {}^{4}\log(5)$
$x = \frac{1}{2} \cdot {}^{4}\log(5)$

f $3 \cdot 5^{2x+1} = 60$
$5^{2x+1} = 20$
$2x + 1 = {}^{5}\log(20)$
$2x = -1 + {}^{5}\log(20)$
$x = -\frac{1}{2} + \frac{1}{2} \cdot {}^{5}\log(20)$

g $3^{x+2} + 3^x = 600$
$3^x \cdot 3^2 + 3^x = 600$
$9 \cdot 3^x + 3^x = 600$
$10 \cdot 3^x = 600$
$3^x = 60$
$x = {}^{3}\log(60)$

h $2^{1+2x} = 4^x + 6$
$2^1 \cdot 2^{2x} = 4^x + 6$
$2 \cdot (2^2)^x = 4^x + 6$
$2 \cdot 4^x = 4^x + 6$
$4^x = 6$
$x = {}^{4}\log(6)$

76 **a** $4^x = 2^{x+2} - 3$
$(2^2)^x = 2^x \cdot 2^2 - 3$
$(2^x)^2 = 4 \cdot 2^x - 3$
b 2^x bij de vergelijking $(2^x)^2 = 4 \cdot 2^x - 3$ vervangen door u geeft $u^2 = 4u - 3$.
c $u^2 = 4u - 3$
$u^2 - 4u + 3 = 0$
$(u-1)(u-3) = 0$
$u = 1 \vee u = 3$
d $u = 1$ geeft $2^x = 1$ oftewel $x = 0$.
$u = 3$ geeft $2^x = 3$ oftewel $x = {}^2\log(3)$.

77 **a** $9^x - 3^{x+1} = 4$
$(3^2)^x - 3^x \cdot 3^1 = 4$
$(3^x)^2 - 3 \cdot 3^x - 4 = 0$
Stel $3^x = u$.
$u^2 - 3u - 4 = 0$
$(u+1)(u-4) = 0$
$u = -1 \vee u = 4$
$3^x = -1 \vee 3^x = 4$
$x = {}^3\log(4)$
b $4^x = 2^x + 42$
$(2^2)^x = 2^x + 42$
$(2^x)^2 - 2^x - 42 = 0$
Stel $2^x = u$.
$u^2 - u - 42 = 0$
$(u+6)(u-7) = 0$
$u = -6 \vee u = 7$
$2^x = -6 \vee 2^x = 7$
$x = {}^2\log(7)$
c $2^x = 24 - 2^{2x-1}$
$2^x = 24 - 2^{2x} \cdot 2^{-1}$
$2^x = 24 - \frac{1}{2} \cdot (2^x)^2$
$\frac{1}{2} \cdot (2^x)^2 + 2^x - 24 = 0$
$(2^x)^2 + 2 \cdot 2^x - 48 = 0$
Stel $2^x = u$.
$u^2 + 2u - 48 = 0$
$(u-6)(u+8) = 0$
$u = 6 \vee u = -8$
$2^x = 6 \vee 2^x = -8$
$x = {}^2\log(6)$
d $9^x = 5 \cdot 3^x + 6$
$(3^2)^x = 5 \cdot 3^x + 6$
$(3^x)^2 - 5 \cdot 3^x - 6 = 0$
Stel $3^x = u$.
$u^2 - 5u - 6 = 0$
$(u+1)(u-6) = 0$
$u = -1 \vee u = 6$
$3^x = -1 \vee 3^x = 6$
$x = {}^3\log(6)$

78 **a** $5^{x-1} + 5^{2x-1} = 4$
$5^x \cdot 5^{-1} + 5^{2x} \cdot 5^{-1} = 4$
$\frac{1}{5} \cdot 5^x + \frac{1}{5} \cdot (5^x)^2 = 4$
$5^x + (5^x)^2 = 20$
$(5^x)^2 + 5^x - 20 = 0$
Stel $5^x = u$.
$u^2 + u - 20 = 0$
$(u-4)(u+5) = 0$
$u = 4 \vee u = -5$
$5^x = 4 \vee 5^x = -5$
$x = {}^5\log(4)$
b $3^x - 2 = 8 \cdot (\frac{1}{3})^x$
$3^x - 2 = 8 \cdot \frac{1}{3^x}$
$(3^x)^2 - 2 \cdot 3^x = 8$
$(3^x)^2 - 2 \cdot 3^x - 8 = 0$
Stel $3^x = u$.
$u^2 - 2u - 8 = 0$
$(u+2)(u-4) = 0$
$u = -2 \vee u = 4$
$3^x = -2 \vee 3^x = 4$
$x = {}^3\log(4)$
c $2^x = 6 - 5 \cdot (\frac{1}{2})^x$
$2^x = 6 - 5 \cdot \frac{1}{2^x}$
$(2^x)^2 = 6 \cdot 2^x - 5$
$(2^x)^2 - 6 \cdot 2^x + 5 = 0$
Stel $2^x = u$.
$u^2 - 6u + 5 = 0$
$(u-1)(u-5) = 0$
$u = 1 \vee u = 5$
$2^x = 1 \vee 2^x = 5$
$x = 0 \vee x = {}^2\log(5)$
d $3^x + 2 \cdot (\frac{1}{3})^{x-2} = 9$
$3^x + 2 \cdot (\frac{1}{3})^x \cdot (\frac{1}{3})^{-2} = 9$
$3^x + 18 \cdot \frac{1}{3^x} = 9$
$(3^x)^2 + 18 = 9 \cdot 3^x$
$(3^x)^2 - 9 \cdot 3^x + 18 = 0$
Stel $3^x = u$.
$u^2 - 9u + 18 = 0$
$(u-3)(u-6) = 0$
$u = 3 \vee u = 6$
$3^x = 3 \vee 3^x = 6$
$x = 1 \vee x = {}^3\log(6)$

Bladzijde 41

79 $3 \cdot 27^x + 2 \cdot (\frac{1}{3})^x = 7 \cdot 3^x$

$3 \cdot (3^3)^x + 2 \cdot \dfrac{1}{3^x} = 7 \cdot 3^x$

$3 \cdot (3^x)^3 + 2 \cdot \dfrac{1}{3^x} = 7 \cdot 3^x$

$3 \cdot (3^x)^4 + 2 = 7 \cdot (3^x)^2$

$3 \cdot (3^x)^4 - 7 \cdot (3^x)^2 + 2 = 0$

Stel $3^x = u$.

$3u^4 - 7u^2 + 2 = 0$

$D = (-7)^2 - 4 \cdot 3 \cdot 2 = 25$

$u^2 = \dfrac{7+5}{6} \vee u^2 = \dfrac{7-5}{6}$

$u^2 = 2 \vee u^2 = \frac{1}{3}$

$u = \sqrt{2} \vee u = -\sqrt{2} \vee u = \sqrt{\frac{1}{3}} \vee u = -\sqrt{\frac{1}{3}}$

$3^x = \sqrt{2} \vee 3^x = -\sqrt{2} \vee 3^x = \sqrt{\frac{1}{3}} \vee 3^x = -\sqrt{\frac{1}{3}}$

$x = {}^3\log(\sqrt{2}) \vee 3^x = 3^{-\frac{1}{2}}$

$x = {}^3\log(\sqrt{2}) \vee x = -\frac{1}{2}$

80 Voor f geldt $y = 2^x$, dus voor f^{inv} geldt $x = 2^y$.

$x = 2^y$ geeft $2^y = x$

$y = {}^2\log(x)$

Dus $g(x) = {}^2\log(x)$ is de inverse van f.

Bladzijde 42

81 **a** $ax + b = 0$ geeft $x = -\dfrac{b}{a}$.

De verticale asymptoot van de grafiek is de lijn $x = -\dfrac{b}{a}$.

b ${}^3\log(2x + 5) = 2$

$2x + 5 = 3^2$

$2x = 4$

$x = 2$

${}^3\log(2x + 5) \leq 2$ geeft $-2\frac{1}{2} < x \leq 2$

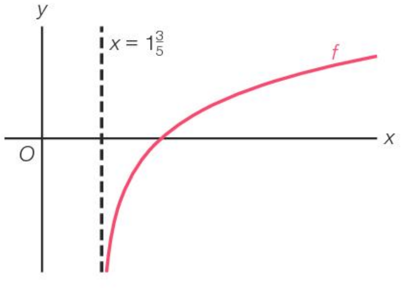
82 **a** $5x - 8 > 0$

$5x > 8$

$x > 1\frac{3}{5}$

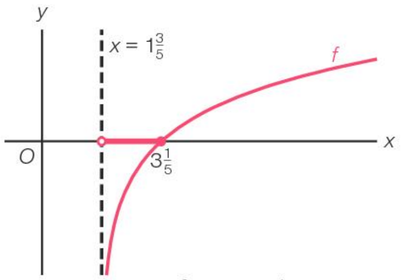
$D_f = \langle 1\frac{3}{5}, \rightarrow \rangle$ en de verticale asymptoot van de grafiek van f is de lijn $x = 1\frac{3}{5}$.

b



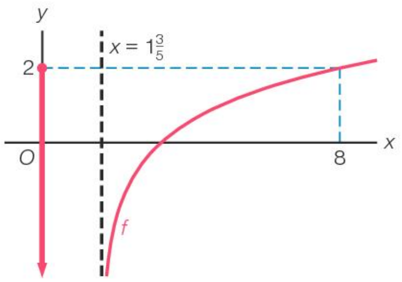
c $f(x) = 0$ geeft $-3 + {}^2\log(5x - 8) = 0$

$${}^2\log(5x - 8) = 3$$
$$5x - 8 = 2^3$$
$$5x = 16$$
$$x = 3\tfrac{1}{5}$$



$f(x) \leq 0$ geeft $1\tfrac{3}{5} < x \leq 3\tfrac{1}{5}$

d $f(8) = -3 + {}^2\log(32) = -3 + 5 = 2$



Voor $x \leq 8$ is $f(x) \leq 2$.

83 **a** $f(x) = 5$ geeft ${}^{\frac{1}{2}}\log(x + 3) = 5$

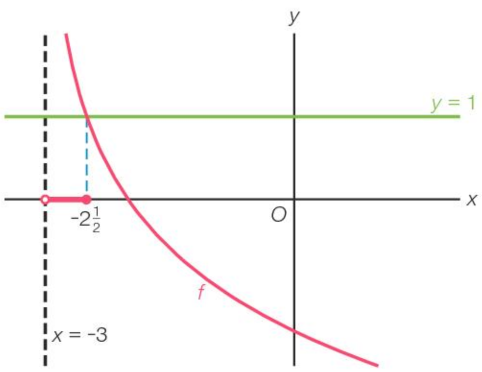
$$x + 3 = (\tfrac{1}{2})^5$$
$$x = -2\tfrac{31}{32}$$

b $f(-1) = {}^{\frac{1}{2}}\log(2) = -1$ en $g(-1) = {}^3\log(6)$

$AB = {}^3\log(6) - -1 = {}^3\log(6) + 1$

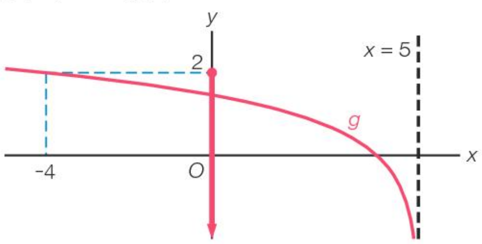
c $f(x) = 1$ geeft ${}^{\frac{1}{2}}\log(x + 3) = 1$

$$x + 3 = (\tfrac{1}{2})^1$$
$$x = -2\tfrac{1}{2}$$



$f(x) \geq 1$ geeft $-3 < x \leq -2\tfrac{1}{2}$

d $g(-4) = {}^3\log(9) = 2$



Voor $x \geq -4$ is $g(x) \leq 2$.

84 $A(4, 1)$ op de grafiek van f geeft ${}^2\log(4 + c) + d = 1$.
$B(7, 3)$ op de grafiek van f geeft ${}^2\log(7 + c) + d = 3$.
${}^2\log(4 + c) + d = 1$ geeft ${}^2\log(4 + c) = 1 - d$
$4 + c = 2^{1-d}$
$c = 2^{1-d} - 4$
${}^2\log(7 + c) + d = 3$ geeft ${}^2\log(7 + c) = 3 - d$
$7 + c = 2^{3-d}$
$c = 2^{3-d} - 7$
$c = 2^{1-d} - 4$ en $c = 2^{3-d} - 7$ geeft $2^{1-d} - 4 = 2^{3-d} - 7$
$2 \cdot 2^{-d} - 4 = 2^3 \cdot 2^{-d} - 7$
$2 \cdot 2^{-d} - 4 = 8 \cdot 2^{-d} - 7$
$-6 \cdot 2^{-d} = -3$
$2^{-d} = \frac{1}{2}$
$d = 1$
$d = 1$ geeft $c = 2^{1-1} - 4 = 2^0 - 4 = 1 - 4 = -3$
Dus $c = -3$ en $d = 1$.

Diagnostische toets

Bladzijde 46

1 **a** $(3a^{-5}b^4)^{-2} = 3^{-2} \cdot a^{10} \cdot b^{-8} = \frac{1}{9} \cdot a^{10} \cdot \frac{1}{b^8} = \frac{a^{10}}{9b^8}$

b $(\frac{2}{3}a^{-2}b)^{-2} = (\frac{2}{3})^{-2} \cdot a^4 \cdot b^{-2} = \frac{1}{(\frac{2}{3})^2} \cdot a^4 \cdot \frac{1}{b^2} = \frac{1}{\frac{4}{9}} \cdot a^4 \cdot \frac{1}{b^2} = \frac{9}{4} \cdot a^4 \cdot \frac{1}{b^2} = \frac{9a^4}{4b^2}$

c $3a^{1\frac{1}{3}}b^{-3} = 3a \cdot \sqrt[3]{a} \cdot \frac{1}{b^3} = \frac{3a \cdot \sqrt[3]{a}}{b^3}$

d $(a^{-\frac{1}{4}})^3 = a^{-\frac{3}{4}} = \frac{1}{a^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{a^3}}$

e $a^{-2}b^{\frac{1}{5}} = \frac{1}{a^2} \cdot \sqrt[5]{b} = \frac{\sqrt[5]{b}}{a^2}$

f $7a^{-\frac{1}{3}}b^{\frac{3}{5}} = 7 \cdot \frac{1}{a^{\frac{1}{3}}} \cdot \sqrt[5]{b^3} = \frac{7 \cdot \sqrt[5]{b^3}}{\sqrt[3]{a}}$

2 **a** $\frac{\sqrt{x}}{x^2} = \frac{x^{\frac{1}{2}}}{x^2} = x^{\frac{1}{2} - 2} = x^{-1\frac{1}{2}}$

b $x^2 \cdot \sqrt[3]{x} = x^2 \cdot x^{\frac{1}{3}} = x^{2\frac{1}{3}}$

c $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{\frac{2}{3}}} = x^{-\frac{2}{3}}$

3 **a** $y = 3x^{-3}(\frac{1}{2}x^2)^3 = 3 \cdot x^{-3} \cdot \frac{1}{8}x^6 = \frac{3}{8}x^3$
Dus $y = \frac{3}{8}x^3$.

b $y = \frac{1}{5x^2 \cdot \sqrt{x}} = \frac{1}{5} \cdot \frac{1}{x^{2\frac{1}{2}}} = \frac{1}{5}x^{-2\frac{1}{2}}$

Dus $y = \frac{1}{5}x^{-2\frac{1}{2}}$.

c $y = \frac{12}{5x^{-3}} \cdot \sqrt[5]{x^3} = \frac{12}{5} \cdot \frac{1}{x^{-3}} \cdot x^{\frac{3}{5}} = 2\frac{2}{5} \cdot x^3 \cdot x^{\frac{3}{5}} = 2\frac{2}{5}x^{3\frac{3}{5}}$
Dus $y = 2\frac{2}{5}x^{3\frac{3}{5}}$.

4 **a** $x^2 \cdot \sqrt[3]{x} = 128$
$x^2 \cdot x^{\frac{1}{3}} = 128$
$x^{2\frac{1}{3}} = 128$
$x = (2^7)^{\frac{3}{7}}$
$x = 2^3$
$x = 8$

b $(2x + 3)^{-\frac{2}{3}} = \frac{4}{9}$
$2x + 3 = (\frac{4}{9})^{-\frac{3}{2}}$
$2x + 3 = ((\frac{2}{3})^2)^{-\frac{3}{2}}$
$2x + 3 = (\frac{2}{3})^{-3}$
$2x + 3 = \frac{1}{(\frac{2}{3})^3}$
$2x + 3 = \frac{1}{\frac{8}{27}}$
$2x + 3 = \frac{27}{8}$
$2x = \frac{3}{8}$
$x = \frac{3}{16}$

c $112 - 2x^{-4} = 5x^{-4}$
$-7x^{-4} = -112$
$x^{-4} = 16$
$x = 16^{-\frac{1}{4}}$
$x = (2^4)^{-\frac{1}{4}}$
$x = \frac{1}{2}$

5 **a** $y = 0{,}02x^{-1\frac{3}{5}}$
$0{,}02x^{-1\frac{3}{5}} = y$
$x^{-\frac{8}{5}} = 50y$
$x = (50y)^{-\frac{5}{8}}$
$x = 50^{-\frac{5}{8}} \cdot y^{-\frac{5}{8}}$
$x \approx 0{,}09 \cdot y^{-0{,}63}$
Dus $x = 0{,}09y^{-0{,}63}$.

b $y = \frac{1}{4}x^2 \cdot \sqrt[3]{x} = \frac{1}{4}x^2 \cdot x^{\frac{1}{3}} = \frac{1}{4}x^{2\frac{1}{3}}$
$\frac{1}{4}x^{2\frac{1}{3}} = y$
$x^{\frac{7}{3}} = 4y$
$x = (4y)^{\frac{3}{7}}$
$x = 4^{\frac{3}{7}} \cdot y^{\frac{3}{7}}$
$x \approx 1{,}81 \cdot y^{0{,}43}$
Dus $x = 1{,}81y^{0{,}43}$.

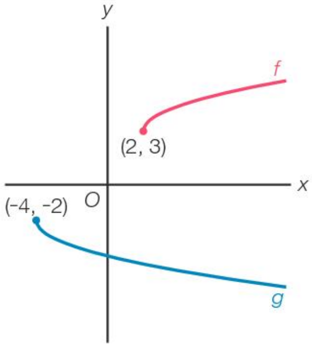
c $y = \frac{20}{x^2 \cdot \sqrt{x}} = \frac{20}{x^{2\frac{1}{2}}} = 20x^{-2\frac{1}{2}}$
$20x^{-2\frac{1}{2}} = y$
$x^{-\frac{5}{2}} = 0{,}05y$
$x = (0{,}05y)^{-\frac{2}{5}}$
$x = 0{,}05^{-\frac{2}{5}} \cdot y^{-\frac{2}{5}}$
$x \approx 3{,}31 \cdot y^{-0{,}4}$
Dus $x = 3{,}31y^{-0{,}4}$.

6 $f(x) = \frac{1}{3}(x + 2)^4 - 6$
$\downarrow$ verm. x-as, -4
$y = -1\frac{1}{3}(x + 2)^4 + 24$
$\downarrow$ translatie $(3, -15)$
$g(x) = -1\frac{1}{3}(x - 3 + 2)^4 + 24 - 15$
oftewel $g(x) = -1\frac{1}{3}(x - 1)^4 + 9$
max. is $g(1) = 9$

7 **a** $y = \sqrt{x}$
$\downarrow$ translatie $(2, 3)$
$f(x) = 3 + \sqrt{x - 2}$

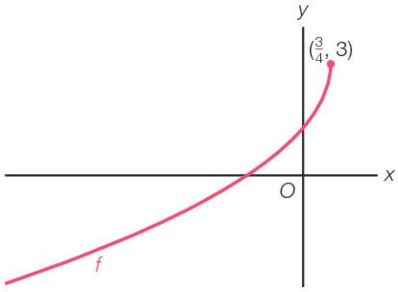
$y = \sqrt{x}$
$\downarrow$ verm. x-as, -1
$y = -\sqrt{x}$
$\downarrow$ translatie $(-4, -2)$
$g(x) = -2 - \sqrt{x + 4}$

b



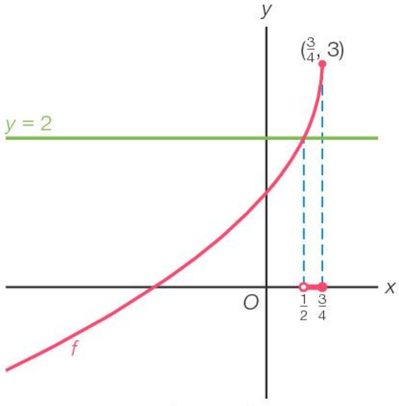
c $D_f = [2, \rightarrow\rangle$, $B_f = [3, \rightarrow\rangle$, $D_g = [-4, \rightarrow\rangle$ en $B_g = \langle\leftarrow, -2]$

8 **a** $3 - 4x \geq 0$
$-4x \geq -3$
$x \leq \frac{3}{4}$
$D_f = \langle\leftarrow, \frac{3}{4}]$ en het randpunt is $(\frac{3}{4}, 3)$.



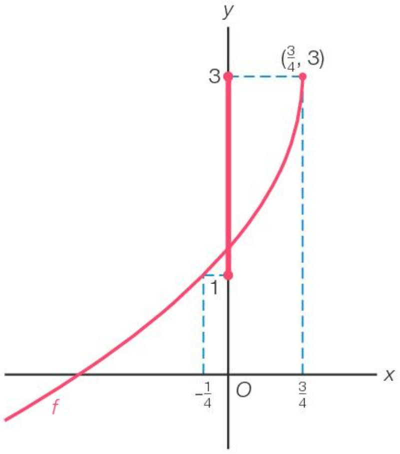
$B_f = \langle\leftarrow, 3]$

b $f(x) = 2$ geeft $3 - \sqrt{3 - 4x} = 2$
$\sqrt{3 - 4x} = 1$
$3 - 4x = 1$
$-4x = -2$
$x = \frac{1}{2}$



$f(x) > 2$ geeft $\frac{1}{2} < x \leq \frac{3}{4}$

c $f(-\frac{1}{4}) = 1$



Voor $x \geq -\frac{1}{4}$ is $1 \leq f(x) \leq 3$.

9 **a** $N = 2\sqrt{-5t + 1}$
$2\sqrt{-5t + 1} = N$
kwadrateren geeft
$4(-5t + 1) = N^2$
$-20t + 4 = N^2$
$-20t = N^2 - 4$
$t = -\frac{1}{20}N^2 + \frac{1}{5}$

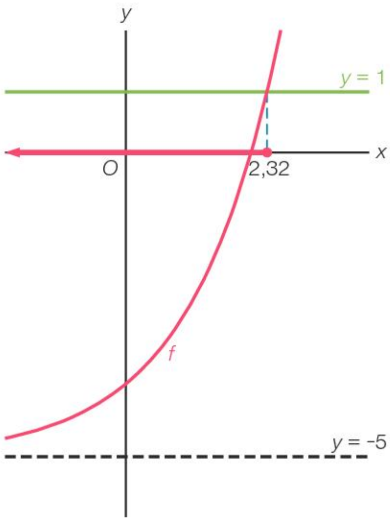
b $2x\sqrt{y} - 6\sqrt{x} = 1$
$2x\sqrt{y} = 6\sqrt{x} + 1$
kwadrateren geeft
$4x^2y = (6\sqrt{x} + 1)^2$
$4x^2y = 36x + 12\sqrt{x} + 1$
$y = \dfrac{36x + 12\sqrt{x} + 1}{4x^2}$

Bladzijde 47

10 **a** $y = 2^x$
↓ verm. x-as, 0,3
$y = 0{,}3 \cdot 2^x$
↓ translatie (−2, −5)
$f(x) = 0{,}3 \cdot 2^{x+2} - 5$

b $B_f = \langle -5, \rightarrow \rangle$
De horizontale asymptoot is de lijn $y = -5$.

c Voer in $y_1 = 0{,}3 \cdot 2^{x+2} - 5$ en $y_2 = 1$.
De optie snijpunt geeft $x \approx 2{,}32$.



$f(x) \leq 1$ geeft $x \leq 2{,}32$

11 **a** $y = 2^x \cdot 2^{-4x+3} = 2^x \cdot 2^{-4x} \cdot 2^3 = 2^{-3x} \cdot 8 = 8 \cdot (2^{-3})^x = 8 \cdot (\frac{1}{8})^x$
Dus $y = 8 \cdot (\frac{1}{8})^x$.

b $y = 108 \cdot 3^{4x-3} = 108 \cdot 3^{4x} \cdot 3^{-3} = 108 \cdot (3^4)^x \cdot \frac{1}{27} = 4 \cdot 81^x$
Dus $y = 4 \cdot 81^x$.

c $y = \dfrac{250}{5^{-2x+3}} = \dfrac{250}{5^{-2x} \cdot 5^3} = \dfrac{250}{5^{-2x} \cdot 125} = 2 \cdot 5^{2x} = 2 \cdot (5^2)^x = 2 \cdot 25^x$
Dus $y = 2 \cdot 25^x$.

12 **a** $5^{x-1} = 125\sqrt{5}$
$5^{x-1} = 5^3 \cdot 5^{\frac{1}{2}}$
$5^{x-1} = 5^{3\frac{1}{2}}$
$x - 1 = 3\frac{1}{2}$
$x = 4\frac{1}{2}$

b $3^{2x-5} = (\frac{1}{27})^x$
$3^{2x-5} = (3^{-3})^x$
$3^{2x-5} = 3^{-3x}$
$2x - 5 = -3x$
$5x = 5$
$x = 1$

c $2 \cdot 4^{2x-1} - 3 = 61$
$2 \cdot 4^{2x-1} = 64$
$4^{2x-1} = 32$
$(2^2)^{2x-1} = 2^5$
$2^{4x-2} = 2^5$
$4x - 2 = 5$
$4x = 7$
$x = 1\frac{3}{4}$

d $9^{x-1} = 27^{x+1}$
$(3^2)^{x-1} = (3^3)^{x+1}$
$3^{2x-2} = 3^{3x+3}$
$2x - 2 = 3x + 3$
$-x = 5$
$x = -5$

e $2^{x+2} + 2^{x-1} = 36$
$2^x \cdot 2^2 + 2^x \cdot 2^{-1} = 36$
$4 \cdot 2^x + \frac{1}{2} \cdot 2^x = 36$
$4\frac{1}{2} \cdot 2^x = 36$
$2^x = 8$
$2^x = 2^3$
$x = 3$

f $2^{x^2} = (\frac{1}{8})^x$
$2^{x^2} = (2^{-3})^x$
$2^{x^2} = 2^{-3x}$
$x^2 = -3x$
$x^2 + 3x = 0$
$x(x + 3) = 0$
$x = 0 \vee x = -3$

13 **a** $f(x) = g(x)$ geeft $2^{x+2} - 3 = 6 - 2^{x-1}$
$2^x \cdot 2^2 - 3 = 6 - 2^x \cdot 2^{-1}$
$4 \cdot 2^x - 3 = 6 - \frac{1}{2} \cdot 2^x$
$4\frac{1}{2} \cdot 2^x = 9$
$2^x = 2$
$x = 1$
$f(x) \geq g(x)$ geeft $x \geq 1$

b $g(4) = 6 - 2^3 = 6 - 8 = -2$
Voor $x \leq 4$ is $-2 \leq g(x) < 6$.

c $f(x) + g(x) = 2^{x+2} - 3 + 6 - 2^{x-1} = 2^x \cdot 2^2 - 3 + 6 - 2^x \cdot 2^{-1} = 4 \cdot 2^x + 3 - \frac{1}{2} \cdot 2^x = 3\frac{1}{2} \cdot 2^x + 3$
Het bereik van $f(x) + g(x)$ is $\langle 3, \rightarrow \rangle$.
Dus de vergelijking $f(x) + g(x) = p$ heeft geen oplossing voor $p \leq 3$.

14 **a** ${}^{3}\log(3\sqrt{3}) = {}^{3}\log(3^1 \cdot 3^{\frac{1}{2}}) = {}^{3}\log(3^{1\frac{1}{2}}) = 1\frac{1}{2}$

b ${}^{2}\log(\frac{1}{4}\sqrt{8}) = {}^{2}\log(2^{-2} \cdot \sqrt{2^3}) = {}^{2}\log(2^{-2} \cdot 2^{1\frac{1}{2}}) = {}^{2}\log(2^{-\frac{1}{2}}) = -\frac{1}{2}$

c ${}^{2}\log(\frac{1}{16} \cdot \sqrt[3]{2}) = {}^{2}\log(2^{-4} \cdot 2^{\frac{1}{3}}) = {}^{2}\log(2^{-3\frac{2}{3}}) = -3\frac{2}{3}$

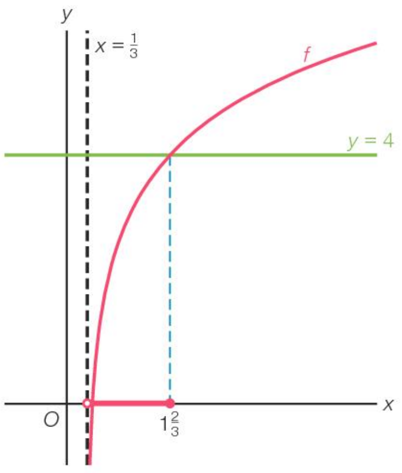
15 **a** ${}^{4}\log(2x - 3) = 2$
$2x - 3 = 4^2$
$2x - 3 = 16$
$2x = 19$
$x = 9\frac{1}{2}$

b ${}^{\frac{1}{2}}\log(x - 3) = -4$
$x - 3 = (\frac{1}{2})^{-4}$
$x - 3 = 16$
$x = 19$

c $5 + 3 \cdot {}^2\log(x) = 20$
$3 \cdot {}^2\log(x) = 15$
${}^2\log(x) = 5$
$x = 2^5$
$x = 32$

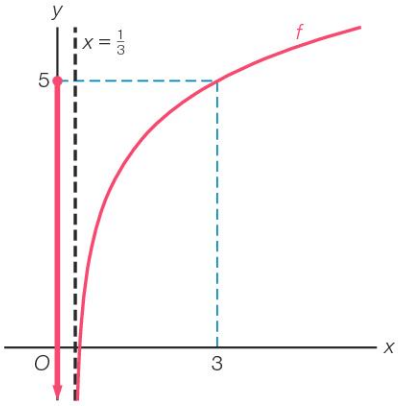
16 **a** $7^{x-3} = 20$
$x - 3 = {}^7\log(20)$
$x = 3 + {}^7\log(20)$
b $4^x - 2^{x+4} = 80$
$(2^2)^x - 2^x \cdot 2^4 = 80$
$(2^x)^2 - 16 \cdot 2^x - 80 = 0$
Stel $2^x = u$.
$u^2 - 16u - 80 = 0$
$(u + 4)(u - 20) = 0$
$u = -4 \vee u = 20$
$2^x = -4 \vee 2^x = 20$
$x = {}^2\log(20)$
c $5^{x+1} - 6 \cdot (\frac{1}{5})^{x-1} = 25$
$5^x \cdot 5 - 6 \cdot (\frac{1}{5})^x \cdot (\frac{1}{5})^{-1} = 25$
$5 \cdot 5^x - 30 \cdot \frac{1}{5^x} = 25$
$5 \cdot (5^x)^2 - 30 = 25 \cdot 5^x$
$(5^x)^2 - 5 \cdot 5^x - 6 = 0$
Stel $5^x = u$.
$u^2 - 5u - 6 = 0$
$(u + 1)(u - 6) = 0$
$u = -1 \vee u = 6$
$5^x = -1 \vee 5^x = 6$
$x = {}^5\log(6)$

17 **a** $3x - 1 > 0$ geeft $x > \frac{1}{3}$
$D_f = \langle \frac{1}{3}, \rightarrow \rangle$ en de verticale asymptoot van de grafiek is de lijn $x = \frac{1}{3}$.
$f(x) = 4$ geeft ${}^2\log(3x - 1) + 2 = 4$
${}^2\log(3x - 1) = 2$
$3x - 1 = 2^2$
$3x - 1 = 4$
$3x = 5$
$x = 1\frac{2}{3}$



$f(x) \leq 4$ geeft $\frac{1}{3} < x \leq 1\frac{2}{3}$

b $f(3) = {}^2\log(8) + 2 = 5$



Voor $x \leq 3$ is $f(x) \leq 5$.

6 Differentiaalrekening

Voorkennis Afgeleide en raaklijn

Bladzijde 52

1 **a** $f(x) = \frac{1}{2}x^4 - 3x^2 + 6x - 1$ geeft $f'(x) = 2x^3 - 6x + 6$

b $g(x) = (5x^2 - 2)^2 = 25x^4 - 20x^2 + 4$ geeft $g'(x) = 100x^3 - 40x$

c $h(x) = \dfrac{x-3}{x^2+4}$ geeft $h'(x) = \dfrac{(x^2+4) \cdot 1 - (x-3) \cdot 2x}{(x^2+4)^2} = \dfrac{x^2 + 4 - 2x^2 + 6x}{(x^2+4)^2} = \dfrac{-x^2 + 6x + 4}{(x^2+4)^2}$

d $k(p) = -\frac{1}{6}p^3 + \frac{1}{5}p^2 - 8$ geeft $k'(p) = -\frac{1}{2}p^2 + \frac{2}{5}p$

e $l(q) = 10 - 5q^2 + 9q^3 - a^4$ geeft $l'(q) = -10q + 27q^2$

f $m(x) = 2x^2 - \dfrac{6}{x+1}$ geeft $m'(x) = 4x - \dfrac{(x+1) \cdot 0 - 6 \cdot 1}{(x+1)^2} = 4x + \dfrac{6}{(x+1)^2}$

Bladzijde 53

2 **a** $f(x) = x^3 - x^2 - 4x + 2$ geeft $f'(x) = 3x^2 - 2x - 4$

Stel k: $y = ax + b$ met $a = f'(0) = -4$.

$\left.\begin{array}{l} y = -4x + b \\ f(0) = 2, \text{ dus } A(0, 2) \end{array}\right\} b = 2$

Dus k: $y = -4x + 2$.

b $f'(x) = 1$ geeft $3x^2 - 2x - 4 = 1$

$3x^2 - 2x - 5 = 0$

$D = (-2)^2 - 4 \cdot 3 \cdot -5 = 64$

$x = \dfrac{2+8}{6} = 1\frac{2}{3} \vee x = \dfrac{2-8}{6} = -1$

$f(-1) = 4$ en $f(1\frac{2}{3}) = -2\frac{22}{27}$, dus $B(-1, 4)$ en $C(1\frac{2}{3}, -2\frac{22}{27})$.

3 **a** $f(x) = \dfrac{2x-1}{x+1}$ geeft $f'(x) = \dfrac{(x+1) \cdot 2 - (2x-1) \cdot 1}{(x+1)^2} = \dfrac{2x + 2 - 2x + 1}{(x+1)^2} = \dfrac{3}{(x+1)^2}$

Stel k: $y = ax + b$ met $a = f'(2) = \dfrac{3}{(2+1)^2} = \frac{1}{3}$.

$\left.\begin{array}{l} y = \frac{1}{3}x + b \\ f(2) = 1, \text{ dus } A(2, 1) \end{array}\right\} \frac{1}{3} \cdot 2 + b = 1$

$\frac{2}{3} + b = 1$

$b = \frac{1}{3}$

Dus k: $y = \frac{1}{3}x + \frac{1}{3}$.

b $f'(x) = \frac{3}{4}$ geeft $\dfrac{3}{(x+1)^2} = \frac{3}{4}$

$(x+1)^2 = 4$

$x + 1 = 2 \vee x + 1 = -2$

$x = 1 \vee x = -3$

$f(-3) = 3\frac{1}{2}$ en $f(1) = \frac{1}{2}$, dus $B(-3, 3\frac{1}{2})$ en $C(1, \frac{1}{2})$.

6.1 Toppen en buigpunten

Bladzijde 54

1 In toppen is de richtingscoëfficient van de raaklijn gelijk aan 0.

$f(x) = \frac{1}{3}x^3 - 1\frac{1}{2}x^2 - 18x + 50$ geeft $f'(x) = x^2 - 3x - 18$

$f'(x) = 0$ geeft $x^2 - 3x - 18 = 0$

$(x+3)(x-6) = 0$

$x = -3 \vee x = 6$

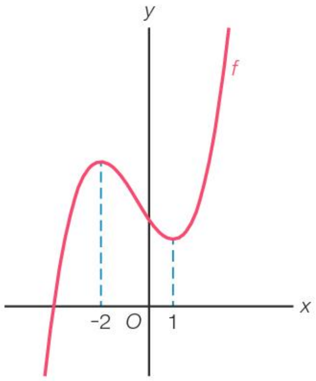
Dus $x_A = -3$ en $x_B = 6$.

2 **a** $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 5$ geeft $f'(x) = x^2 + x - 2$

$f'(x) = 0$ geeft $x^2 + x - 2 = 0$

$(x - 1)(x + 2) = 0$

$x = 1 \vee x = -2$



max. is $f(-2) = 8\frac{1}{3}$ en min. is $f(1) = 3\frac{5}{6}$.

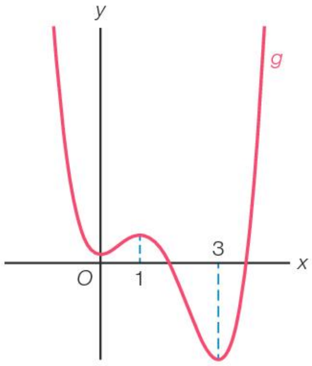
b $g(x) = 3x^4 - 16x^3 + 18x^2 + 2$ geeft $g'(x) = 12x^3 - 48x^2 + 36x$

$g'(x) = 0$ geeft $12x^3 - 48x^2 + 36x = 0$

$12x(x^2 - 4x + 3) = 0$

$12x(x - 1)(x - 3) = 0$

$x = 0 \vee x = 1 \vee x = 3$



min. is $g(0) = 2$, max. is $g(1) = 7$ en min. is $g(3) = -25$.
Het bereik is $B_g = [-25, \rightarrow\rangle$.

3 **a** $f(x) = \frac{5x}{x^2 + 4}$ geeft $f'(x) = \frac{(x^2 + 4) \cdot 5 - 5x \cdot 2x}{(x^2 + 4)^2} = \frac{5x^2 + 20 - 10x^2}{(x^2 + 4)^2} = \frac{-5x^2 + 20}{(x^2 + 4)^2}$

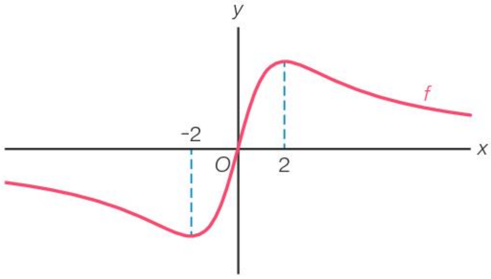
$f'(x) = 0$ geeft $\frac{-5x^2 + 20}{(x^2 + 4)^2} = 0$

$-5x^2 + 20 = 0$

$-5x^2 = -20$

$x^2 = 4$

$x = 2 \vee x = -2$



min. is $f(-2) = -1\frac{1}{4}$ en max. is $f(2) = 1\frac{1}{4}$.

$f(x) = 0$ geeft $5x = 0$

$x = 0$

De grafiek snijdt de x-as alleen in (0, 0), dus $B_f = [-1\frac{1}{4}, 1\frac{1}{4}]$.

b Stel k: $y = ax + b$ met $a = f'(6) = \dfrac{-5 \cdot 6^2 + 20}{(6^2 + 4)^2} = -\frac{1}{10}$.

$$\left.\begin{array}{l} y = -\frac{1}{10}x + b \\ f(6) = \frac{3}{4}, \text{ dus } A(6, \frac{3}{4}) \end{array}\right\} -\frac{1}{10} \cdot 6 + b = \frac{3}{4}$$

$$-\frac{3}{5} + b = \frac{3}{4}$$

$$b = 1\frac{7}{20}$$

Dus k: $y = -\frac{1}{10}x + 1\frac{7}{20}$.

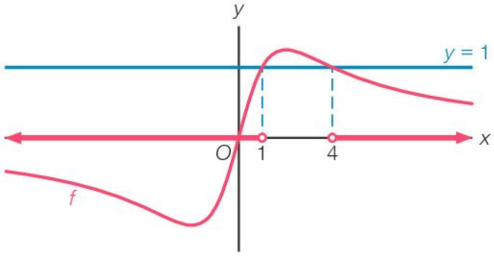
c $f(x) = 1$ geeft $\dfrac{5x}{x^2 + 4} = 1$

$5x = x^2 + 4$

$x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$

$x = 1 \vee x = 4$



$f(x) < 1$ geeft $x < 1 \vee x > 4$

4 **a** $f_{-2}(x) = (x^2 - 2)(x^2 - 16) = x^4 - 18x^2 + 32$ geeft $f_{-2}'(x) = 4x^3 - 36x$

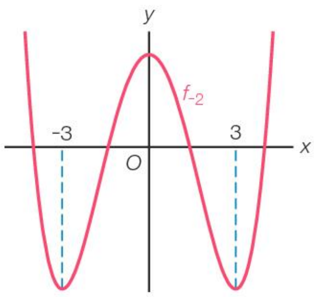
$f_{-2}'(x) = 0$ geeft $4x^3 - 36x = 0$

$4x(x^2 - 9) = 0$

$4x = 0 \vee x^2 - 9 = 0$

$x = 0 \vee x^2 = 9$

$x = 0 \vee x = 3 \vee x = -3$



min. is $f_{-2}(-3) = -49$, max. is $f_{-2}(0) = 32$ en min. is $f_{-2}(3) = -49$.

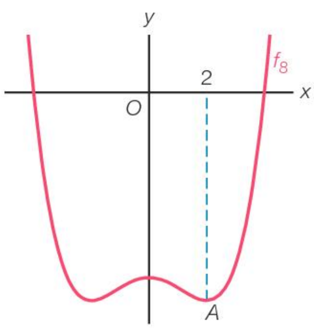
$B_f = [-49, \rightarrow\rangle$

b $f_p(x) = (x^2 + p)(x^2 - 16) = x^4 - 16x^2 + px^2 - 16p$ geeft $f_p'(x) = 4x^3 - 32x + 2px$

$f_p'(2) = 0$ geeft $32 - 64 + 4p = 0$

$4p = 32$

$p = 8$



Dus voor $p = 8$.

5 $f_p(x) = (x^2 + p)(x^2 - 16) = x^4 - 16x^2 + px^2 - 16p$ geeft $f_p{}'(x) = 4x^3 - 32x + 2px$

Voor $x = x_B$ geldt $\left.\begin{array}{l} 4x^3 - 32x + 2px = 0 \\ p = 14x \end{array}\right\}$ $4x^3 - 32x + 2 \cdot 14x \cdot x = 0$

$4x^3 + 28x^2 - 32x = 0$

$4x(x^2 + 7x - 8) = 0$

$4x(x - 1)(x + 8) = 0$

$x = 0 \vee x = 1 \vee x = -8$

$x = 0$ geeft $p = 0$

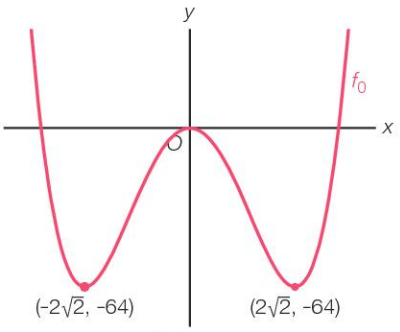
$f_0(x) = x^4 - 16x^2$ geeft $f_0{}'(x) = 4x^3 - 32x$

$f_0{}'(x) = 0$ geeft $4x^3 - 32x = 0$

$4x(x^2 - 8) = 0$

$4x = 0 \vee x^2 = 8$

$x = 0 \vee x = 2\sqrt{2} \vee x = -2\sqrt{2}$



$B_{f_0} = [-64, \rightarrow\rangle$

$x = 1$ geeft $p = 14$

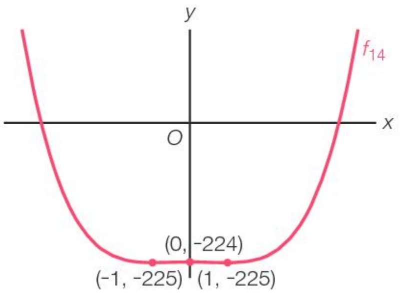
$f_{14}(x) = x^4 - 16x^2 + 14x^2 - 16 \cdot 14 = x^4 - 2x^2 - 224$ geeft $f_{14}{}'(x) = 4x^3 - 4x$

$f_{14}{}'(x) = 0$ geeft $4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$4x = 0 \vee x^2 = 1$

$x = 0 \vee x = 1 \vee x = -1$



$B_{f_{14}} = [-225, \rightarrow\rangle$

$x = -8$ geeft $p = -112$

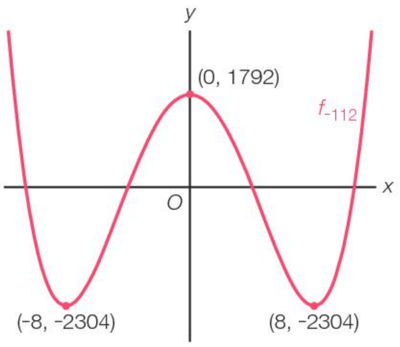
$f_{-112}(x) = x^4 - 16x^2 - 112x^2 - 16 \cdot -112 = x^4 - 128x^2 + 1792$ geeft $f_{-112}{}'(x) = 4x^3 - 256x$

$f_{-112}{}'(x) = 0$ geeft $4x^3 - 256x = 0$

$4x(x^2 - 64) = 0$

$4x = 0 \vee x^2 = 64$

$x = 0 \vee x = 8 \vee x = -8$



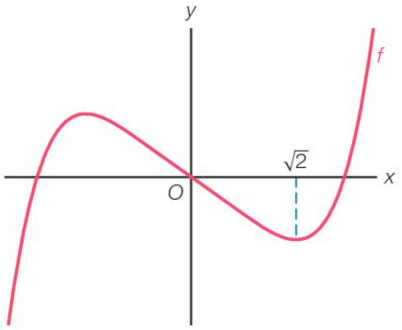
$B_{f_{-112}} = [-2304, \rightarrow\rangle$

6 **a** $f(x) = \frac{1}{4}x^4 - x^2 - 4x + 3$ geeft $f'(x) = x^3 - 2x - 4$
b $f'(2) = 2^3 - 2 \cdot 2 - 4 = 8 - 4 - 4 = 0$
c In de schets is te zien dat de grafiek een top heeft voor $x = 2$. Dus f heeft een extreme waarde voor $x = 2$.

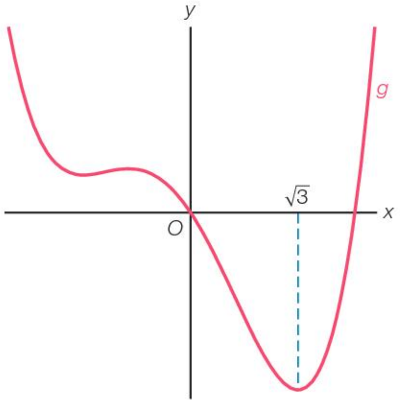
Bladzijde 57

7 **a** $f'(x) = 4x^3 - 18x^2 + 24x - 10$
$f'(1) = 4 - 18 + 24 - 10 = 0$
Uit de schets blijkt dat er geen extreme waarde is voor $x = 1$. Er is wel een horizontale raaklijn.
b $f(1) = 1 - 6 + 12 - 10 + 7 = 4$ en $f(2\frac{1}{2}) = 2\frac{5}{16}$.
Dus de horizontale raaklijnen zijn $y = 4$ en $y = 2\frac{5}{16}$.

8 **a** $f(x) = \frac{1}{2}x^5 - \frac{1}{2}x^3 - 7x$ geeft $f'(x) = 2\frac{1}{2}x^4 - 1\frac{1}{2}x^2 - 7$
$f'(\sqrt{2}) = 2\frac{1}{2} \cdot (\sqrt{2})^4 - 1\frac{1}{2} \cdot (\sqrt{2})^2 - 7 = 2\frac{1}{2} \cdot 4 - 1\frac{1}{2} \cdot 2 - 7 = 10 - 3 - 7 = 0$



$f'(\sqrt{2}) = 0$ en in de schets is te zien dat de grafiek een top heeft voor $x = \sqrt{2}$.
Dus f heeft een extreme waarde voor $x = \sqrt{2}$.
b $g(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - 1\frac{1}{2}x^2 - 3x$ geeft $g'(x) = x^3 + x^2 - 3x - 3$
$g'(\sqrt{3}) = (\sqrt{3})^3 + (\sqrt{3})^2 - 3\sqrt{3} - 3 = 3\sqrt{3} + 3 - 3\sqrt{3} - 3 = 0$



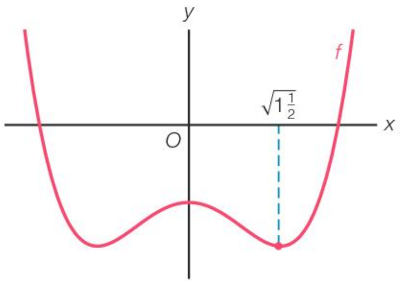
$g'(\sqrt{3}) = 0$ en in de schets is te zien dat de grafiek een top heeft voor $x = \sqrt{3}$.
Dus g heeft een extreme waarde voor $x = \sqrt{3}$.

9 **a** $f(x) = (x^2+1)(x^2-4) = x^4 - 4x^2 + x^2 - 4 = x^4 - 3x^2 - 4$ geeft $f'(x) = 4x^3 - 6x$

$f'(1) = 4 \cdot 1^3 - 6 \cdot 1 = 4 - 6 = -2$

$f'(1)$ is niet gelijk aan 0, dus f heeft geen extreme waarde voor $x = 1$.

b $f'(\sqrt{1\frac{1}{2}}) = 4 \cdot (\sqrt{1\frac{1}{2}})^3 - 6 \cdot \sqrt{1\frac{1}{2}} = 4 \cdot 1\frac{1}{2}\sqrt{1\frac{1}{2}} - 6\sqrt{1\frac{1}{2}} = 6\sqrt{1\frac{1}{2}} - 6\sqrt{1\frac{1}{2}} = 0$

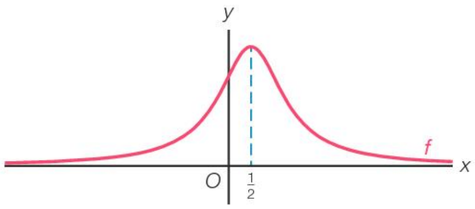


$f'(\sqrt{1\frac{1}{2}}) = 0$ en in de schets is te zien dat de grafiek een top heeft voor $x = \sqrt{1\frac{1}{2}}$.

Dus f heeft een extreme waarde voor $x = \sqrt{1\frac{1}{2}}$.

10 **a** $f(x) = \dfrac{x+1}{x^3+1}$ geeft $f'(x) = \dfrac{(x^3+1) \cdot 1 - (x+1) \cdot 3x^2}{(x^3+1)^2} = \dfrac{x^3 + 1 - 3x^3 - 3x^2}{(x^3+1)^2} = \dfrac{-2x^3 - 3x^2 + 1}{(x^3+1)^2}$

$f'(\frac{1}{2}) = \dfrac{-2 \cdot (\frac{1}{2})^3 - 3 \cdot (\frac{1}{2})^2 + 1}{((\frac{1}{2})^3 + 1)^2} = \dfrac{-2 \cdot \frac{1}{8} - 3 \cdot \frac{1}{4} + 1}{(\frac{1}{8} + 1)^2} = \dfrac{-\frac{1}{4} - \frac{3}{4} + 1}{(\frac{9}{8})^2} = \dfrac{0}{\frac{81}{64}} = 0$



$f'(\frac{1}{2}) = 0$ en in de schets is te zien dat de grafiek een top heeft voor $x = \frac{1}{2}$.

Dus f heeft een extreme waarde voor $x = \frac{1}{2}$.

b $(x+1)(x^2 - x + 1) = x^3 - x^2 + x + x^2 - x + 1 = x^3 + 1$

c $f(x) = \dfrac{x+1}{x^3+1} = \dfrac{x+1}{(x+1)(x^2-x+1)} = \dfrac{1}{x^2-x+1}$ mits $x \neq -1$ geeft

$f'(x) = \dfrac{(x^2-x+1) \cdot 0 - 1 \cdot (2x-1)}{(x^2-x+1)^2} = \dfrac{-2x+1}{(x^2-x+1)^2}$

$f'(x) = 0$ geeft $\dfrac{-2x+1}{(x^2-x+1)^2} = 0$

$-2x + 1 = 0$

$-2x = -1$

$x = \frac{1}{2}$

vold.

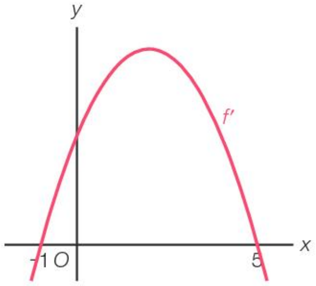
11 **a** dalend op $\langle \leftarrow, -1 \rangle$ en $\langle 5, \rightarrow \rangle$

stijgend op $\langle -1, 5 \rangle$

toenemend stijgend op $\langle -1, 2 \rangle$

afnemend stijgend op $\langle 2, 5 \rangle$

b $f'(x) = -3x^2 + 12x + 15$
$f'(x) = 0$ geeft $-3x^2 + 12x + 15 = 0$
$x^2 - 4x - 5 = 0$
$(x + 1)(x - 5) = 0$
$x = -1 \vee x = 5$

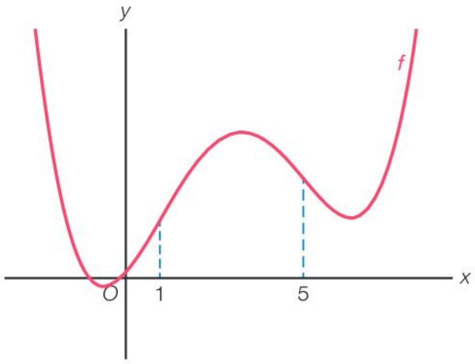


c $x_P = \frac{-1 + 5}{2} = 2$

d Bij $x = x_p$ gaat de grafiek over van toenemend stijgend in afnemend stijgend, want links van $x = x_p$ nemen de hellingen toe en rechts van $x = x_p$ nemen de hellingen af.

Bladzijde 59

12 **a** $f(x) = x^4 - 12x^3 + 30x^2 + 48x + 5$
$f'(x) = 4x^3 - 36x^2 + 60x + 48$
$f''(x) = 12x^2 - 72x + 60$
$f''(x) = 0$ geeft $12x^2 - 72x + 60 = 0$
$x^2 - 6x + 5 = 0$
$(x - 1)(x - 5) = 0$
$x = 1 \vee x = 5$
$f(1) = 72$ en $f(5) = 120$.



Uit de schets volgt dat de buigpunten (1, 72) en (5, 120) zijn.

b $g(x) = \frac{1}{3}x^3 - 3x^2 + 6x + 4$
$g'(x) = x^2 - 6x + 6$
$g''(x) = 2x - 6$
$g''(x) = 0$ geeft $2x - 6 = 0$
$2x = 6$
$x = 3$
Stel k: $y = ax + b$ met $a = g'(3) = 3^2 - 6 \cdot 3 + 6 = -3$.

$$\left.\begin{array}{l} y = -3x + b \\ g(3) = 4, \text{ dus buigpunt } (3, 4) \end{array}\right\} \begin{array}{l} -3 \cdot 3 + b = 4 \\ -9 + b = 4 \\ b = 13 \end{array}$$

Dus k: $y = -3x + 13$.

13 **a** $f(x) = -\frac{1}{12}x^4 - \frac{1}{3}x^3$

$f'(x) = -\frac{1}{3}x^3 - x^2$

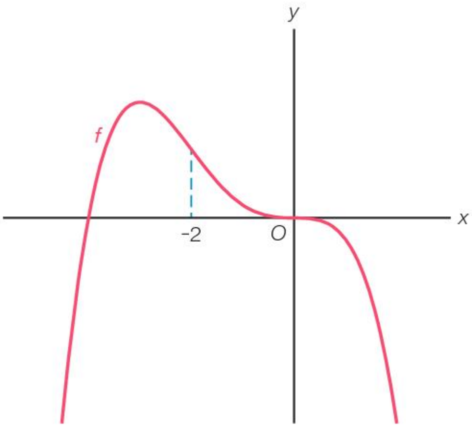
$f''(x) = -x^2 - 2x$

$f''(x) = 0$ geeft $-x^2 - 2x = 0$

$-x(x + 2) = 0$

$x = 0 \vee x = -2$

$f(0) = 0$ en $f(-2) = 1\frac{1}{3}$.



Uit de schets volgt dat $(-2, 1\frac{1}{3})$ en $(0, 0)$ de buigpunten zijn.

b $f''(0) = 0$ en $f'(0) = 0$, dus in het punt $(0, 0)$ is er sprake van een horizontale buigraaklijn.

14 **a** $f_5(x) = \frac{1}{4}x^4 - 2x^3 + 5x^2 - 5x - 5$

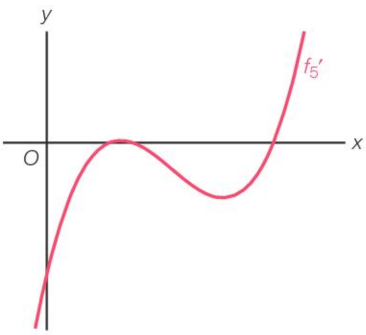
$f_5'(x) = x^3 - 6x^2 + 10x - 5$

$f_5''(x) = 3x^2 - 12x + 10$

$f_5''(x) = 0$ geeft $3x^2 - 12x + 10 = 0$

$D = (-12)^2 - 4 \cdot 3 \cdot 10 = 24 > 0$, dus twee oplossingen

Dus f_5' heeft twee extremen.



De grafiek van f_5 heeft twee buigpunten, omdat f_5' twee extremen heeft.

b $f_6(x) = \frac{1}{4}x^4 - 2x^3 + 6x^2 - 5x - 5$

$f_6'(x) = x^3 - 6x^2 + 12x - 5$

$f_6''(x) = 3x^2 - 12x + 12$

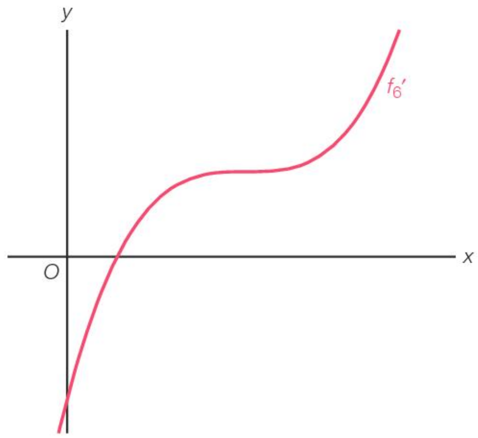
$f_6''(x) = 0$ geeft $3x^2 - 12x + 12 = 0$

$3(x^2 - 4x + 4) = 0$

$3(x - 2)^2 = 0$

De vergelijking $f_6''(x) = 0$ heeft één oplossing.

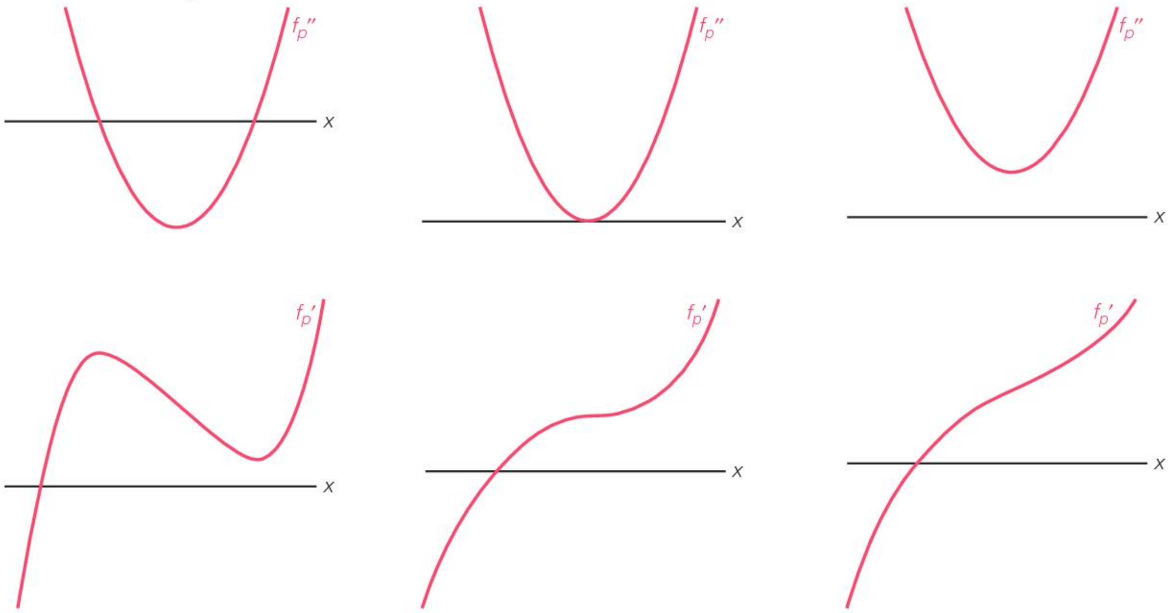
De grafiek van f_6' heeft dus één horizontale raaklijn.



De grafiek van f_6 heeft geen buigpunten omdat f_6' geen extremen heeft.

c $f_p(x) = \frac{1}{4}x^4 - 2x^3 + px^2 - 5x - 5$
$f_p'(x) = x^3 - 6x^2 + 2px - 5$
$f_p''(x) = 3x^2 - 12x + 2p$
$f''(x) = 0$ mag geen of één oplossing hebben, oftewel $D \leq 0$.
$(-12)^2 - 4 \cdot 3 \cdot 2\,p \leq 0$
$144 - 24p \leq 0$
$-24p \leq -144$
$p \geq 6$

d $f_p''(x) = 3x^2 - 12x + 2p$
De grafiek van f_p'' is een dalparabool met twee, één of nul snijpunten met de x-as.



Heeft de grafiek van f_p'' twee snijpunten met de x-as, dan is de grafiek van f_p' stijgend-dalend-stijgend, dus f_p' heeft twee extremen, dus de grafiek van f_p heeft twee buigpunten.

Heeft de grafiek van f_p'' één raakpunt met de x-as, dan is de grafiek van f_p' stijgend-stijgend, dus f_p' heeft geen extremen, dus de grafiek van f_p heeft geen buigpunten.

Heeft de grafiek van f_p'' geen snijpunten met de x-as, dan is de grafiek van f_p' stijgend, dus f_p' heeft geen extremen, dus de grafiek van f_p heeft geen buigpunten.

Dus de grafiek van f_p heeft óf twee óf geen buigpunten.

15 **a** $f(x) = (\frac{1}{2}x^3 - 4)^2 - 5 = \frac{1}{4}x^6 - 4x^3 + 16 - 5 = \frac{1}{4}x^6 - 4x^3 + 11$ geeft

$f'(x) = 1\frac{1}{2}x^5 - 12x^2$

$f'(x) = 0$ geeft $1\frac{1}{2}x^5 - 12x^2 = 0$

$3x^5 - 24x^2 = 0$

$3x^2(x^3 - 8) = 0$

$3x^2 = 0 \vee x^3 = 8$

$x = 0 \vee x = 2$

In de figuur in het leerboek is te zien dat de grafiek een top heeft voor $x = 2$.

$f(2) = -5$, dus top $(2, -5)$.

b $f'(x) = 1\frac{1}{2}x^5 - 12x^2$ geeft $f''(x) = 7\frac{1}{2}x^4 - 24x$

$f''(0) = 0$, in de figuur in het leerboek is te zien dat A een buigpunt is.

c $f''(x) = 0$ geeft $7\frac{1}{2}x^4 - 24x = 0$

$x(7\frac{1}{2}x^3 - 24) = 0$

$x = 0 \vee 7\frac{1}{2}x^3 = 24$

$x = 0 \vee x^3 = 3\frac{1}{5}$

$x = 0 \vee x = \sqrt[3]{3\frac{1}{5}}$

$f(\sqrt[3]{3\frac{1}{5}}) = (\frac{1}{2} \cdot 3\frac{1}{5} - 4)^2 - 5 = \frac{19}{25}$

Dus $B(\sqrt[3]{3\frac{1}{5}}, \frac{19}{25})$.

16 $f_p(x) = x^4 + px^3 + \frac{3}{4}x^2 + 10$

$f_p'(x) = 4x^3 + 3px^2 + 1\frac{1}{2}x$

$f_p''(x) = 12x^2 + 6px + 1\frac{1}{2}$

De grafiek van f_p heeft twee buigpunten als f_p' twee extremen heeft,
dus als $f_p''(x) = 0$ twee oplossingen heeft.

twee oplossingen, dus $D > 0$
$D = (6p)^2 - 4 \cdot 12 \cdot 1\frac{1}{2} = 36p^2 - 72$ } $36p^2 - 72 > 0$

$36p^2 > 72$

$p^2 > 2$

$p < -\sqrt{2} \vee p > \sqrt{2}$.

Dus de grafiek van f_p heeft twee buigpunten voor $p < -\sqrt{2} \vee p > \sqrt{2}$.

17 **a** $f(x) = ax^3 + bx^2 + cx + d$ geeft $f'(x) = 3ax^2 + 2bx + c$

Gegeven is dat f een derdegraadsfunctie is, dus $a \neq 0$.

Hieruit volgt dat de grafiek van f' een parabool is, dus f' heeft één extreem.

Dus de grafiek van elke derdegraadsfunctie heeft één buigpunt.

b Omdat de grafiek van f twee toppen heeft, heeft de grafiek van f' twee snijpunten met de x-as.

$f'(x) = 0$ geeft $3ax^2 + 2bx + c = 0$

twee oplossingen, dus $D > 0$
$D = (2b)^2 - 4 \cdot 3a \cdot c = 4b^2 - 12ac$ } $4b^2 - 12ac > 0$

$4b^2 > 12ac$

$b^2 > 3ac$

c $f'(x) = 3ax^2 + 2bx + c$ geeft $f''(x) = 6ax + 2b$

$f''(x) = 0$ geeft $6ax + 2b = 0$

$6ax = -2b$

$x = -\frac{b}{3a}$

Dus $x_R = -\frac{b}{3a}$.

Voor een derdegraadsfunctie geldt dat $a \neq 0$, dus er is precies één buigpunt.
De x-coördinaten van de toppen zijn de oplossingen van $f'(x) = 0$.

$f'(x) = 0$ geeft $3ax^2 + 2bx + c = 0$

$$D = 4b^2 - 12ac$$

$$x = \frac{-2b + \sqrt{D}}{6a} \vee x = \frac{-2b - \sqrt{D}}{6a}$$

$$x_P + x_Q = \frac{-2b + \sqrt{D}}{6a} + \frac{-2b - \sqrt{D}}{6a} = \frac{-2b + \sqrt{D} - 2b - \sqrt{D}}{6a} = \frac{-4b}{6a} = \frac{-2b}{3a} = 2 \cdot -\frac{b}{3a} = 2x_R$$

Dus $2x_R = x_P + x_Q$ oftewel $x_R = \frac{x_P + x_Q}{2}$.

6.2 De afgeleide van machtsfuncties

Bladzijde 62

18 **a** $\frac{4}{x^2} = 4x^{-2}$ $\quad \frac{6}{x^3} = 6x^{-3}$ $\quad \frac{5}{x^4} = 5x^{-4}$ $\quad \frac{1}{3x^2} = \frac{1}{3}x^{-2}$

b $x^{-4} = \frac{1}{x^4}$ $\quad 3x^{-2} = \frac{3}{x^2}$ $\quad -2x^{-3} = -\frac{2}{x^3}$ $\quad \frac{1}{7}x^{-6} = \frac{1}{7x^6}$

c $\frac{x^3 + 5x^2}{x} = \frac{x^3}{x} + \frac{5x^2}{x} = x^2 + 5x$

$\frac{4x^2 + 7x}{x^3} = \frac{4x^2}{x^3} + \frac{7x}{x^3} = 4x^{-1} + 7x^{-2}$

$\frac{2x^5 + 5x^2}{3x^4} = \frac{2x^5}{3x^4} + \frac{5x^2}{3x^4} = \frac{2}{3}x + 1\frac{2}{3}x^{-2}$

d $\frac{1}{2x} + \frac{2}{x^2} = \frac{x}{2x^2} + \frac{4}{2x^2} = \frac{x + 4}{2x^2}$

$\frac{1}{2}x + \frac{3}{x^2} = \frac{x}{2} + \frac{3}{x^2} = \frac{x^3}{2x^2} + \frac{6}{2x^2} = \frac{x^3 + 6}{2x^2}$

$\frac{2}{3}x^2 - \frac{3}{4x} = \frac{2x^2}{3} - \frac{3}{4x} = \frac{8x^3}{12x} - \frac{9}{12x} = \frac{8x^3 - 9}{12x}$

19 **a** $f(x) = \frac{1}{x^2}$ geeft $f'(x) = \frac{x^2 \cdot [1]' - 1 \cdot [x^2]'}{(x^2)^2} = \frac{x^2 \cdot 0 - 1 \cdot 2x}{x^4} = \frac{-2x}{x^4} = \frac{-2}{x^3}$

b $f(x) = x^{-2} = \frac{1}{x^2}$ geeft $f'(x) = -\frac{2}{x^3} = -2x^{-3}$

c $g(x) = x^{-5} = \frac{1}{x^5}$ geeft $g'(x) = \frac{x^5 \cdot [1]' - 1 \cdot [x^5]'}{(x^5)^2} = \frac{x^5 \cdot 0 - 1 \cdot 5x^4}{x^{10}} = \frac{-5x^4}{x^{10}} = -5x^{-6}$

Bladzijde 63

20 **a** $n = 0$ geeft $f(x) = x^0 = 1$ en hieruit volgt $f'(x) = 0$.
$n = 0$ geeft $f'(x) = 0 \cdot x^{0-1} = 0$
Dus $f(x) = x^n$ geeft $f'(x) = nx^{n-1}$ klopt voor $n = 0$.
$n = 1$ geeft $f(x) = x^1 = x$ en hieruit volgt $f'(x) = 1$.
$n = 1$ geeft $f'(x) = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1 \cdot 1 = 1$
Dus $f(x) = x^n$ geeft $f'(x) = nx^{n-1}$ klopt voor $n = 1$.

b Dat lukte omdat de noemers uit slechts één term bestaan.
De functies g en h zijn te differentiëren door uit te delen.

21 **a** $f(x) = \frac{1}{x^6} = x^{-6}$ geeft $f'(x) = -6x^{-7} = \frac{-6}{x^7}$

b $g(x) = 5 - \frac{3}{x^2} = 5 - 3x^{-2}$ geeft $g'(x) = 6x^{-3} = \frac{6}{x^3}$

c $h(x) = ax^4 - \frac{b}{x^4} = ax^4 - bx^{-4}$ geeft $h'(x) = 4ax^3 + 4bx^{-5} = 4ax^3 + \frac{4b}{x^5}$

22 **a** $f(x) = \frac{2x-1}{3x^2} = \frac{2x}{3x^2} - \frac{1}{3x^2} = \frac{2}{3}x^{-1} - \frac{1}{3}x^{-2}$ geeft $f'(x) = -\frac{2}{3}x^{-2} + \frac{2}{3}x^{-3} = -\frac{2}{3x^2} + \frac{2}{3x^3} = -\frac{2x}{3x^3} + \frac{2}{3x^3} = \frac{2-2x}{3x^3}$

b $g(x) = \frac{3x^2}{2x-1}$ geeft $g'(x) = \frac{(2x-1) \cdot 6x - 3x^2 \cdot 2}{(2x-1)^2} = \frac{12x^2 - 6x - 6x^2}{(2x-1)^2} = \frac{6x^2 - 6x}{(2x-1)^2}$

c $h(x) = \frac{3x^6 - 3}{x^3} = \frac{3x^6}{x^3} - \frac{3}{x^3} = 3x^3 - 3x^{-3}$ geeft $h'(x) = 9x^2 + 9x^{-4} = 9x^2 + \frac{9}{x^4} = \frac{9x^6}{x^4} + \frac{9}{x^4} = \frac{9x^6+9}{x^4}$

23 **a** $f(x) = 5x^2 - \frac{5}{x^2} = 5x^2 - 5x^{-2}$ geeft $f'(x) = 10x + 10x^{-3} = 10x + \frac{10}{x^3}$

b $g(x) = \frac{5}{2x^2} - \frac{2x^2}{5} = \frac{5}{2}x^{-2} - \frac{2}{5}x^2$ geeft $g'(x) = -5x^{-3} - \frac{4}{5}x = -\frac{5}{x^3} - \frac{4x}{5}$

c $h(x) = 6 - \frac{x^2-1}{x} = 6 - \left(\frac{x^2}{x} - \frac{1}{x}\right) = 6 - x + x^{-1}$ geeft $h'(x) = -1 - x^{-2} = -1 - \frac{1}{x^2}$

Bladzijde 64

24 **a** $f(x) = 0$ geeft $\frac{3x+3}{x} = 0$

$3x + 3 = 0$

$3x = -3$

$x = -1$

Dus $A(-1, 0)$.

$f(x) = \frac{3x+3}{x} = \frac{3x}{x} + \frac{3}{x} = 3 + 3x^{-1}$ geeft $f'(x) = -3x^{-2} = -\frac{3}{x^2}$

Stel k: $y = ax + b$ met $a = f'(-1) = -\frac{3}{(-1)^2} = -3$.

$\left.\begin{array}{l} y = -3x + b \\ \text{door } A(-1, 0) \end{array}\right\}$ $-3 \cdot -1 + b = 0$

$3 + b = 0$

$b = -3$

Dus k: $y = -3x - 3$.

b $\text{rc}_{\text{raaklijn}} = -\frac{3}{4}$, dus $f'(x) = -\frac{3}{4}$

$-\frac{3}{x^2} = -\frac{3}{4}$

$x^2 = 4$

$x = 2 \vee x = -2$

$f(2) = 4\frac{1}{2}$ en $f(-2) = 1\frac{1}{2}$, dus de raakpunten zijn $(2, 4\frac{1}{2})$ en $(-2, 1\frac{1}{2})$.

25 **a** $f(x) = \frac{x}{x^2-1}$ geeft $f'(x) = \frac{(x^2-1) \cdot 1 - x \cdot 2x}{(x^2-1)^2} = \frac{x^2 - 1 - 2x^2}{(x^2-1)^2} = \frac{-x^2-1}{(x^2-1)^2}$

Stel k: $y = ax + b$ met $a = f'(2) = \frac{-4-1}{(4-1)^2} = -\frac{5}{9}$.

$\left.\begin{array}{l} y = -\frac{5}{9}x + b \\ f(2) = \frac{2}{3}, \text{ dus } A(2, \frac{2}{3}) \end{array}\right\}$ $-\frac{5}{9} \cdot 2 + b = \frac{2}{3}$

$-1\frac{1}{9} + b = \frac{2}{3}$

$b = 1\frac{7}{9}$

Dus k: $y = -\frac{5}{9}x + 1\frac{7}{9}$.

b $g(x) = \frac{x^2 - 1}{x} = \frac{x^2}{x} - \frac{1}{x} = x - x^{-1}$ geeft $g'(x) = 1 + x^{-2} = 1 + \frac{1}{x^2} = \frac{x^2 + 1}{x^2}$

Stel l: $y = ax + b$ met $a = g'(2) = \frac{4 + 1}{4} = 1\frac{1}{4}$.

$$\left.\begin{array}{l} y = 1\frac{1}{4}x + b \\ g(2) = 1\frac{1}{2}, \text{ dus } B(2, 1\frac{1}{2}) \end{array}\right\} 1\frac{1}{4} \cdot 2 + b = 1\frac{1}{2}$$

$$2\frac{1}{2} + b = 1\frac{1}{2}$$

$$b = -1$$

Dus l: $y = 1\frac{1}{4}x - 1$.

c $g'(x) = 1 + x^{-2}$ geeft $g''(x) = -2x^{-3} = \frac{-2}{x^3}$

$g''(x) = 0$ geeft $\frac{-2}{x^3} = 0$ en deze vergelijking heeft geen oplossingen.

g' heeft geen extreme waarden, dus de grafiek van g heeft geen buigpunten.

26 **a** $f(x) = \frac{x^2 + 4}{x} = \frac{x^2}{x} + \frac{4}{x} = x + 4x^{-1}$ geeft $f'(x) = 1 - 4x^{-2} = 1 - \frac{4}{x^2} = \frac{x^2 - 4}{x^2}$

$f'(x) = 0$ geeft $\frac{x^2 - 4}{x^2} = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = 2 \vee x = -2$

Zie de figuur in het leerboek.

max. is $f(-2) = -4$ en min. is $f(2) = 4$.

b $rc_{raaklijn} = -3$ geeft $f'(x) = -3$

$\frac{x^2 - 4}{x^2} = -3$

$x^2 - 4 = -3x^2$

$4x^2 = 4$

$x^2 = 1$

$x = 1 \vee x = -1$

$f(1) = 5$ en $f(-1) = -5$, dus de raakpunten zijn $(1, 5)$ en $(-1, -5)$.

c $f'(x) = 2$ geeft $\frac{x^2 - 4}{x^2} = 2$

$x^2 - 4 = 2x^2$

$x^2 = -4$

geen opl.

Dus er is geen raaklijn met richtingscoëfficiënt 2.

d $f'(x) = \frac{5}{9}$ geeft $\frac{x^2 - 4}{x^2} = \frac{5}{9}$

$9x^2 - 36 = 5x^2$

$4x^2 = 36$

$x^2 = 9$

$x = 3 \vee x = -3$

$$\left.\begin{array}{l} y = \frac{5}{9}x + b \\ f(3) = 4\frac{1}{3}, \text{ dus door } (3, 4\frac{1}{3}) \end{array}\right\} \frac{5}{9} \cdot 3 + b = 4\frac{1}{3}$$

$$1\frac{2}{3} + b = 4\frac{1}{3}$$

$$b = 2\frac{2}{3}$$

Dus de lijn k: $y = \frac{5}{9}x + 2\frac{2}{3}$ raakt de grafiek van f.

27 $f_a(x) = \frac{x^2}{a} + \frac{a}{x^2} = \frac{1}{a} \cdot x^2 + a \cdot x^{-2}$ geeft $f_a'(x) = \frac{1}{a} \cdot 2x + a \cdot -2x^{-3} = \frac{2x}{a} - \frac{2a}{x^3}$

$f'(x) = 0$ geeft $\frac{2x}{a} = \frac{2a}{x^3}$

$2x^4 = 2a^2$

$x^4 = a^2$

$x^2 = a \vee x^2 = -a$

vold. vold. niet

$y_{\text{toppen}} = \frac{a}{a} + \frac{a}{a} = 1 + 1 = 2$ en dus liggen de toppen op de lijn $y = 2$.

28 **a** $\sqrt{x} = x^{\frac{1}{2}}$

b $x^2 \cdot \sqrt{x} = x^2 \cdot x^{\frac{1}{2}} = x^{2\frac{1}{2}}$

c $\frac{1}{x\sqrt{x}} = \frac{1}{x^1 \cdot x^{\frac{1}{2}}} = \frac{1}{x^{1\frac{1}{2}}} = x^{-1\frac{1}{2}}$

d $x^3 \cdot \sqrt{x} = x^3 \cdot x^{\frac{1}{2}} = x^{3\frac{1}{2}}$

29 **a** $\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

b $2\frac{1}{2}x^{1\frac{1}{2}} = 2\frac{1}{2} \cdot x \cdot x^{\frac{1}{2}} = 2\frac{1}{2}x\sqrt{x}$

c $-1\frac{1}{2}x^{-2\frac{1}{2}} = -\frac{3}{2} \cdot \frac{1}{x^{2\frac{1}{2}}} = -\frac{3}{2 \cdot x^2 \cdot x^{\frac{1}{2}}} = -\frac{3}{2x^2 \cdot \sqrt{x}}$

d $1\frac{1}{3}x^{\frac{1}{3}} = 1\frac{1}{3} \cdot \sqrt[3]{x}$

Bladzijde 65

30 **a** $x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x$

$[x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}]' = [x]'$

$[x^{\frac{1}{2}}]' \cdot x^{\frac{1}{2}} + x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 1$

$2 \cdot x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 1$

b $2 \cdot x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 1$

$x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = \frac{1}{2}$

$[x^{\frac{1}{2}}]' = \frac{\frac{1}{2}}{x^{\frac{1}{2}}}$

$[x^{\frac{1}{2}}]' = \frac{1}{2}x^{-\frac{1}{2}}$

Bladzijde 66

31 **a** $f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}}$ geeft $f'(x) = 1 + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = 1 + \frac{1}{2\sqrt{x}}$

b $g(x) = x \cdot \sqrt[3]{x} = x \cdot x^{\frac{1}{3}} = x^{1\frac{1}{3}}$ geeft $g'(x) = 1\frac{1}{3}x^{\frac{1}{3}} = 1\frac{1}{3} \cdot \sqrt[3]{x}$

c $h(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$ geeft $h'(x) = -\frac{1}{2}x^{-1\frac{1}{2}} = -\frac{1}{2x^{1\frac{1}{2}}} = -\frac{1}{2x \cdot x^{\frac{1}{2}}} = -\frac{1}{2x\sqrt{x}}$

d $k(x) = x^3 \cdot \sqrt[5]{x^3} = x^3 \cdot x^{\frac{3}{5}} = x^{3\frac{3}{5}}$ geeft $k'(x) = 3\frac{3}{5}x^{2\frac{3}{5}} = 3\frac{3}{5}x^2 \cdot x^{\frac{3}{5}} = 3\frac{3}{5}x^2 \cdot \sqrt[5]{x^3}$

32 **a** $f(x) = x^2 \cdot \sqrt[4]{x} = x^2 \cdot x^{\frac{1}{4}} = x^{2\frac{1}{4}}$ geeft $f'(x) = 2\frac{1}{4}x^{1\frac{1}{4}} = 2\frac{1}{4}x \cdot x^{\frac{1}{4}} = 2\frac{1}{4}x \cdot \sqrt[4]{x}$

b $g(x) = \dfrac{4x^2+1}{x\sqrt{x}} = \dfrac{4x^2}{x \cdot x^{\frac{1}{2}}} + \dfrac{1}{x \cdot x^{\frac{1}{2}}} = \dfrac{4x^2}{x^{1\frac{1}{2}}} + \dfrac{1}{x^{1\frac{1}{2}}} = 4x^{\frac{1}{2}} + x^{-1\frac{1}{2}}$ geeft

$g'(x) = 2x^{-\frac{1}{2}} - 1\frac{1}{2}x^{-2\frac{1}{2}} = \dfrac{2}{x^{\frac{1}{2}}} - \dfrac{3}{2} \cdot \dfrac{1}{x^{2\frac{1}{2}}} = \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x^2 \cdot x^{\frac{1}{2}}} = \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x^2 \cdot \sqrt{x}} = \dfrac{4x^2-3}{2x^2 \cdot \sqrt{x}}$

c $h(x) = (x^2+1)(1+\sqrt{x}) = x^2 + x^2 \cdot \sqrt{x} + 1 + \sqrt{x} = x^2 + x^2 \cdot x^{\frac{1}{2}} + 1 + x^{\frac{1}{2}} = x^2 + x^{2\frac{1}{2}} + 1 + x^{\frac{1}{2}}$ geeft

$h'(x) = 2x + 2\frac{1}{2}x^{1\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} = 2x + 2\frac{1}{2}x \cdot x^{\frac{1}{2}} + \frac{1}{2} \cdot \dfrac{1}{x^{\frac{1}{2}}} = 2x + 2\frac{1}{2}x\sqrt{x} + \dfrac{1}{2\sqrt{x}}$

d $k(x) = \dfrac{x-4}{\sqrt[3]{x}} = \dfrac{x}{x^{\frac{1}{3}}} - \dfrac{4}{x^{\frac{1}{3}}} = x^{\frac{2}{3}} - 4x^{-\frac{1}{3}}$ geeft

$k'(x) = \frac{2}{3}x^{-\frac{1}{3}} + \frac{4}{3}x^{-1\frac{1}{3}} = \dfrac{2}{3} \cdot \dfrac{1}{x^{\frac{1}{3}}} + \dfrac{4}{3} \cdot \dfrac{1}{x^{1\frac{1}{3}}} = \dfrac{2}{3} \cdot \dfrac{1}{x^{\frac{1}{3}}} + \dfrac{4}{3} \cdot \dfrac{1}{x \cdot x^{\frac{1}{3}}} = \dfrac{2}{3 \cdot \sqrt[3]{x}} + \dfrac{4}{3x \cdot \sqrt[3]{x}} = \dfrac{2x+4}{3x \cdot \sqrt[3]{x}}$

33 **a** $f(x) = \dfrac{1}{x\sqrt{x}} = \dfrac{1}{x \cdot x^{\frac{1}{2}}} = \dfrac{1}{x^{1\frac{1}{2}}} = x^{-1\frac{1}{2}}$ geeft $f'(x) = -1\frac{1}{2} \cdot x^{-2\frac{1}{2}} = -\dfrac{3}{2} \cdot \dfrac{1}{x^{2\frac{1}{2}}} = -\dfrac{3}{2 \cdot x^2 \cdot x^{\frac{1}{2}}} = -\dfrac{3}{2x^2 \cdot \sqrt{x}}$

b $g(x) = \sqrt{x} - \dfrac{1}{\sqrt{x}} = x^{\frac{1}{2}} - \dfrac{1}{x^{\frac{1}{2}}} = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$ geeft

$g'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-1\frac{1}{2}} = \dfrac{1}{2} \cdot \dfrac{1}{x^{\frac{1}{2}}} + \dfrac{1}{2} \cdot \dfrac{1}{x^{1\frac{1}{2}}} = \dfrac{1}{2\sqrt{x}} + \dfrac{1}{2 \cdot x \cdot x^{\frac{1}{2}}} = \dfrac{1}{2\sqrt{x}} + \dfrac{1}{2x\sqrt{x}}$

c $h(x) = \dfrac{x^2-2}{x\sqrt{x}} = \dfrac{x^2}{x \cdot x^{\frac{1}{2}}} - \dfrac{2}{x \cdot x^{\frac{1}{2}}} = \dfrac{x^2}{x^{1\frac{1}{2}}} - \dfrac{2}{x^{1\frac{1}{2}}} = x^{\frac{1}{2}} - 2x^{-1\frac{1}{2}}$ geeft

$h'(x) = \frac{1}{2}x^{-\frac{1}{2}} + 3x^{-2\frac{1}{2}} = \dfrac{1}{2} \cdot \dfrac{1}{x^{\frac{1}{2}}} + \dfrac{3}{x^{2\frac{1}{2}}} = \dfrac{1}{2\sqrt{x}} + \dfrac{3}{x^2 \cdot x^{\frac{1}{2}}} = \dfrac{1}{2\sqrt{x}} + \dfrac{3}{x^2 \cdot \sqrt{x}} = \dfrac{x^2+6}{2x^2 \cdot \sqrt{x}}$

d $k(x) = x^2(x\sqrt{x} - 3) = x^3 \cdot \sqrt{x} - 3x^2 = x^3 \cdot x^{\frac{1}{2}} - 3x^2 = x^{3\frac{1}{2}} - 3x^2$ geeft

$k'(x) = 3\frac{1}{2}x^{2\frac{1}{2}} - 6x = 3\frac{1}{2}x^2 \cdot x^{\frac{1}{2}} - 6x = 3\frac{1}{2}x^2 \cdot \sqrt{x} - 6x$

34 **a** $f(x) = (x\sqrt{x} - 3)^2 = x^3 - 6x\sqrt{x} + 9 = x^3 - 6x \cdot x^{\frac{1}{2}} + 9 = x^3 - 6x^{1\frac{1}{2}} + 9$ geeft $f'(x) = 3x^2 - 9x^{\frac{1}{2}} = 3x^2 - 9\sqrt{x}$

b $g(x) = \dfrac{2x-3}{x^2 \cdot \sqrt{x}} = \dfrac{2x}{x^2 \cdot x^{\frac{1}{2}}} - \dfrac{3}{x^2 \cdot x^{\frac{1}{2}}} = \dfrac{2x}{x^{2\frac{1}{2}}} - \dfrac{3}{x^{2\frac{1}{2}}} = 2x^{-1\frac{1}{2}} - 3x^{-2\frac{1}{2}}$ geeft

$g'(x) = -3x^{-2\frac{1}{2}} + 7\frac{1}{2}x^{-3\frac{1}{2}} = -\dfrac{3}{x^{2\frac{1}{2}}} + \dfrac{15}{2} \cdot \dfrac{1}{x^{3\frac{1}{2}}} = -\dfrac{3}{x^2 \cdot x^{\frac{1}{2}}} + \dfrac{15}{2x^3 \cdot x^{\frac{1}{2}}} = -\dfrac{3}{x^2 \cdot \sqrt{x}} + \dfrac{15}{2x^3 \cdot \sqrt{x}} = \dfrac{-6x+15}{2x^3 \cdot \sqrt{x}}$

c $h(x) = (x - \sqrt[3]{x})^2 = (x - x^{\frac{1}{3}})^2 = x^2 - 2 \cdot x \cdot x^{\frac{1}{3}} + (x^{\frac{1}{3}})^2 = x^2 - 2x^{1\frac{1}{3}} + x^{\frac{2}{3}}$ geeft

$h'(x) = 2x - 2\frac{2}{3}x^{\frac{1}{3}} + \frac{2}{3}x^{-\frac{1}{3}} = 2x - 2\frac{2}{3} \cdot \sqrt[3]{x} + \dfrac{2}{3} \cdot \dfrac{1}{x^{\frac{1}{3}}} = 2x - 2\frac{2}{3} \cdot \sqrt[3]{x} + \dfrac{2}{3 \cdot \sqrt[3]{x}}$

d $k(x) = \dfrac{x^2+4}{\sqrt[4]{x}} = \dfrac{x^2}{x^{\frac{1}{4}}} + \dfrac{4}{x^{\frac{1}{4}}} = x^{1\frac{3}{4}} + 4x^{-\frac{1}{4}}$ geeft

$k'(x) = 1\frac{3}{4}x^{\frac{3}{4}} - x^{-1\frac{1}{4}} = 1\frac{3}{4} \cdot \sqrt[4]{x^3} - \dfrac{1}{x^{1\frac{1}{4}}} = 1\frac{3}{4} \cdot \sqrt[4]{x^3} - \dfrac{1}{x \cdot x^{\frac{1}{4}}} = 1\frac{3}{4} \cdot \sqrt[4]{x^3} - \dfrac{1}{x \cdot \sqrt[4]{x}} = \dfrac{1\frac{3}{4}x^2 - 1}{x \cdot \sqrt[4]{x}} = \dfrac{7x^2-4}{4x \cdot \sqrt[4]{x}}$

35 $f(x) = 3 \cdot \sqrt[3]{x^2} = 3x^{\frac{2}{3}}$ geeft $f'(x) = 2x^{-\frac{1}{3}} = \dfrac{2}{x^{\frac{1}{3}}} = \dfrac{2}{\sqrt[3]{x}}$

Stel k: $y = ax + b$ met $a = f'(\frac{1}{8}) = \dfrac{2}{\sqrt[3]{\frac{1}{8}}} = \dfrac{2}{\frac{1}{2}} = 4$.

$$\left.\begin{array}{l} y = 4x + b \\ f(\frac{1}{8}) = \frac{3}{4}, \text{ dus } A(\frac{1}{8}, \frac{3}{4}) \end{array}\right\} 4 \cdot \tfrac{1}{8} + b = \tfrac{3}{4}$$

$\frac{1}{2} + b = \frac{3}{4}$

$b = \frac{1}{4}$

Dus k: $y = 4x + \frac{1}{4}$.

Stel l: $y = ax + b$ met $a = f'(8) = \dfrac{2}{\sqrt[3]{8}} = \dfrac{2}{2} = 1$.

$$\left.\begin{array}{l} y = x + b \\ f(8) = 12, \text{ dus } B(8, 12) \end{array}\right\} \begin{array}{l} 8 + b = 12 \\ b = 4 \end{array}$$

Dus l: $y = x + 4$.

Snijden van k en l geeft $4x + \frac{1}{4} = x + 4$

$3x = 3\frac{3}{4}$

$$\left.\begin{array}{l} x = 1\frac{1}{4} \\ y = x + 4 \end{array}\right\} y = 1\tfrac{1}{4} + 4 = 5\tfrac{1}{4}$$

Dus $C(1\frac{1}{4}, 5\frac{1}{4})$.

36 **a** $f(x) = x\sqrt{x} - 3x = x^{1\frac{1}{2}} - 3x$ geeft $f'(x) = 1\frac{1}{2}x^{\frac{1}{2}} - 3 = 1\frac{1}{2}\sqrt{x} - 3$

$f'(x) = 0$ geeft $1\frac{1}{2}\sqrt{x} - 3 = 0$

$1\frac{1}{2}\sqrt{x} = 3$

$\sqrt{x} = 2$

$x = 4$

$f(4) = 4\sqrt{4} - 3 \cdot 4 = -4$

min. is $f(4) = -4$.

b Stel k: $y = ax$ met $a = f'(0) = 1\frac{1}{2}\sqrt{0} - 3 = -3$.

Dus k: $y = -3x$.

c $rc_l = 3$, dus $f'(x) = 3$

$1\frac{1}{2}\sqrt{x} - 3 = 3$

$1\frac{1}{2}\sqrt{x} = 6$

$\sqrt{x} = 4$

$x = 16$

$$\left.\begin{array}{l} l\colon y = 3x + b \\ f(16) = 16, \text{ dus } A(16, 16) \end{array}\right\} \begin{array}{l} 3 \cdot 16 + b = 16 \\ 48 + b = 16 \\ b = -32 \end{array}$$

Dus $A(16, 16)$ en l: $y = 3x - 32$.

Bladzijde 67

37 **a** $f(x) = \dfrac{x^3 + 2}{\sqrt{x}} = \dfrac{x^3}{x^{\frac{1}{2}}} + \dfrac{2}{x^{\frac{1}{2}}} = x^{2\frac{1}{2}} + 2x^{-\frac{1}{2}}$ geeft

$f'(x) = 2\frac{1}{2}x^{1\frac{1}{2}} - x^{-1\frac{1}{2}} = 2\frac{1}{2}x \cdot x^{\frac{1}{2}} - \dfrac{1}{x^{1\frac{1}{2}}} = 2\frac{1}{2}x\sqrt{x} - \dfrac{1}{x \cdot x^{\frac{1}{2}}} = 2\frac{1}{2}x\sqrt{x} - \dfrac{1}{x\sqrt{x}} = \dfrac{2\frac{1}{2}x^3 - 1}{x\sqrt{x}}$

Stel k: $y = ax + b$ met $a = f'(1) = \dfrac{2\frac{1}{2} \cdot 1^3 - 1}{1\sqrt{1}} = 1\frac{1}{2}$.

$$\left.\begin{array}{l} y = 1\tfrac{1}{2}x + b \\ f(1) = 3 \text{, dus } A(1, 3) \end{array}\right\} 1\tfrac{1}{2} \cdot 1 + b = 3$$

$$1\tfrac{1}{2} + b = 3$$

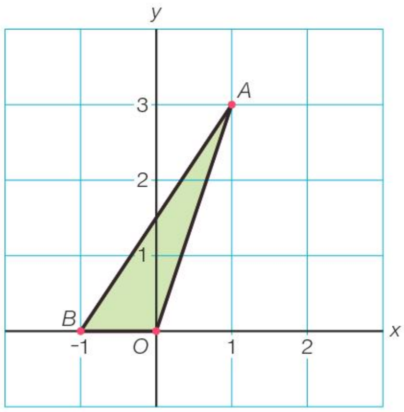
$$b = 1\tfrac{1}{2}$$

Dus k: $y = 1\tfrac{1}{2}x + 1\tfrac{1}{2}$.

Snijden met de x-as, dus $y = 0$ geeft $1\tfrac{1}{2}x + 1\tfrac{1}{2} = 0$

$$1\tfrac{1}{2}x = -1\tfrac{1}{2}$$

$x = -1$, dus $B(-1, 0)$



$O(\triangle OAB) = \tfrac{1}{2} \cdot 1 \cdot 3 = 1\tfrac{1}{2}$

b $f'(x) = 0$ geeft $\dfrac{2\tfrac{1}{2}x^3 - 1}{x\sqrt{x}} = 0$

$$2\tfrac{1}{2}x^3 - 1 = 0$$

$$2\tfrac{1}{2}x^3 = 1$$

$$x^3 = \tfrac{2}{5}$$

$$x = \sqrt[3]{\tfrac{2}{5}}$$

Dus $p = \tfrac{2}{5}$.

38 **a** $s(t) = 10t\sqrt{t} = 10t^{1\frac{1}{2}}$ geeft $s'(t) = 15t^{\frac{1}{2}} = 15\sqrt{t}$

$s'(8) = 15\sqrt{8} = 15 \cdot 2\sqrt{2} = 30\sqrt{2}$

Dus de snelheid na 8 seconden is $30\sqrt{2}$ m/s.

b 108 km/uur $= \dfrac{108}{3{,}6} = 30$ m/s

$s'(t) = 30$ geeft $15\sqrt{t} = 30$

$$\sqrt{t} = 2$$

$$t = 4$$

Dus na 4 seconden is de snelheid 108 km/uur.

c $s(9) = 10 \cdot 9\sqrt{9} = 270$ en $s'(9) = 15 \cdot \sqrt{9} = 45$

Dus in de eerste 9 seconden legt de trein 270 meter af en in de volgende 51 seconden $51 \cdot 45 = 2295$ meter.

Dus in de eerste minuut legt de trein $270 + 2295 = 2565$ meter af.

39 $f_p(x) = (2x - p\sqrt{x})^2 = 4x^2 - 4px\sqrt{x} + p^2x = 4x^2 - 4px^{1\frac{1}{2}} + p^2x$ geeft

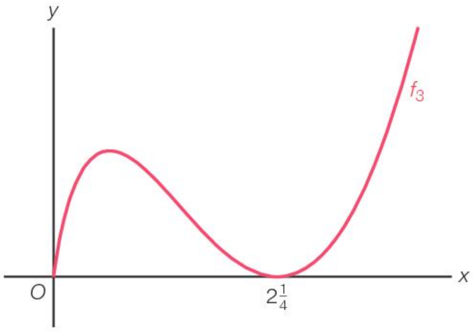
$f_p'(x) = 8x - 6px^{\frac{1}{2}} + p^2 = 8x - 6p\sqrt{x} + p^2$

$f_p'(2\frac{1}{4}) = 0$ geeft $18 - 6p \cdot 1\frac{1}{2} + p^2 = 0$

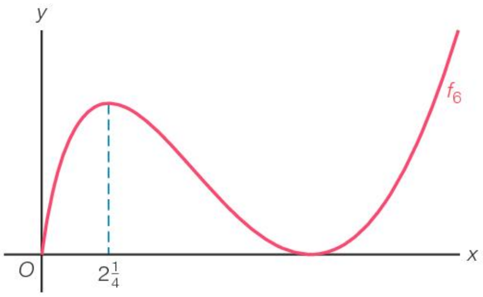
$p^2 - 9p + 18 = 0$

$(p - 3)(p - 6) = 0$

$p = 3 \vee p = 6$



$p = 3$ geeft min. is $f_3(2\frac{1}{4}) = 0$. Dus $p = 3$ voldoet niet.



$p = 6$ geeft max. is $f_6(2\frac{1}{4}) = 20\frac{1}{4}$.

6.3 De kettingregel

Bladzijde 69

40 **a** $v(3) = 3^2 - 5 \cdot 3 = 9 - 15 = -6$

$u(v(3)) = u(-6) = (-6)^4 = 1296$

b $v(4) = 4^2 - 5 \cdot 4 = 16 - 20 = -4$

$u(v(4)) = u(-4) = (-4)^4 = 256$

41 **a** $f(x) = u(v(x))$ met $u(v) = v^6$ en $v(x) = 3 - x^5$.

b $f(x) = u(v(x))$ met $u(v) = \sqrt{v}$ en $v(x) = x^2 + 1$.

c $f(x) = u(v(x))$ met $u(v) = \dfrac{2}{v^3}$ en $v(x) = x + 8$.

Bladzijde 71

42 **a** $f(x) = (4x + 3)^3$ geeft $f'(x) = 3(4x + 3)^2 \cdot 4 = 12(4x + 3)^2$

b $g(x) = 6(\frac{1}{2}x - 4)^5$ geeft $g'(x) = 30(\frac{1}{2}x - 4)^4 \cdot \frac{1}{2} = 15(\frac{1}{2}x - 4)^4$

c $h(x) = 3x^2 - (\frac{1}{4}x - 2)^3$ geeft $h'(x) = 6x - 3(\frac{1}{4}x - 2)^2 \cdot \frac{1}{4} = 6x - \frac{3}{4}(\frac{1}{4}x - 2)^2$

d $j(x) = (4x^2 - 3)^4$ geeft $j'(x) = 4(4x^2 - 3)^3 \cdot 8x = 32x(4x^2 - 3)^3$

e $k(x) = 5x - \dfrac{4}{(3x + 2)^3} = 5x - 4(3x + 2)^{-3}$ geeft

$k'(x) = 5 + 12(3x + 2)^{-4} \cdot 3 = 5 + 36(3x + 2)^{-4} = 5 + \dfrac{36}{(3x + 2)^4}$

f $l(x) = \sqrt{4x + 1}$ geeft $l'(x) = \dfrac{1}{2\sqrt{4x + 1}} \cdot 4 = \dfrac{2}{\sqrt{4x + 1}}$

43 **a** $f(x) = -2(2x+1)^4$ geeft $f'(x) = -8(2x+1)^3 \cdot 2 = -16(2x+1)^3$

b $g(x) = \dfrac{1}{(3x-2)^2} = (3x-2)^{-2}$ geeft $g'(x) = -2(3x-2)^{-3} \cdot 3 = \dfrac{-6}{(3x-2)^3}$

c $h(x) = \sqrt{2x^2+4x}$ geeft $h'(x) = \dfrac{1}{2\sqrt{2x^2+4x}} \cdot (4x+4) = \dfrac{2x+2}{\sqrt{2x^2+4x}}$

d $j(x) = \dfrac{1}{\sqrt{4x-1}} = (4x-1)^{-\frac{1}{2}}$ geeft $j'(x) = -\frac{1}{2}(4x-1)^{-1\frac{1}{2}} \cdot 4 = \dfrac{-2}{(4x-1)\sqrt{4x-1}}$

e $k(x) = (x^2+3)\sqrt{x^2+3} = (x^2+3)^{1\frac{1}{2}}$ geeft $k'(x) = 1\frac{1}{2}(x^2+3)^{\frac{1}{2}} \cdot 2x = 3x\sqrt{x^2+3}$

f $l(x) = \dfrac{1}{\sqrt{x^2+2x+3}} = (x^2+2x+3)^{-\frac{1}{2}}$ geeft

$l'(x) = -\frac{1}{2}(x^2+2x+3)^{-1\frac{1}{2}} \cdot (2x+2) = \dfrac{-x-1}{(x^2+2x+3)\sqrt{x^2+2x+3}}$

44 **a** $f(x) = 4(x^3+7x-2)^2$ geeft $f'(x) = 8(x^3+7x-2) \cdot (3x^2+7) = 8(3x^2+7)(x^3+7x-2)$

b $g(x) = -\dfrac{6}{(x^2+3x)^3} = -6(x^2+3x)^{-3}$ geeft $g'(x) = 18(x^2+3x)^{-4} \cdot (2x+3) = \dfrac{18(2x+3)}{(x^2+3x)^4}$

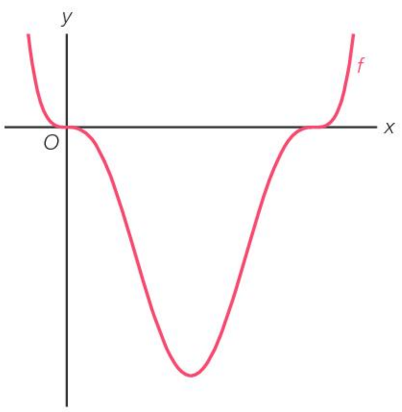
c $h(x) = \sqrt[3]{x^3+3x} = (x^3+3x)^{\frac{1}{3}}$ geeft $h'(x) = \frac{1}{3}(x^3+3x)^{-\frac{2}{3}} \cdot (3x^2+3) = \dfrac{x^2+1}{\sqrt[3]{(x^3+3x)^2}}$

d $j(x) = \dfrac{1}{(4-x)\sqrt{4-x}} = (4-x)^{-1\frac{1}{2}}$ geeft $j'(x) = -1\frac{1}{2}(4-x)^{-2\frac{1}{2}} \cdot -1 = \dfrac{3}{2(4-x)^2 \cdot \sqrt{4-x}}$

e $k(x) = 5\sqrt{2x^4+x^2} + 4x^2$ geeft $k'(x) = 5 \cdot \dfrac{1}{2\sqrt{2x^4+x^2}} \cdot (8x^3+2x) + 8x = \dfrac{5(4x^3+x)}{\sqrt{2x^4+x^2}} + 8x$

f $l(x) = \dfrac{x^2+4}{\sqrt{x^2+4}} = \sqrt{x^2+4}$ geeft $l'(x) = \dfrac{1}{2\sqrt{x^2+4}} \cdot 2x = \dfrac{x}{\sqrt{x^2+4}}$

45 **a**



b Raaklijn horizontaal dus $f'(x) = 0$.

$f(x) = (\frac{1}{2}x^2 - 2x)^3$ geeft $f'(x) = 3(\frac{1}{2}x^2 - 2x)^2 \cdot (x-2)$

$f'(x) = 0$ geeft $3(\frac{1}{2}x^2 - 2x)^2 \cdot (x-2) = 0$

$\frac{1}{2}x^2 - 2x = 0 \vee x - 2 = 0$

$x^2 - 4x = 0 \vee x = 2$

$x(x-4) = 0 \vee x = 2$

$x = 0 \vee x = 4 \vee x = 2$

c Stel k: $y = ax + b$ met $a = f'(3) = 3(\frac{1}{2} \cdot 3^2 - 2 \cdot 3)^2 \cdot (3-2) = 6\frac{3}{4}$

$\left.\begin{array}{l} y = 6\frac{3}{4}x + b \\ f(3) = -3\frac{3}{8}, \text{ dus } A(3, -3\frac{3}{8}) \end{array}\right\}$ $6\frac{3}{4} \cdot 3 + b = -3\frac{3}{8}$

$20\frac{1}{4} + b = -3\frac{3}{8}$

$b = -23\frac{5}{8}$

Dus k: $y = 6\frac{3}{4}x - 23\frac{5}{8}$ en $y_B = -23\frac{5}{8}$.

46 **a** $f(x) = (\frac{1}{4}x - 1)^4 - x + 2$ geeft $f'(x) = 4(\frac{1}{4}x - 1)^3 \cdot \frac{1}{4} - 1 = (\frac{1}{4}x - 1)^3 - 1$

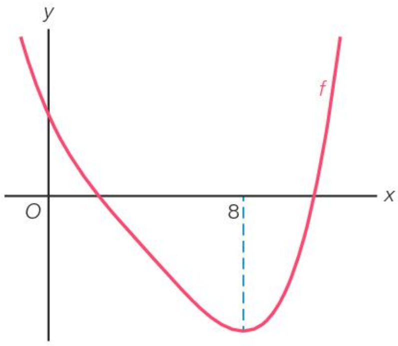
$f'(x) = 0$ geeft $(\frac{1}{4}x - 1)^3 - 1 = 0$

$(\frac{1}{4}x - 1)^3 = 1$

$\frac{1}{4}x - 1 = 1$

$\frac{1}{4}x = 2$

$x = 8$



min. is $f(8) = -5$ en $B_f = [-5, \rightarrow\rangle$.

b Stel k: $y = ax + b$ met $a = rc_k = rc_l = -2$.

$f'(x) = -2$ geeft $(\frac{1}{4}x - 1)^3 - 1 = -2$

$(\frac{1}{4}x - 1)^3 = -1$

$\frac{1}{4}x - 1 = -1$

$\frac{1}{4}x = 0$

$x = 0$

$\left.\begin{array}{l} y = -2x + b \\ f(0) = 3, \text{ dus door } (0, 3) \end{array}\right\} b = 3$

Dus k: $y = -2x + 3$.

47 **a** $g(x) = (\frac{1}{4}x - 1)^2 + ax + b = \frac{1}{16}x^2 - \frac{1}{2}x + 1 + ax + b = \frac{1}{16}x^2 + (a - \frac{1}{2})x + b + 1$

Ook geldt $g(x) = \frac{1}{16}x^2 - 1\frac{1}{2}x + 15$.

Dus $a - \frac{1}{2} = -1\frac{1}{2}$ oftewel $a = -1$ en $b + 1 = 15$ oftewel $b = 14$.

b $f(x) = g(x)$ geeft $(\frac{1}{4}x - 1)^4 - x + 2 = (\frac{1}{4}x - 1)^2 - x + 14$

$(\frac{1}{4}x - 1)^4 - (\frac{1}{4}x - 1)^2 - 12 = 0$

Stel $(\frac{1}{4}x - 1)^2 = u$.

$u^2 - u - 12 = 0$

$(u + 3)(u - 4) = 0$

$u = -3 \vee x = 4$

$(\frac{1}{4}x - 1)^2 = -3 \vee (\frac{1}{4}x - 1)^2 = 4$

geen oplossing $\frac{1}{4}x - 1 = 2 \vee \frac{1}{4}x - 1 = -2$

$\frac{1}{4}x = 3 \vee \frac{1}{4}x = -1$

$x = 12 \vee x = -4$

Bladzijde 72

48 **a** $f(3) = \frac{1}{4}(2 \cdot 3 - 5)^3 + 2 = \frac{1}{4} \cdot 1^3 + 2 = 2\frac{1}{4}$

$g(3) = -\frac{1}{4}(3 \cdot 3 - 10)^4 + 2\frac{1}{2} = -\frac{1}{4} \cdot 1 + 2\frac{1}{2} = 2\frac{1}{4}$

$f(3) = g(3)$, dus A ligt zowel op de grafiek van f als op die van g.

b $f(x) = \frac{1}{4}(2x-5)^3 + 2$ geeft $f'(x) = \frac{3}{4}(2x-5)^2 \cdot 2 = 1\frac{1}{2}(2x-5)^2$

$f'(3) = 1\frac{1}{2}(2 \cdot 3 - 5)^2 = 1\frac{1}{2}$

$g(x) = -\frac{1}{4}(3x-10)^4 + 2\frac{1}{2}$ geeft $g'(x) = -(3x-10)^3 \cdot 3 = -3(3x-10)^3$

$g'(3) = -3(3 \cdot 3 - 10)^3 = 3$

$f'(3) \neq g'(3)$, dus de raaklijnen in A hebben niet dezelfde richting.
Emma heeft dus geen gelijk.

c $\text{rc}_{\text{raaklijn}} = 13\frac{1}{2}$, dus $f'(x) = 13\frac{1}{2}$ geeft $1\frac{1}{2}(2x-5)^2 = 13\frac{1}{2}$

$(2x-5)^2 = 9$

$2x - 5 = 3 \vee 2x - 5 = -3$

$2x = 8 \vee 2x = 2$

$x = 4 \vee x = 1$

$f(4) = 8\frac{3}{4}$ en $f(1) = -4\frac{3}{4}$

Dus de punten zijn $(4, 8\frac{3}{4})$ en $(1, -4\frac{3}{4})$.

49 **a** $f(x) = \sqrt{x^2+9} - x^2 + 5x$ geeft $f'(x) = \dfrac{1}{2\sqrt{x^2+9}} \cdot 2x - 2x + 5 = \dfrac{x}{\sqrt{x^2+9}} - 2x + 5$

$f'(\sqrt{7}) = \dfrac{\sqrt{7}}{\sqrt{7+9}} - 2 \cdot \sqrt{7} + 5 = \frac{1}{4}\sqrt{7} - 2\sqrt{7} + 5 = 5 - 1\frac{3}{4}\sqrt{7} \neq 0$

Dus f heeft geen extreme waarde voor $x = \sqrt{7}$.

b Stel k: $y = ax + b$ met $a = \text{rc}_k = \text{rc}_l = 5$.

$f'(x) = 5$ geeft $\dfrac{x}{\sqrt{x^2+9}} - 2x + 5 = 5$

$\dfrac{x}{\sqrt{x^2+9}} = 2x$

kwadrateren geeft

$\dfrac{x^2}{x^2+9} = 4x^2$

$4x^2(x^2+9) = x^2$

$4x^4 + 36x^2 = x^2$

$4x^4 + 35x^2 = 0$

$x^2(4x^2 + 35) = 0$

$x^2 = 0 \vee 4x^2 = -35$

$x = 0$ geen opl.

$\left.\begin{array}{l} y = 5x + b \\ f(0) = 3, \text{ dus door } (0, 3) \end{array}\right\} b = 3$

Dus k: $y = 5x + 3$.

c $f(0) = 3$, dus $A(0, 3)$.

Stel m: $y = ax + b$ met $a = f'(4) = \dfrac{4}{\sqrt{4^2+9}} - 8 + 5 = \dfrac{4}{\sqrt{25}} - 3 = \frac{4}{5} - 3 = -2\frac{1}{5}$.

$\left.\begin{array}{l} y = -2\frac{1}{5}x + b \\ f(4) = 9, \text{ dus } B(4, 9) \end{array}\right\} -2\frac{1}{5} \cdot 4 + b = 9$

$-8\frac{4}{5} + b = 9$

$b = 17\frac{4}{5}$

Dus m: $y = -2\frac{1}{5}x + 17\frac{4}{5}$ en $C(0, 17\frac{4}{5})$.

$AC = 17\frac{4}{5} - 3 = 14\frac{4}{5}$

50 **a** $f(x) = \dfrac{5x^3 + 10}{\sqrt{x}} = 5x^{2\frac{1}{2}} + 10x^{-\frac{1}{2}}$ geeft $f'(x) = 12\frac{1}{2}x^{1\frac{1}{2}} - 5x^{-1\frac{1}{2}} = 12\frac{1}{2}x\sqrt{x} - \dfrac{5}{x\sqrt{x}} = \dfrac{12\frac{1}{2}x^3 - 5}{x\sqrt{x}}$

$f'(x) = 0$ geeft $\dfrac{12\frac{1}{2}x^3 - 5}{x\sqrt{x}} = 0$

$12\frac{1}{2}x^3 - 5 = 0$

$12\frac{1}{2}x^3 = 5$

$x^3 = \dfrac{5}{12\frac{1}{2}} = \frac{2}{5}$

$x = \sqrt[3]{\frac{2}{5}}$

$f(\sqrt[3]{\frac{2}{5}}) = \dfrac{5 \cdot \frac{2}{5} + 10}{\sqrt{\sqrt[3]{\frac{2}{5}}}} = \dfrac{12}{((\frac{2}{5})^{\frac{1}{3}})^{\frac{1}{2}}} = \dfrac{12}{(\frac{2}{5})^{\frac{1}{6}}} = \dfrac{12}{\sqrt[6]{\frac{2}{5}}}$

Dus $a = 12$, $b = 6$ en $c = \frac{2}{5}$.

b k raakt de grafiek van $g(x) = \dfrac{5x^3 + 10}{\sqrt{x}} + d$.

$g(x) = \dfrac{5x^3 + 10}{\sqrt{x}} + d$ geeft $g'(x) = \dfrac{12\frac{1}{2}x^3 - 5}{x\sqrt{x}}$

Stel k: $y = ax + b$ met $a = g'(1) = \dfrac{12\frac{1}{2} - 5}{1} = 7\frac{1}{2}$.

$\left.\begin{array}{l} y = 7\frac{1}{2}x + b \\ g(1) = 15 + d, \text{ dus } A(1,\ 15 + d) \end{array}\right\} 7\frac{1}{2} \cdot 1 + b = 15 + d$

$b = 7\frac{1}{2} + d$

k door de oorsprong, dus $7\frac{1}{2} + d = 0$

$d = -7\frac{1}{2}$

Bladzijde 73

51 **a** De tijd nodig van A naar B via C is $\frac{2}{80} + \frac{10}{100} = \frac{1}{8}$ uur = 450 seconden.

De afstand AB is $\sqrt{2^2 + 10^2} = \sqrt{104}$ km.

De tijd nodig van A direct naar B is $\dfrac{\sqrt{104}}{80}$ uur = 458,9... seconden.

Het verschil is 458,9... − 450 ≈ 9 seconden.

b $t = \dfrac{AD}{80} + \dfrac{BD}{100} = \dfrac{\sqrt{x^2 + 2^2}}{80} + \dfrac{10 - x}{100} = \frac{1}{80}\sqrt{x^2 + 4} + \dfrac{10}{100} - \dfrac{x}{100} = \frac{1}{80}\sqrt{x^2 + 4} + 0{,}1 - 0{,}01x$

Dus $t = \frac{1}{80}\sqrt{x^2 + 4} + 0{,}1 - 0{,}01x$.

c $t(x) = \frac{1}{80}\sqrt{x^2 + 4} + 0{,}1 - 0{,}01x$ geeft $t'(x) = \frac{1}{80} \cdot \dfrac{1}{2\sqrt{x^2 + 4}} \cdot 2x - 0{,}01 = \dfrac{x}{80\sqrt{x^2 + 4}} - 0{,}01$

$t'(x) = 0$ geeft $\dfrac{x}{80\sqrt{x^2 + 4}} - 0{,}01 = 0$

$\dfrac{x}{80\sqrt{x^2 + 4}} = 0{,}01$

$x = 0{,}8\sqrt{x^2 + 4}$

kwadrateren geeft

$x^2 = 0{,}64(x^2 + 4)$

$x^2 = 0{,}64x^2 + 2{,}56$

$0{,}36x^2 = 2{,}56$

$x^2 = \frac{64}{9}$

$x = 2\frac{2}{3}$

$t(2\frac{2}{3}) = \frac{1}{80}\sqrt{(2\frac{2}{3})^2 + 4} + 0{,}1 - 0{,}01 \cdot 2\frac{2}{3} = 0{,}115$ uur = 414 seconden

De snelste route is 450 − 414 = 36 seconden sneller dan via C naar B.

52 $f(x) = (ax-2)^4 + \frac{1}{2}ax$ geeft $f'(x) = 4(ax-2)^3 \cdot a + \frac{1}{2}a = 4a(ax-2)^3 + \frac{1}{2}a$

Een extreme waarde voor $x = 3$, dus $f'(3) = 0$.

Dit geeft $4a(a \cdot 3 - 2)^3 + \frac{1}{2}a = 0$

$4a(3a-2)^3 + \frac{1}{2}a = 0$

$a(4(3a-2)^3 + \frac{1}{2}) = 0$

$a = 0 \vee 4(3a-2)^3 + \frac{1}{2} = 0$

$a = 0 \vee 4(3a-2)^3 = -\frac{1}{2}$

$a = 0 \vee (3a-2)^3 = -\frac{1}{8}$

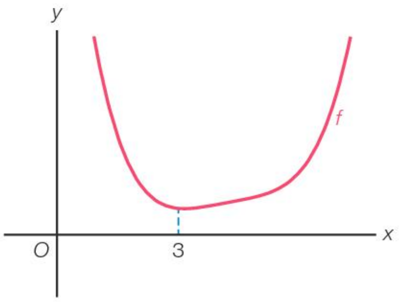
$a = 0 \vee 3a - 2 = -\frac{1}{2}$

$a = 0 \vee 3a = 1\frac{1}{2}$

$a = 0 \vee a = \frac{1}{2}$

$a = 0$ geeft $f(x) = (-2)^4 + 0 = 16$ en deze functie heeft geen extreme waarde voor $x = 3$.

$a = \frac{1}{2}$ geeft $f(x) = (\frac{1}{2}x - 2)^4 + \frac{1}{4}x$



f heeft een extreme waarde voor $x = 3$ als $a = \frac{1}{2}$.

53 $f(x) = x\sqrt{2x+1}$ is het product van de factoren x en $\sqrt{2x+1}$.
Dus volgens de productregel is $f'(x) = [x]' \cdot \sqrt{2x+1} + x \cdot [\sqrt{2x+1}]'$.
De afgeleide van $\sqrt{2x+1}$ bereken je met de kettingregel.

Bladzijde 74

54 **a** $f(x) = x\sqrt{2x+1}$ geeft

$$f'(x) = 1 \cdot \sqrt{2x+1} + x \cdot \frac{1}{2\sqrt{2x+1}} \cdot 2 = \sqrt{2x+1} + \frac{x}{\sqrt{2x+1}} = \frac{2x+1}{\sqrt{2x+1}} + \frac{x}{\sqrt{2x+1}} = \frac{3x+1}{\sqrt{2x+1}}$$

b $g(x) = \dfrac{x+6}{\sqrt{8x+9}}$ geeft

$$g'(x) = \frac{\sqrt{8x+9} \cdot 1 - (x+6) \cdot \frac{1}{2\sqrt{8x+9}} \cdot 8}{(\sqrt{8x+9})^2} = \frac{\sqrt{8x+9} - \frac{4(x+6)}{\sqrt{8x+9}}}{8x+9} \cdot \frac{\sqrt{8x+9}}{\sqrt{8x+9}} = \frac{8x+9-4x-24}{(8x+9)\sqrt{8x+9}} = \frac{4x-15}{(8x+9)\sqrt{8x+9}}$$

55 **a** $f(x) = x\sqrt{3x+4}$ geeft

$$f'(x) = 1 \cdot \sqrt{3x+4} + x \cdot \frac{1}{2\sqrt{3x+4}} \cdot 3 = \sqrt{3x+4} + \frac{3x}{2\sqrt{3x+4}} = \frac{2(3x+4)}{2\sqrt{3x+4}} + \frac{3x}{2\sqrt{3x+4}}$$

$$= \frac{6x+8+3x}{2\sqrt{3x+4}} = \frac{9x+8}{2\sqrt{3x+4}}$$

b $g(x) = x(3x+1)^3$ geeft

$g'(x) = 1 \cdot (3x+1)^3 + x \cdot 3(3x+1)^2 \cdot 3 = (3x+1)^3 + 9x(3x+1)^2$

$= (3x+1)^2(3x+1+9x) = (3x+1)^2(12x+1)$

c $h(x) = \dfrac{\sqrt{x^2+1}}{2x+1}$ geeft

$$h'(x) = \frac{(2x+1) \cdot \dfrac{1}{2\sqrt{x^2+1}} \cdot 2x - \sqrt{x^2+1} \cdot 2}{(2x+1)^2} = \frac{\dfrac{(2x+1) \cdot x}{\sqrt{x^2+1}} - 2\sqrt{x^2+1}}{(2x+1)^2} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$$= \frac{(2x+1) \cdot x - 2(x^2+1)}{(2x+1)^2 \cdot \sqrt{x^2+1}} = \frac{2x^2+x-2x^2-2}{(2x+1)^2 \cdot \sqrt{x^2+1}} = \frac{x-2}{(2x+1)^2 \cdot \sqrt{x^2+1}}$$

56 Voor $-\frac{4}{3} < x < 0$ is $x\sqrt{3x+4} < 0$ en $\sqrt{3x^3+4x^2} > 0$, dus $x\sqrt{3x+4} \neq \sqrt{3x^3+4x^2}$. Dit is de eerste fout.

$2\sqrt{3x^3+4x^2} = 2 \cdot |x| \cdot \sqrt{3x+4} \neq 2x\sqrt{3x+4}$, dus $\dfrac{1}{2\sqrt{3x^3+4x^2}} \cdot (9x^2+8x) \neq \dfrac{9x^2+8x}{2x\sqrt{3x+4}}$.

Dit is de tweede fout.

$\dfrac{9x^2+8x}{2x\sqrt{3x+4}}$ is niet gedefinieerd voor $x = 0$ en $\dfrac{9x+8}{2\sqrt{3x+4}}$ wel, dus $\dfrac{9x^2+8x}{2x\sqrt{3x+4}} \neq \dfrac{9x+8}{2\sqrt{3x+4}}$.

Dit is de derde fout.

57 **a** $f(x) = \frac{1}{2}x\sqrt{3x+1}$ geeft

$$f'(x) = \tfrac{1}{2} \cdot \sqrt{3x+1} + \tfrac{1}{2}x \cdot \frac{1}{2\sqrt{3x+1}} \cdot 3 = \tfrac{1}{2}\sqrt{3x+1} + \frac{3x}{4\sqrt{3x+1}} = \frac{2(3x+1)}{4\sqrt{3x+1}} + \frac{3x}{4\sqrt{3x+1}}$$

$$= \frac{6x+2+3x}{4\sqrt{3x+1}} = \frac{9x+2}{4\sqrt{3x+1}}$$

b Het randpunt is $(-\frac{1}{3}, 0)$.

Stel k: $y = ax + b$ met $a = \text{rc}_l = f'(0) = \dfrac{0+2}{4\sqrt{0+1}} = \frac{1}{2}$.

$\left.\begin{array}{l} y = \frac{1}{2}x + b \\ \text{door } (-\frac{1}{3}, 0) \end{array}\right\} \frac{1}{2} \cdot -\frac{1}{3} + b = 0$

$b = \frac{1}{6}$

Dus k: $y = \frac{1}{2}x + \frac{1}{6}$.

c $f'(x) = 1\frac{3}{8}$ geeft

$\dfrac{9x+2}{4\sqrt{3x+1}} = \dfrac{11}{8}$

$72x + 16 = 44\sqrt{3x+1}$

$18x + 4 = 11\sqrt{3x+1}$

kwadrateren geeft

$324x^2 + 144x + 16 = 121(3x+1)$

$324x^2 + 144x + 16 = 363x + 121$

$324x^2 - 219x - 105 = 0$

$D = (-219)^2 - 4 \cdot 324 \cdot -105 = 184\,041$

$x = \dfrac{219+429}{648} = 1 \vee x = \dfrac{219-429}{648} = -0{,}324...$

vold. vold. niet

$f(1) = 1$, dus $A(1, 1)$.

Bladzijde 75

58 **a** $8 - 2x \geq 0$
$-2x \geq -8$
$x \leq 4$
Dus $D_f = \langle \leftarrow, 4]$.

b $f(x) = x\sqrt{8 - 2x}$ geeft
$$f'(x) = 1 \cdot \sqrt{8-2x} + x \cdot \frac{1}{2\sqrt{8-2x}} \cdot -2 = \sqrt{8-2x} - \frac{x}{\sqrt{8-2x}} = \frac{8-2x}{\sqrt{8-2x}} - \frac{x}{\sqrt{8-2x}} = \frac{8-3x}{\sqrt{8-2x}}$$

c $f'(x) = 0$ geeft $\frac{8-3x}{\sqrt{8-2x}} = 0$
$8 - 3x = 0$
$3x = 8$
$x = \frac{8}{3}$
$$f(\tfrac{8}{3}) = \tfrac{8}{3}\sqrt{\tfrac{8}{3}} = \tfrac{8}{3} \cdot \frac{\sqrt{8}}{\sqrt{3}} = \tfrac{8}{3} \cdot \frac{2\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \tfrac{8}{3} \cdot \frac{2\sqrt{6}}{3} = \tfrac{16}{9}\sqrt{6} = 1\tfrac{7}{9}\sqrt{6}$$
Dus de top is $(2\frac{2}{3}, 1\frac{7}{9}\sqrt{6})$ en $B_f = \langle \leftarrow, 1\frac{7}{9}\sqrt{6}]$.

d $f'(x) = 1$ geeft $\frac{8-3x}{\sqrt{8-2x}} = 1$
$8 - 3x = \sqrt{8-2x}$
kwadrateren geeft
$64 - 48x + 9x^2 = 8 - 2x$
$9x^2 - 46x + 56 = 0$
$D = (-46)^2 - 4 \cdot 9 \cdot 56 = 100$
$x = \frac{46+10}{18} = 3\frac{1}{9} \vee x = \frac{46-10}{18} = 2$
vold. niet vold.
$f(2) = 4$, dus $A(2, 4)$.

59 **a** $f(x) = \frac{4x+4}{\sqrt{x^2+4}}$ geeft
$$f'(x) = \frac{\sqrt{x^2+4} \cdot 4 - (4x+4) \cdot \frac{1}{2\sqrt{x^2+4}} \cdot 2x}{x^2+4} = \frac{4\sqrt{x^2+4} - \frac{x(4x+4)}{\sqrt{x^2+4}}}{x^2+4} \cdot \frac{\sqrt{x^2+4}}{\sqrt{x^2+4}} = \frac{4(x^2+4) - x(4x+4)}{(x^2+4)\sqrt{x^2+4}}$$
$$= \frac{4x^2 + 16 - 4x^2 - 4x}{(x^2+4)\sqrt{x^2+4}} = \frac{16-4x}{(x^2+4)\sqrt{x^2+4}}$$
$f'(x) = 0$ geeft $\frac{16-4x}{(x^2+4)\sqrt{x^2+4}} = 0$
$16 - 4x = 0$
$-4x = -16$
$x = 4$
$$f(4) = \frac{16+4}{\sqrt{4^2+4}} = \frac{20}{\sqrt{20}} = \sqrt{20} = 2\sqrt{5}$$
De coördinaten van de top zijn $(4, 2\sqrt{5})$.

b $f(x) = 0$ geeft $4x + 4 = 0$

$4x = -4$

$x = -1$

Dus $A(-1, 0)$.

Stel k: $y = ax + b$ met $a = f'(-1) = \dfrac{16 + 4}{((-1)^2 + 4)\sqrt{(-1)^2 + 4}} = \dfrac{20}{5\sqrt{5}} = \dfrac{4}{\sqrt{5}} = \frac{4}{5}\sqrt{5}$

$\left.\begin{array}{l} y = \frac{4}{5}\sqrt{5} \cdot x + b \\ \text{door } A(-1, 0) \end{array}\right\} \frac{4}{5}\sqrt{5} \cdot -1 + b = 0$

$b = \frac{4}{5}\sqrt{5}$

Dus k: $y = \frac{4}{5}\sqrt{5} \cdot x + \frac{4}{5}\sqrt{5}$ en het snijpunt met de y-as is $(0, \frac{4}{5}\sqrt{5})$.

$f(0) = \dfrac{0 + 4}{\sqrt{0^2 + 4}} = 2$, dus $B(0, 2)$.

Dus bewering I is niet waar, de lijn k gaat niet door B.

Stel l: $y = ax + b$ met $a = f'(0) = \dfrac{16 - 0}{(0^2 + 4)\sqrt{0^2 + 4}} = 2$.

$\left.\begin{array}{l} y = 2x + b \\ \text{door } B(0, 2) \end{array}\right\} b = 2$

Dus l: $y = 2x + 2$.

l snijden met de x-as, dus $y = 0$ geeft $2x + 2 = 0$

$2x = -2$

$x = -1$

Dus het snijpunt met de x-as is $(-1, 0)$.

Dus bewering II is waar, de lijn l gaat door A.

60 **a** $f(x) = 2x\sqrt{9 - 2x} - 3$ geeft

$f'(x) = 2 \cdot \sqrt{9 - 2x} + 2x \cdot \dfrac{1}{2\sqrt{9 - 2x}} \cdot -2 = 2\sqrt{9 - 2x} - \dfrac{2x}{\sqrt{9 - 2x}} = \dfrac{2(9 - 2x)}{\sqrt{9 - 2x}} - \dfrac{2x}{\sqrt{9 - 2x}}$

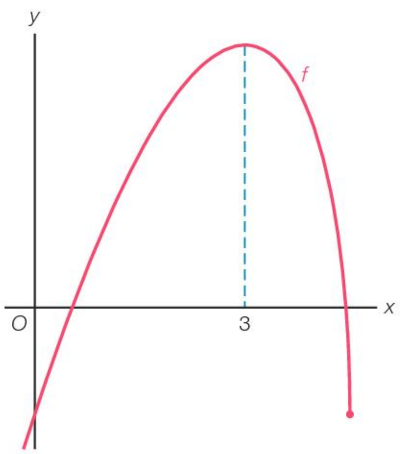
$= \dfrac{18 - 4x - 2x}{\sqrt{9 - 2x}} = \dfrac{18 - 6x}{\sqrt{9 - 2x}}$

$f'(x) = 0$ geeft $\dfrac{18 - 6x}{\sqrt{9 - 2x}} = 0$

$18 - 6x = 0$

$-6x = -18$

$x = 3$



max. is $f(3) = 6\sqrt{3} - 3$.

b $9 - 2x \geq 0$
$-2x \geq -9$
$x \leq 4\frac{1}{2}$
Dus $D_f = \langle\leftarrow, 4\frac{1}{2}]$ en $B_f = \langle\leftarrow, 6\sqrt{3} - 3]$.

c Raaklijn evenwijdig met $y = 1\frac{1}{2}x$, dus $f'(x) = 1\frac{1}{2}$.
$f'(x) = 1\frac{1}{2}$ geeft $\frac{18 - 6x}{\sqrt{9 - 2x}} = \frac{3}{2}$
$36 - 12x = 3\sqrt{9 - 2x}$
$12 - 4x = \sqrt{9 - 2x}$
kwadrateren geeft
$144 - 96x + 16x^2 = 9 - 2x$
$16x^2 - 94x + 135 = 0$
$D = (-94)^2 - 4 \cdot 16 \cdot 135 = 196$
$x = \frac{94 + 14}{32} = 3\frac{3}{8} \vee x = \frac{94 - 14}{32} = 2\frac{1}{2}$
vold. niet vold.
$f(2\frac{1}{2}) = 7$, dus $A(2\frac{1}{2}, 7)$.

61 $f_p(x) = \frac{px}{\sqrt{x^2 + 9}}$ geeft $f_p'(x) = \frac{\sqrt{x^2+9} \cdot p - px \cdot \frac{1}{2\sqrt{x^2+9}} \cdot 2x}{x^2 + 9} = \frac{p(x^2 + 9) - px^2}{(x^2 + 9)\sqrt{x^2 + 9}} = \frac{9p}{(x^2 + 9)\sqrt{x^2 + 9}}$
$f_p'(0) = \frac{9p}{9\sqrt{9}} = \frac{1}{3}p$
$f_p'(0) = 1$ geeft $\frac{1}{3}p = 1$
$p = 3$
$f_3(0) = 0$, dus voor $p = 3$ raakt de lijn $y = x$ de grafiek van f_p in de oorsprong.

6.4 Functies met parameters

Bladzijde 77

62 Raaklijn k heeft richtingscoëfficiënt $2\frac{1}{2}$, dus $f_p'(x) = 2\frac{1}{2}$.
Raken in het punt A met $x_A = 3$, dus $f_p'(3) = 2\frac{1}{2}$.
$f_p(x) = \frac{1}{4}x^2 + px + 2$ geeft $f_p'(x) = \frac{1}{2}x + p$
$f_p'(3) = 2\frac{1}{2}$ geeft $\frac{1}{2} \cdot 3 + p = 2\frac{1}{2}$
$1\frac{1}{2} + p = 2\frac{1}{2}$
$p = 1$

63 **a** $f_p(x) = (x^2 + p) \cdot \sqrt{x} = x^{2\frac{1}{2}} + p\sqrt{x}$ geeft $f_p'(x) = 2\frac{1}{2}x^{1\frac{1}{2}} + \frac{p}{2\sqrt{x}} = \frac{5x\sqrt{x}}{2} + \frac{p}{2\sqrt{x}} = \frac{5x^2}{2\sqrt{x}} + \frac{p}{2\sqrt{x}} = \frac{5x^2 + p}{2\sqrt{x}}$

b $f_p'(4) = rc_k$
$\frac{5 \cdot 4^2 + p}{2\sqrt{4}} = 18$
$\frac{80 + p}{4} = 18$
$80 + p = 72$
$p = -8$

c $f_{-8}(4) = (16 - 8) \cdot \sqrt{4} = 16$, dus $A(4, 16)$.
$y = 18x + q$ door $A(4, 16)$ } $18 \cdot 4 + q = 16$
$72 + q = 16$
$q = -56$

64 $f_p(x) = \sqrt{2x^2 + p}$ geeft $f_p'(x) = \frac{1}{2\sqrt{2x^2+p}} \cdot 4x = \frac{2x}{\sqrt{2x^2+p}}$

$f_p'(1) = \frac{2}{3}$ geeft $\frac{2}{\sqrt{2+p}} = \frac{2}{3}$

$\sqrt{2+p} = 3$

$2 + p = 9$

$p = 7$

$\left.\begin{array}{l} y = \frac{2}{3}x + q \\ f_7(1) = 3, \text{ dus } A(1, 3) \end{array}\right\} \frac{2}{3} + q = 3$

$q = 2\frac{1}{3}$

Dus $p = 7$ en $q = 2\frac{1}{3}$.

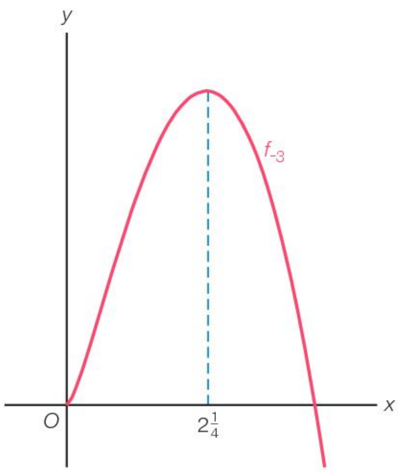
65 **a** $f_p(x) = 6x\sqrt{x} + px^2 = 6x^{1\frac{1}{2}} + px^2$ geeft $f_p'(x) = 9x^{\frac{1}{2}} + 2px = 9\sqrt{x} + 2px$

$f_p'(2\frac{1}{4}) = 0$ geeft $9 \cdot \sqrt{2\frac{1}{4}} + 2p \cdot 2\frac{1}{4} = 0$

$9 \cdot 1\frac{1}{2} + 4\frac{1}{2}p = 0$

$4\frac{1}{2}p = -13\frac{1}{2}$

$p = -3$



Dus f_p heeft een maximum voor $x = 2\frac{1}{4}$ als $p = -3$.

b k: $y = 5x + q$ raakt de grafiek van f_p in A met $x_A = 1$ geeft $f_p'(1) = \text{rc}_k$

$9 \cdot \sqrt{1} + 2p \cdot 1 = 5$

$9 + 2p = 5$

$2p = -4$

$p = -2$

$f_{-2}(x) = 6x\sqrt{x} - 2x^2$

$\left.\begin{array}{l} k: y = 5x + q \\ f_{-2}(1) = 4, \text{ dus } A(1, 4) \end{array}\right\} 5 \cdot 1 + q = 4$

$q = -1$

Dus $p = -2$ en $q = -1$.

66 **a** $f_p(x) = \frac{4x + p}{x^2 + 1}$ geeft

$$f_p'(x) = \frac{(x^2 + 1) \cdot 4 - (4x + p) \cdot 2x}{(x^2 + 1)^2} = \frac{4x^2 + 4 - 8x^2 - 2px}{(x^2 + 1)^2} = \frac{-4x^2 - 2px + 4}{(x^2 + 1)^2}$$

$$a = f_p'(0) = \frac{-4 \cdot 0^2 - 2p \cdot 0 + 4}{(0^2 + 1)^2} = \frac{4}{1} = 4$$

k: $y = 4x + 4$ snijdt de y-as in $(0, 4)$
$f_p(0) = p$, dus door $(0, p)$ } $p = 4$

Dus $a = 4$ en $p = 4$.

b l: $y = -x + b$ raakt de grafiek in A met $x_A = -1$, dus $f_p'(-1) = rc_l$.

$$f_p'(-1) = -1 \text{ geeft } \frac{-4 \cdot (-1)^2 - 2p \cdot -1 + 4}{((-1)^2 + 1)^2} = -1$$

$$\frac{-4 + 2p + 4}{(1 + 1)^2} = -1$$

$$\frac{2p}{4} = -1$$

$2p = -4$

$p = -2$

$$f_{-2}(x) = \frac{4x - 2}{x^2 + 1}$$

l: $y = -x + b$
$f_{-2}(-1) = \frac{4 \cdot -1 - 2}{(-1)^2 + 1} = -3$, dus $A(-1, -3)$ } $1 + b = -3$, $b = -4$

Dus l: $y = -x - 4$.

c Een extreme waarde voor $x = 2$ geeft $f_p'(2) = 0$

$$\frac{-4 \cdot 2^2 - 2p \cdot 2 + 4}{(2^2 + 1)^2} = 0$$

$-12 - 4p = 0$

$-4p = 12$

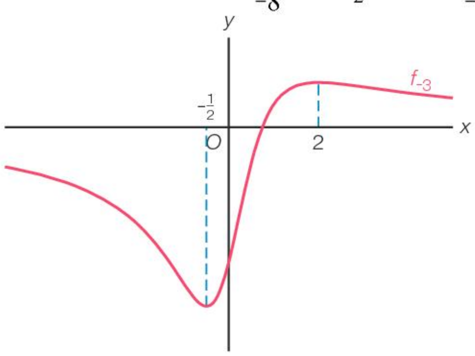
$p = -3$

$$f_{-3}'(x) = \frac{-4x^2 + 6x + 4}{(x^2 + 1)^2}$$

$f_{-3}'(x) = 0$ geeft $-4x^2 + 6x + 4 = 0$

$D = 6^2 - 4 \cdot -4 \cdot 4 = 100$

$$x = \frac{-6 + 10}{-8} = -\tfrac{1}{2} \vee x = \frac{-6 - 10}{-8} = 2$$



min. is $f_{-3}(-\tfrac{1}{2}) = -4$.

67 **a** $f_p(x) = \frac{1}{3}x^3 + px^2 - 3x - p$ geeft $f_p{}'(x) = x^2 + 2px - 3$

f_p heeft twee extremen als $f_p{}'(x) = 0$ twee oplossingen heeft.

$D = (2p)^2 - 4 \cdot 1 \cdot -3 = 4p^2 + 12$

$4p^2 + 12$ is voor elke p groter dan nul, dus $f_p{}'(x) = 0$ heeft voor elke p twee oplossingen.

b f_p heeft een extreme waarde voor $x = 3$, dus $f_p{}'(3) = 0$

$3^2 + 2p \cdot 3 - 3 = 0$

$9 + 6p - 3 = 0$

$6p = -6$

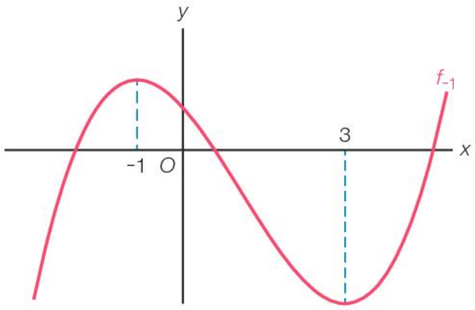
$p = -1$

$f_{-1}(x) = \frac{1}{3}x^3 - x^2 - 3x + 1$ en $f_{-1}{}'(x) = x^2 - 2x - 3$

$f_{-1}{}'(x) = 0$ geeft $x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1 \vee x = 3$



max. is $f_{-1}(-1) = 2\frac{2}{3}$.

c $l: y = -x + q$ raakt de grafiek van f_p in B met $x_B = -2$, dus $f_p{}'(-2) = \text{rc}_l$

$(-2)^2 + 2p \cdot -2 - 3 = -1$

$-4p = -2$

$p = \frac{1}{2}$

$f_{\frac{1}{2}}(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 3x - \frac{1}{2}$

$$\left.\begin{array}{l} l: y = -x + q \\ f_{\frac{1}{2}}(-2) = 4\frac{5}{6}, \text{ dus } B(-2, 4\frac{5}{6}) \end{array}\right\} 2 + q = 4\frac{5}{6}$$

$q = 2\frac{5}{6}$

Dus $p = \frac{1}{2}$ en $q = 2\frac{5}{6}$.

68 $f_{p,q}(2) = 2$ geeft $\dfrac{2+p}{\sqrt{4+q}} = 2$

$2 + p = 2\sqrt{4+q}$

$p = -2 + 2\sqrt{4+q}$

$f_{p,q}(x) = \dfrac{x+p}{\sqrt{x^2+q}}$ geeft

$$f_{p,q}{}'(x) = \frac{\sqrt{x^2+q} \cdot 1 - (x+p) \cdot \dfrac{1}{2\sqrt{x^2+q}} \cdot 2x}{x^2+q} = \frac{\sqrt{x^2+q} - \dfrac{x(x+p)}{\sqrt{x^2+q}}}{x^2+q} = \frac{x^2+q-x(x+p)}{(x^2+q)\sqrt{x^2+q}} = \frac{q-px}{(x^2+q)\sqrt{x^2+q}}$$

$f_{p,q}{}'(2) = -\frac{1}{9}$ geeft $\dfrac{q-2p}{(4+q)\sqrt{4+q}} = -\frac{1}{9}$

$q - 2p = -\frac{1}{9}(4+q)\sqrt{4+q}$

$-2p = -q - \frac{1}{9}(4+q)\sqrt{4+q}$

$p = \frac{1}{2}q + \frac{1}{18}(4+q)\sqrt{4+q}$

Voer in $y_1 = -2 + 2\sqrt{4+x}$ en $y_2 = \frac{1}{2}x + \frac{1}{18}(4+x)\sqrt{4+x}$.

De optie snijpunt geeft $x = -4$ en $y = -2$ en ook $x = 5$ en $y = 4$.

$p = -2$ en $q = -4$ voldoet niet, want $A(2, 2)$ ligt niet op de grafiek van $f_{-2,-4}(x) = \dfrac{x-2}{\sqrt{x^2-4}}$.

Dus $p = 4$ en $q = 5$.

69 $f_p'(x) = -\frac{1}{4}x^2 + px + 3$ geeft $f_p'(x) = -\frac{1}{2}x + p$

$f_p'(x) = 0$ geeft $-\frac{1}{2}x + p = 0$

$p = \frac{1}{2}x$

Bladzijde 80

70 **a** Je hoeft alleen de formule op te stellen van de kromme waarop alle toppen liggen. Je hoeft niet aan te tonen dat alle punten van die kromme een top zijn.

b $f_p'(0) = 3 \cdot 0^2 + 2p \cdot 0 + 2 = 2 \neq 0$, dus voor elke waarde van p geldt dat $f_p'(0) \neq 0$, dus (0, 3) is geen top van een van de grafieken van f_p.

c De formule van de kromme waarop alle toppen liggen, wil niet zeggen dat elk punt op de kromme per se een top is. Het punt (0, 3) ligt dus wel op de kromme, maar hoeft dus geen top te zijn.

Bladzijde 81

71 $f_p(x) = \frac{1}{3}x^3 + px^2 + 3x + 5$ geeft $f_p'(x) = x^2 + 2px + 3$

$f_p'(x) = 0$ geeft $x^2 + 2px + 3 = 0$

$2px = -x^2 - 3$

$$\left.\begin{array}{l}\text{voor } x \neq 0 \text{ geldt } p = \dfrac{-x^2-3}{2x} \\ y = \frac{1}{3}x^3 + px^2 + 3x + 5\end{array}\right\} y = \frac{1}{3}x^3 + \frac{-x^2-3}{2x} \cdot x^2 + 3x + 5$$

$y = \frac{1}{3}x^3 - \frac{1}{2}x^3 - 1\frac{1}{2}x + 3x + 5$

$y = -\frac{1}{6}x^3 + 1\frac{1}{2}x + 5$

Dus de formule van de kromme waarop alle toppen liggen is $y = -\frac{1}{6}x^3 + 1\frac{1}{2}x + 5$.

72 **a** Extreme waarde voor $x = 1$, dus $f_p'(1) = 0$.

$f_p(x) = \dfrac{x+p}{x^2+4}$ geeft $f_p'(x) = \dfrac{(x^2+4) \cdot 1 - (x+p) \cdot 2x}{(x^2+4)^2} = \dfrac{x^2 + 4 - 2x^2 - 2px}{(x^2+4)^2} = \dfrac{-x^2 - 2px + 4}{(x^2+4)^2}$

$f_p'(1) = 0$ geeft $\dfrac{-1^2 - 2p \cdot 1 + 4}{(1^2+4)^2} = 0$

$-1 - 2p + 4 = 0$

$-2p = -3$

$p = 1\frac{1}{2}$

$f_{1\frac{1}{2}}'(x) = \dfrac{-x^2 - 3x + 4}{(x^2+4)^2}$

$f_{1\frac{1}{2}}'(x) = 0$ geeft $\dfrac{-x^2 - 3x + 4}{(x^2+4)^2} = 0$

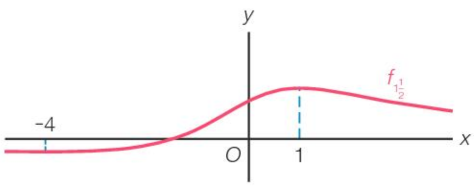
$-x^2 - 3x + 4 = 0$

$x^2 + 3x - 4 = 0$

$(x-1)(x+4) = 0$

$x = 1 \vee x = -4$

$f_{1\frac{1}{2}}(-4) = \dfrac{-4 + 1\frac{1}{2}}{(-4)^2 + 4} = \dfrac{-2\frac{1}{2}}{20} = -\frac{1}{8}$



Dus $p = 1\frac{1}{2}$ en min. is $f_{1\frac{1}{2}}(-4) = -\frac{1}{8}$.

b $f_p'(x) = 0$ geeft $\dfrac{-x^2 - 2px + 4}{(x^2+4)^2} = 0$

$-x^2 - 2px + 4 = 0$

$-2px = x^2 - 4$

$\left.\begin{array}{l} \text{voor } x \neq 0 \text{ geldt } p = \dfrac{-x^2+4}{2x} \\ y = \dfrac{x+p}{x^2+4} \end{array}\right\} y = \dfrac{x + \dfrac{-x^2+4}{2x}}{x^2+4}$

$y = \dfrac{2x^2 - x^2 + 4}{2x(x^2+4)}$

$y = \dfrac{x^2+4}{2x(x^2+4)}$

$y = \dfrac{1}{2x}$

Dus alle toppen liggen op de kromme $y = \dfrac{1}{2x}$.

73 **a** $f_p(x) = (p - x^2) \cdot \sqrt{x} = p\sqrt{x} - x^{2\frac{1}{2}}$ geeft $f_p'(x) = \dfrac{p}{2\sqrt{x}} - 2\frac{1}{2}x^{1\frac{1}{2}} = \dfrac{p}{2\sqrt{x}} - \dfrac{5x\sqrt{x}}{2} = \dfrac{p - 5x^2}{2\sqrt{x}}$

$f_p'(x) = 0$ geeft $\dfrac{p - 5x^2}{2\sqrt{x}} = 0$

$p - 5x^2 = 0$

$\left.\begin{array}{l} p = 5x^2 \\ y = (p - x^2) \cdot \sqrt{x} \end{array}\right\} \begin{array}{l} y = (5x^2 - x^2) \cdot \sqrt{x} \\ y = 4x^2 \cdot \sqrt{x} \end{array}$

Dus de formule van de kromme waarop alle toppen liggen is $y = 4x^2 \cdot \sqrt{x}$.

b $y = 8$

$4x^2 \cdot \sqrt{x} = 8$

$x^2\sqrt{x} = 2$

$x^5 = 4$

$\left.\begin{array}{l} x = 4^{\frac{1}{5}} \\ p = 5x^2 \end{array}\right\} p = 5 \cdot (4^{\frac{1}{5}})^2 = 5 \cdot 4^{\frac{2}{5}} = 5 \cdot \sqrt[5]{16}$

74 **a** $f(1) = \frac{1}{2} \cdot 1^2 + 1 + 1\frac{1}{2} = 3$ en $g(1) = -1^2 + 4 \cdot 1 = 3$

Dus $A(1, 3)$ ligt op de grafiek van f en op de grafiek van g.

b $f(x) = \frac{1}{2}x^2 + x + 1\frac{1}{2}$ geeft $f'(x) = x + 1$

Stel k: $y = ax + b$ met $a = f'(1) = 2$.

$\left.\begin{array}{l} k: y = 2x + b \\ \text{door } A(1, 3) \end{array}\right\} \begin{array}{l} 2 + b = 3 \\ b = 1 \end{array}$

Dus k: $y = 2x + 1$.

$g(x) = -x^2 + 4x$ geeft $g'(x) = -2x + 4$

Stel l: $y = ax + b$ met $a = g'(1) = 2$.

$\left.\begin{array}{l} l: y = 2x + b \\ \text{door } A(1, 3) \end{array}\right\} \begin{array}{l} 2 + b = 3 \\ b = 1 \end{array}$

Dus l: $y = 2x + 1$.

c A is het punt waar de raaklijnen van de grafieken van f en g samenvallen.

Bladzijde 82

75 De grafieken hebben voor $x = -3$ geen punt gemeenschappelijk, dus ze raken elkaar niet. Wel geldt dat voor $x = -3$ de raaklijn van de grafiek van f en de raaklijn van de grafiek van g evenwijdig zijn.

76 **a** $f(x) = x^3 + 4x^2 + 2x + 1$ geeft $f'(x) = 3x^2 + 8x + 2$
$g(x) = x^2 + 11x + 28$ geeft $g'(x) = 2x + 11$
$f(x) = g(x) \wedge f'(x) = g'(x)$
$x^3 + 4x^2 + 2x + 1 = x^2 + 11x + 28 \wedge 3x^2 + 8x + 2 = 2x + 11$
$x^3 + 3x^2 - 9x - 27 = 0 \wedge 3x^2 + 6x - 9 = 0$
$3x^2 + 6x - 9 = 0$ geeft $x^2 + 2x - 3 = 0$
$(x - 1)(x + 3) = 0$
$x = 1 \vee x = -3$
Substitutie van $x = 1$ in $x^3 + 3x^2 - 9x - 27 = 0$ geeft
$1 + 3 - 9 - 27 = 0$ klopt niet.
Substitutie van $x = -3$ in $x^3 + 3x^2 - 9x - 27 = 0$ geeft
$-27 + 27 + 27 - 27 = 0$ klopt, dus de grafieken raken elkaar voor $x = -3$.
$f(-3) = 4$, dus het raakpunt is $(-3, 4)$.

b $g'(-3) = -6 + 11 = 5$
$\left.\begin{array}{l} y = 5x + b \\ \text{door } (-3, 4) \end{array}\right\} \begin{array}{l} 5 \cdot -3 + b = 4 \\ b = 19 \end{array}$
Dus de gemeenschappelijke raaklijn is $y = 5x + 19$.

77 **a** $f(x) = \sqrt{2x}$ geeft $f'(x) = \dfrac{1}{2\sqrt{2x}} \cdot 2 = \dfrac{1}{\sqrt{2x}}$
$g_1(x) = x^2 + 1$ geeft $g_1'(x) = 2x$
$f(x) = g_1(x) \wedge f'(x) = g_1'(x)$
$\sqrt{2x} = x^2 + 1 \wedge \dfrac{1}{\sqrt{2x}} = 2x$
$\sqrt{2x} = x^2 + 1 \wedge 2x\sqrt{2x} = 1$
$2x\sqrt{2x} = 1$ kwadrateren geeft $(2x)^3 = 1$
$2x = 1$
$x = \frac{1}{2}$
Substitutie van $x = \frac{1}{2}$ in $\sqrt{2x} = x^2 + 1$ geeft
$\sqrt{1} = \frac{1}{4} + 1$ klopt niet.
Dus de grafieken van f en g_1 raken elkaar niet.

b $f(x) = g_p(x) \wedge f'(x) = g_p'(x)$
$\sqrt{2x} = x^2 + p \wedge \dfrac{1}{\sqrt{2x}} = 2x$
$x = \frac{1}{2}$
Substitutie van $x = \frac{1}{2}$ in $\sqrt{2x} = x^2 + p$ geeft $\sqrt{1} = \frac{1}{4} + p$
$p = \frac{3}{4}$
Dus de grafieken raken elkaar voor $p = \frac{3}{4}$.

78 **a** De grafieken van $f(x) = x^3 - 2x^2$ en $g(x) = \frac{1}{3}x^3 - 2\frac{2}{3}$ raken elkaar als

$f(x) = g(x) \wedge f'(x) = g'(x)$

$x^3 - 2x^2 = \frac{1}{3}x^3 - 2\frac{2}{3} \wedge 3x^2 - 4x = x^2$

$3x^2 - 4x = x^2$ geeft $2x^2 - 4x = 0$

$2x(x - 2) = 0$

$x = 0 \vee x = 2$

$x = 0 \wedge x^3 - 2x^2 = \frac{1}{3}x^3 - 2\frac{2}{3}$ geeft $0 = -2\frac{2}{3}$, dus voldoet niet.

$x = 2 \wedge x^3 - 2x^2 = \frac{1}{3}x^3 - 2\frac{2}{3}$ geeft $8 - 8 = \frac{1}{3} \cdot 8 - 2\frac{2}{3}$ klopt.

Dus de grafieken raken elkaar voor $x = 2$.

$f(2) = 0$, dus het raakpunt is $(2, 0)$.

b $h(x) = -x^2 + 8x - 12$ geeft $h'(x) = -2x + 8$

$k_p(x) = x^2 + px$ geeft $k_p'(x) = 2x + p$

$h(x) = k_p(x) \wedge h'(x) = k_p'(x)$

$-x^2 + 8x - 12 = x^2 + px \wedge -2x + 8 = 2x + p$

$-2x^2 + (8 - p)x - 12 = 0 \wedge -4x + 8 = p$

Substitutie van $p = -4x + 8$ in $-2x^2 + (8 - p)x - 12 = 0$ geeft $-2x^2 + (8 - (-4x + 8))x - 12 = 0$

$-2x^2 + (8 + 4x - 8)x - 12 = 0$

$-2x^2 + 4x^2 - 12 = 0$

$2x^2 = 12$

$x^2 = 6$

$x = \sqrt{6} \vee x = -\sqrt{6}$

$\left.\begin{array}{l} x = \sqrt{6} \\ p = -4x + 8 \end{array}\right\} p = -4\sqrt{6} + 8$

$\left.\begin{array}{l} x = -\sqrt{6} \\ p = -4x + 8 \end{array}\right\} p = 4\sqrt{6} + 8$

Dus $p = -4\sqrt{6} + 8 \vee p = 4\sqrt{6} + 8$.

79 $f_{p,q}(x) = \dfrac{x + p}{\sqrt{x^2 + q}}$ geeft

$$f_{p,q}'(x) = \frac{\sqrt{x^2+q} \cdot 1 - (x+p) \cdot \dfrac{1}{2\sqrt{x^2+q}} \cdot 2x}{x^2+q} = \frac{\sqrt{x^2+q} - \dfrac{x(x+p)}{\sqrt{x^2+q}}}{x^2+q} = \frac{x^2+q-x(x+p)}{(x^2+q)\sqrt{x^2+q}} = \frac{q-px}{(x^2+q)\sqrt{x^2+q}}$$

$g(x) = -1\frac{3}{8}x^2 + 2\frac{7}{8}x$ geeft $g'(x) = -2\frac{3}{4}x + 2\frac{7}{8}$

De grafieken van $f_{p,q}$ en g raken elkaar voor $x = 1$, dus

$f_{p,q}(1) = g(1) \wedge f_{p,q}'(1) = g'(1)$

$\dfrac{1+p}{\sqrt{1+q}} = -1\frac{3}{8} + 2\frac{7}{8} \wedge \dfrac{q-p}{(1+q)\sqrt{1+q}} = -2\frac{3}{4} + 2\frac{7}{8}$

$\dfrac{1+p}{\sqrt{1+q}} = \frac{3}{2} \wedge \dfrac{q-p}{(1+q)\sqrt{1+q}} = \frac{1}{8}$

$2 + 2p = 3\sqrt{1+q} \wedge 8q - 8p = (1+q)\sqrt{1+q}$

$2p = -2 + 3\sqrt{1+q} \wedge -8p = -8q + (1+q)\sqrt{1+q}$

$p = -1 + 1\frac{1}{2}\sqrt{1+q} \wedge p = q - \frac{1}{8}(1+q)\sqrt{1+q}$

Hieruit volgt $-1 + 1\frac{1}{2}\sqrt{1+q} = q - \frac{1}{8}(1+q)\sqrt{1+q}$.

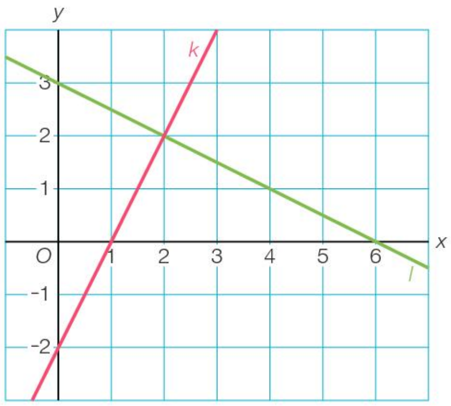
Voer in $y_1 = -1 + 1\frac{1}{2}\sqrt{1+x}$ en $y_2 = x - \frac{1}{8}(1+x)\sqrt{1+x}$.

De optie snijpunt geeft $x = -1$ en $y = -1$, $x = 3$ en $y = 2$, en $x = 35$ en $y = 8$.

$p = -1$ en $q = -1$ voldoet niet, want het punt A met $x_A = 1$ ligt niet op de grafiek van $f_{-1,-1}(x) = \dfrac{x-1}{\sqrt{x^2-1}}$.

Dus $p = 2$ en $q = 3$ of $p = 8$ en $q = 35$.

80 **a**



b Ja, bij de lijn k ga je 1 naar rechts en 2 omhoog, bij de lijn l ga je 2 naar rechts en 1 omlaag.

c m loodrecht op k, oftewel m evenwijdig met l.

Stel m: $y = ax + b$ met $a = \text{rc}_m = \text{rc}_l = -\frac{1}{2}$.

$$\left.\begin{array}{l} y = -\frac{1}{2}x + b \\ \text{door } A(4, 0) \end{array}\right\} \begin{array}{l} -\frac{1}{2} \cdot 4 + b = 0 \\ b = 2 \end{array}$$

Dus m: $y = -\frac{1}{2}x + 2$.

d $\text{rc}_p = \frac{1}{4}$, dus bij p ga je 4 naar rechts en 1 omhoog.
Dus bij n ga je 1 naar rechts en 4 omlaag.
Stel n: $y = ax + b$ met $a = \text{rc}_n = -4$.

$$\left.\begin{array}{l} y = -4x + b \\ \text{door } B(1, 3) \end{array}\right\} \begin{array}{l} -4 \cdot 1 + b = 3 \\ b = 7 \end{array}$$

Dus n: $y = -4x + 7$.

Bladzijde 84

81 **a** $f(x) = \sqrt{x}$ geeft $f'(x) = \dfrac{1}{2\sqrt{x}}$ en $g(x) = -\frac{1}{2}x^2 + 10$ geeft $g'(x) = -x$.

$f(x) = g(x) \wedge f'(x) \cdot g'(x) = -1$

$\sqrt{x} = -\frac{1}{2}x^2 + 10 \wedge \dfrac{1}{2\sqrt{x}} \cdot -x = -1$

$\dfrac{1}{2\sqrt{x}} \cdot -x = -1$ geeft $\dfrac{x}{2\sqrt{x}} = 1$

$\frac{1}{2}\sqrt{x} = 1$

$\sqrt{x} = 2$

$x = 4$

Substitutie van $x = 4$ in $\sqrt{x} = -\frac{1}{2}x^2 + 10$ geeft $\sqrt{4} = -\frac{1}{2} \cdot 4^2 + 10$

$2 = -8 + 10$

$2 = 2$

Dit klopt, dus de grafieken snijden elkaar loodrecht.

b $h_p(x) = p\sqrt{x}$ geeft $h_p'(x) = \dfrac{p}{2\sqrt{x}}$ en $k(x) = -\frac{1}{6}x^2 + 8\frac{2}{3}$ geeft $k'(x) = -\frac{1}{3}x$.

$h_p(x) = k(x) \wedge h_p'(x) \cdot k'(x) = -1$

$p\sqrt{x} = -\frac{1}{6}x^2 + 8\frac{2}{3} \wedge \dfrac{p}{2\sqrt{x}} \cdot -\frac{1}{3}x = -1$

$p\sqrt{x} = -\frac{1}{6}x^2 + 8\frac{2}{3} \wedge -\frac{1}{6}p\sqrt{x} = -1$

$p\sqrt{x} = -\frac{1}{6}x^2 + 8\frac{2}{3} \wedge p\sqrt{x} = 6$

Hieruit volgt $6 = -\frac{1}{6}x^2 + 8\frac{2}{3}$

$\frac{1}{6}x^2 = 2\frac{2}{3}$

$x^2 = 16$

$x = 4 \vee x = -4$

$x = 4$ geeft $p\sqrt{4} = 6$

$2p = 6$

$p = 3$

$x = -4$ voldoet niet.

$k(4) = 6$

Dus $p = 3$ en het snijpunt is $(4, 6)$.

Bladzijde 85

82 **a** $f(x) = x^2 - 4x$ geeft $f'(x) = 2x - 4$

$f'(5) = 10 - 4 = 6$

$\text{rc}_k \cdot 6 = -1$, dus $\text{rc}_k = -\frac{1}{6}$.

$\left.\begin{array}{l} k\text{: } y = -\frac{1}{6}x + b \\ \text{door } A(5, 5) \end{array}\right\} -\frac{5}{6} + b = 5$

$b = 5\frac{5}{6}$

Dus k: $y = -\frac{1}{6}x + 5\frac{5}{6}$.

b $g(x) = \dfrac{2x-1}{x+2}$ geeft $g'(x) = \dfrac{(x+2) \cdot 2 - (2x-1) \cdot 1}{(x+2)^2} = \dfrac{2x + 4 - 2x + 1}{(x+2)^2} = \dfrac{5}{(x+2)^2}$

$g(x) = -5x + p \wedge g'(x) \cdot \text{rc}_l = -1$

$\dfrac{2x-1}{x+2} = -5x + p \wedge \dfrac{5}{(x+2)^2} \cdot -5 = -1$

$p = 5x + \dfrac{2x-1}{x+2} \wedge \dfrac{25}{(x+2)^2} = 1$

$\dfrac{25}{(x+2)^2} = 1$ geeft $(x+2)^2 = 25$

$x + 2 = 5 \vee x + 2 = -5$

$x = 3 \vee x = -7$

$\left.\begin{array}{l} x = 3 \\ p = 5x + \dfrac{2x-1}{x+2} \end{array}\right\} p = 15 + \dfrac{5}{5} = 16$

$\left.\begin{array}{l} x = -7 \\ p = 5x + \dfrac{2x-1}{x+2} \end{array}\right\} p = -35 + \dfrac{-15}{-5} = -32$

Dus $p = 16 \vee p = -32$.

83 **a** $f(x) = x^2 - x$ geeft $f'(x) = 2x - 1$ en $g_p(x) = \frac{p}{x} = px^{-1}$ geeft $g_p'(x) = -px^{-2} = \frac{-p}{x^2}$.

$f(x) = g_p(x) \wedge f'(x) = g_p'(x)$

$x^2 - x = \frac{p}{x} \wedge (2x - 1) = \frac{-p}{x^2}$

$x^3 - x^2 = p \wedge 2x^3 - x^2 = -p$

$p = x^3 - x^2 \wedge p = -2x^3 + x^2$

$x^3 - x^2 = -2x^3 + x^2$

$3x^3 - 2x^2 = 0$

$x^2(3x - 2) = 0$

$x = 0 \quad \vee \quad x = \frac{2}{3}$

vold. niet vold.

$\left.\begin{array}{l} x = \frac{2}{3} \\ p = x^3 - x^2 \end{array}\right\} p = \frac{8}{27} - \frac{4}{9} = -\frac{4}{27}$

$f(\frac{2}{3}) = \frac{4}{9} - \frac{2}{3} = -\frac{2}{9}$

Dus $p = -\frac{4}{27}$ en $A(\frac{2}{3}, -\frac{2}{9})$.

b $f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = -1$

$x^2 - x = \frac{p}{x} \wedge (2x - 1) \cdot \frac{-p}{x^2} = -1$

$x^3 - x^2 = p \wedge p(2x - 1) = x^2$

$(x^3 - x^2)(2x - 1) = x^2$

$x^2(x - 1)(2x - 1) = x^2$

$x \neq 0$, dus $(x - 1)(2x - 1) = 1$

$2x^2 - 3x + 1 = 1$

$2x^2 - 3x = 0$

$x(2x - 3) = 0$

$x = 0 \quad \vee \quad x = \frac{3}{2}$

vold. niet vold.

$\left.\begin{array}{l} x = \frac{3}{2} \\ p = x^3 - x^2 \end{array}\right\} p = \frac{27}{8} - \frac{9}{4} = \frac{9}{8}$

$f(\frac{3}{2}) = \frac{9}{4} - \frac{3}{2} = \frac{3}{4}$

Dus $p = 1\frac{1}{8}$ en $B(1\frac{1}{2}, \frac{3}{4})$.

84 $f_{p,q}(x) = \frac{x + p}{\sqrt{x^2 + q}}$ geeft $f_{p,q}'(x) = \frac{\sqrt{x^2 + q} \cdot 1 - (x + p) \cdot \frac{1}{2\sqrt{x^2 + q}} \cdot 2x}{(\sqrt{x^2 + q})^2} = \frac{x^2 + q - x(x + p)}{(x^2 + q)\sqrt{x^2 + q}} = \frac{q - px}{(x^2 + q)\sqrt{x^2 + q}}$

$g(x) = x^2 - 3x - 1\frac{1}{2}$ geeft $g'(x) = 2x - 3$

De grafieken van $f_{p,q}$ en g snijden elkaar loodrecht voor $x = 4$, dus

$f_{p,q}(4) = g(4) \wedge f_{p,q}'(4) \cdot g'(4) = -1$

$\frac{4 + p}{\sqrt{16 + q}} = 2\frac{1}{2} \wedge \frac{q - 4p}{(16 + q)\sqrt{16 + q}} \cdot 5 = -1$

$4 + p = 2\frac{1}{2}\sqrt{16 + q} \wedge 5q - 20p = -(16 + q)\sqrt{16 + q}$

$p = -4 + 2\frac{1}{2}\sqrt{16 + q} \wedge -20p = -5q - (16 + q)\sqrt{16 + q}$

$p = -4 + 2\frac{1}{2}\sqrt{16 + q} \wedge p = \frac{1}{4}q + \frac{1}{20}(16 + q)\sqrt{16 + q}$

Hieruit volgt $-4 + 2\frac{1}{2}\sqrt{16 + q} = \frac{1}{4}q + \frac{1}{20}(16 + q)\sqrt{16 + q}$.

Voer in $y_1 = -4 + 2\frac{1}{2}\sqrt{16 + x}$ en $y_2 = \frac{1}{4}x + \frac{1}{20}(16 + x)\sqrt{16 + x}$.

De optie snijpunt geeft $x = -16$ en $y = -4$, en $x = 9$ en $y = 8\frac{1}{2}$.

$p = -4$ en $q = -16$ voldoet niet, want het punt A met $x_A = 4$ ligt niet op de grafiek van $f_{-4,-16}(x) = \frac{x - 4}{\sqrt{x^2 - 16}}$.

Dus $p = 8\frac{1}{2}$ en $q = 9$.

Diagnostische toets

Bladzijde 88

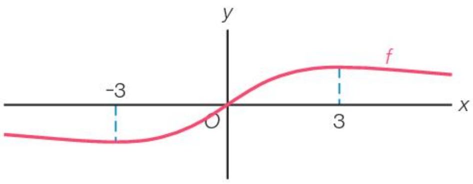
1 $f(x) = \frac{6x}{x^2+9}$ geeft $f'(x) = \frac{(x^2+9) \cdot 6 - 6x \cdot 2x}{(x^2+9)^2} = \frac{6x^2+54-12x^2}{(x^2+9)^2} = \frac{-6x^2+54}{(x^2+9)^2}$

$f'(x) = 0$ geeft $-6x^2 + 54 = 0$

$-6x^2 = -54$

$x^2 = 9$

$x = 3 \vee x = -3$



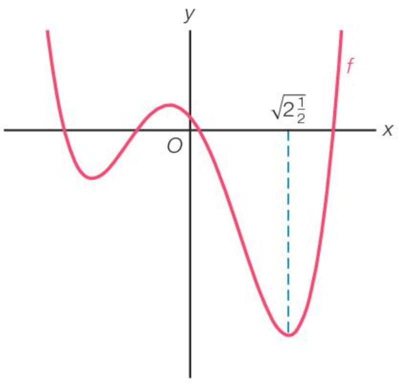
min. is $f(-3) = -1$ en max. is $f(3) = 1$.

$f(x) = 0$ geeft $6x = 0$, dus $x = 0$.

De grafiek snijdt de x-as alleen in (0, 0), dus $B_f = [-1, 1]$.

2 $f(x) = 9x^4 + 4x^3 - 45x^2 - 30x + 5$ geeft $f'(x) = 36x^3 + 12x^2 - 90x - 30$

$f'(\sqrt{2\frac{1}{2}}) = 36 \cdot 2\frac{1}{2}\sqrt{2\frac{1}{2}} + 12 \cdot 2\frac{1}{2} - 90\sqrt{2\frac{1}{2}} - 30 = 90\sqrt{2\frac{1}{2}} + 30 - 90\sqrt{2\frac{1}{2}} - 30 = 0$



$f'(\sqrt{2\frac{1}{2}}) = 0$ en in de schets is te zien dat de grafiek een top heeft voor $x = \sqrt{2\frac{1}{2}}$.

Dus f heeft een extreme waarde voor $x = \sqrt{2\frac{1}{2}}$.

3 **a** $f(x) = 4 - (x^3+1)^2 = 4 - (x^6 + 2x^3 + 1) = -x^6 - 2x^3 + 3$ geeft $f'(x) = -6x^5 - 6x^2$

$f'(x) = 0$ geeft $-6x^5 - 6x^2 = 0$

$-6x^2(x^3 + 1) = 0$

$-6x^2 = 0 \vee x^3 = -1$

$x = 0 \vee x = -1$

$f(-1) = 4$, dus de top is $(-1, 4)$.

b $f'(x) = -6x^5 - 6x^2$ geeft $f''(x) = -30x^4 - 12x$

$f''(x) = 0$ geeft $-30x^4 - 12x = 0$

$-6x(5x^3 + 2) = 0$

$-6x = 0 \vee 5x^3 = -2$

$x = 0 \vee x^3 = -\frac{2}{5}$

$x = 0 \vee x = \sqrt[3]{-\frac{2}{5}}$

$f(0) = 3$ en $f(\sqrt[3]{-\frac{2}{5}}) = 4 - (-\frac{2}{5} + 1)^2 = 4 - \frac{9}{25} = 3\frac{16}{25}$

Dus de buigpunten zijn (0, 3) en $(\sqrt[3]{-\frac{2}{5}}, 3\frac{16}{25})$.

4 **a** $f(x) = \frac{5}{x^6} = 5x^{-6}$ geeft $f'(x) = -30x^{-7} = -\frac{30}{x^7}$

b $g(x) = \frac{x^6 - 2x^4}{x^5} = x - 2x^{-1}$ geeft $g'(x) = 1 + 2x^{-2} = 1 + \frac{2}{x^2} = \frac{x^2 + 2}{x^2}$

c $h(x) = 4x^3 - \frac{3x - 2}{x^3} = 4x^3 - 3x^{-2} + 2x^{-3}$ geeft

$h'(x) = 12x^2 + 6x^{-3} - 6x^{-4} = 12x^2 + \frac{6}{x^3} - \frac{6}{x^4} = 12x^2 + \frac{6x - 6}{x^4}$

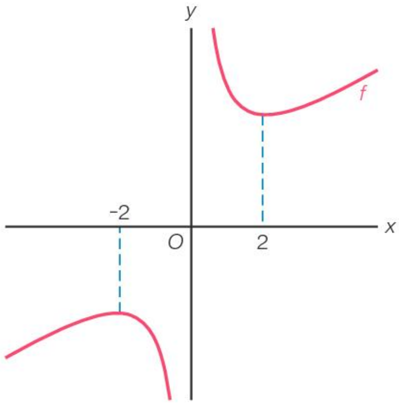
5 **a** $f(x) = 2x + \frac{8}{x} + 1 = 2x + 8x^{-1} + 1$ geeft $f'(x) = 2 - 8x^{-2} = 2 - \frac{8}{x^2} = \frac{2x^2 - 8}{x^2}$

$f'(x) = 0$ geeft $2x^2 - 8 = 0$

$2x^2 = 8$

$x^2 = 4$

$x = 2 \vee x = -2$



max. is $f(-2) = -7$ en min. is $f(2) = 9$.

b $f'(x) = -6$ geeft $\frac{2x^2 - 8}{x^2} = -6$

$2x^2 - 8 = -6x^2$

$8x^2 = 8$

$x^2 = 1$

$x = 1 \vee x = -1$

$f(1) = 11$ en $f(-1) = -9$

Dus de raakpunten zijn (1, 11) en (−1, −9).

c $f'(x) = 3$ geeft $\frac{2x^2 - 8}{x^2} = 3$

$2x^2 - 8 = 3x^2$

$-x^2 = 8$

$x^2 = -8$

geen oplossingen

Er is dus geen raaklijn met richtingscoëfficiënt 3.

6 **a** $f(x) = \frac{8x^6 - x^4}{x\sqrt{x}} = \frac{8x^6 - x^4}{x^{1\frac{1}{2}}} = 8x^{4\frac{1}{2}} - x^{2\frac{1}{2}}$ geeft $f'(x) = 36x^{3\frac{1}{2}} - 2\frac{1}{2}x^{1\frac{1}{2}} = 36x^3 \cdot \sqrt{x} - 2\frac{1}{2}x\sqrt{x}$

b $g(x) = \frac{6x - x^2 \cdot \sqrt[3]{x}}{x^3} = \frac{6x - x^{2\frac{1}{3}}}{x^3} = 6x^{-2} - x^{-\frac{2}{3}}$ geeft $g'(x) = -12x^{-3} + \frac{2}{3}x^{-1\frac{2}{3}} = \frac{-12}{x^3} + \frac{2x^{1\frac{1}{3}}}{3x^3} = \frac{-36 + 2x \cdot \sqrt[3]{x}}{3x^3}$

c $h(x) = (x^2 + 2\sqrt{x})^2 = x^4 + 4x^2 \cdot \sqrt{x} + 4x = x^4 + 4x^{2\frac{1}{2}} + 4x$ geeft

$h'(x) = 4x^3 + 10x^{1\frac{1}{2}} + 4 = 4x^3 + 10x\sqrt{x} + 4$

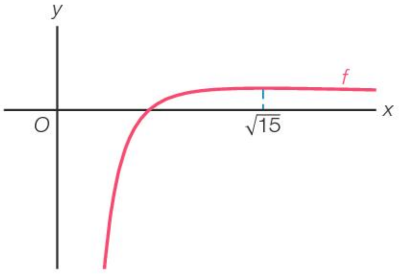
7 $f(x) = \frac{x^2-3}{x^2 \cdot \sqrt{x}} = \frac{x^2-3}{x^{2\frac{1}{2}}} = x^{-\frac{1}{2}} - 3x^{-2\frac{1}{2}}$ geeft $f'(x) = -\frac{1}{2}x^{-1\frac{1}{2}} + 7\frac{1}{2}x^{-3\frac{1}{2}} = \frac{-x^2}{2x^{3\frac{1}{2}}} + \frac{15}{2x^{3\frac{1}{2}}} = \frac{-x^2+15}{2x^3 \cdot \sqrt{x}}$

$f'(x) = 0$ geeft $-x^2 + 15 = 0$

$x^2 = 15$

$x = \sqrt{15} \vee x = -\sqrt{15}$

vold. vold. niet



$y_{\text{top}} = f(\sqrt{15}) = \frac{15-3}{15 \cdot \sqrt{\sqrt{15}}} = \frac{12}{15 \cdot \sqrt[4]{15}} = \frac{4}{5 \cdot \sqrt[4]{15}}$

Dus $a = 4$, $b = 5$ en $c = 15$ zijn mogelijke waarden.

Bladzijde 89

8 **a** $f(x) = x^3 - \frac{5}{(2x^2-x)^4} = x^3 - 5(2x^2-x)^{-4}$ geeft $f'(x) = 3x^2 + 20(2x^2-x)^{-5} \cdot (4x-1) = 3x^2 + \frac{80x-20}{(2x^2-x)^5}$

b $g(x) = \sqrt{x^2-6x+15}$ geeft $g'(x) = \frac{1}{2\sqrt{x^2-6x+15}} \cdot (2x-6) = \frac{x-3}{\sqrt{x^2-6x+15}}$

c $h(x) = (x^2+2)\sqrt{x^2+2} = (x^2+2)^{1\frac{1}{2}}$ geeft $h'(x) = 1\frac{1}{2}(x^2+2)^{\frac{1}{2}} \cdot 2x = 3x\sqrt{x^2+2}$

9 **a** $f(x) = \frac{1}{3}(x^2-2)^3 - x^2$ geeft $f'(x) = (x^2-2)^2 \cdot 2x - 2x = 2x(x^2-2)^2 - 2x$

$f'(x) = 0$ geeft $2x(x^2-2)^2 - 2x = 0$

$2x((x^2-2)^2 - 1) = 0$

$2x = 0 \vee (x^2-2)^2 = 1$

$x = 0 \vee x^2 - 2 = 1 \vee x^2 - 2 = -1$

$x = 0 \vee x^2 = 3 \vee x^2 = 1$

$x = 0 \vee x = \sqrt{3} \vee x = -\sqrt{3} \vee x = 1 \vee x = -1$

b Stel k: $y = ax + b$ met $a = f'(2) = 4 \cdot 2^2 - 4 = 12$.

$\left.\begin{array}{l} y = 12x + b \\ f(2) = -1\frac{1}{3}, \text{ dus } A(2, -1\frac{1}{3}) \end{array}\right\}$ $12 \cdot 2 + b = -1\frac{1}{3}$

$24 + b = -1\frac{1}{3}$

$b = -25\frac{1}{3}$

Dus k: $y = 12x - 25\frac{1}{3}$.

$y = 0$ geeft $12x - 25\frac{1}{3} = 0$

$12x = \frac{76}{3}$

$x = \frac{76}{36} = 2\frac{1}{9}$

Dus $x_B = 2\frac{1}{9}$.

10 **a** $f(x) = x\sqrt{50 - x^2}$ geeft

$$f'(x) = 1 \cdot \sqrt{50 - x^2} + x \cdot \frac{1}{2\sqrt{50 - x^2}} \cdot -2x = \sqrt{50 - x^2} - \frac{x^2}{\sqrt{50 - x^2}} = \frac{50 - x^2 - x^2}{\sqrt{50 - x^2}} = \frac{50 - 2x^2}{\sqrt{50 - x^2}}$$

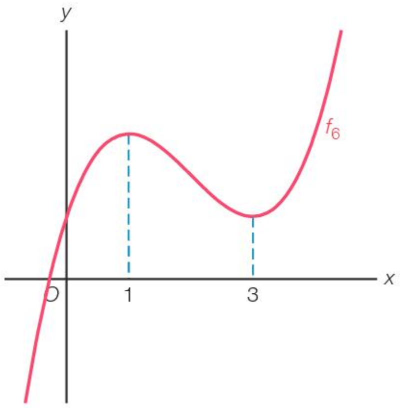
$f'(x) = 0$ geeft $50 - 2x^2 = 0$
$x^2 = 25$
$x = 5 \vee x = -5$
min. is $f(-5) = -25$ en max. is $f(5) = 25$.
Dus $B_f = [-25, 25]$.

b Stel k: $y = ax + b$ met $a = f'(1) = \frac{50 - 2}{\sqrt{50 - 1}} = \frac{48}{\sqrt{49}} = 6\frac{6}{7}$.

$$\left.\begin{array}{l} y = 6\frac{6}{7}x + b \\ f(1) = 7, \text{ dus } A(1, 7) \end{array}\right\} \begin{array}{l} 6\frac{6}{7} \cdot 1 + b = 7 \\ b = \frac{1}{7} \end{array}$$

Dus k: $y = 6\frac{6}{7}x + \frac{1}{7}$.

11 **a** $f_p(x) = \frac{2}{3}x^3 - 4x^2 + px + 2$ geeft $f_p'(x) = 2x^2 - 8x + p$
$f_p'(3) = 0$ geeft $18 - 24 + p = 0$
$p = 6$
$f_6(x) = \frac{2}{3}x^3 - 4x^2 + 6x + 2$ en $f_6'(x) = 2x^2 - 8x + 6$
$f_6'(x) = 0$ geeft $2x^2 - 8x + 6 = 0$
$x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
$x = 1 \vee x = 3$



$f_6(1) = \frac{2}{3} - 4 + 6 + 2 = 4\frac{2}{3}$
Dus max. is $f_6(1) = 4\frac{2}{3}$.

b Voor raken geldt $f_p(-1) = 12 + q \wedge f_p'(-1) = 12$
$f_p'(-1) = 12$ geeft $2 + 8 + p = 12$
$p = 2$
$f_2(x) = \frac{2}{3}x^3 - 4x^2 + 2x + 2$
$f_2(-1) = -12 + q$ geeft $-\frac{2}{3} - 4 - 2 + 2 = -12 + q$
$q = 7\frac{1}{3}$
Dus $p = 2$ en $q = 7\frac{1}{3}$.

12 $f_p(x) = -\frac{1}{3}x^3 + px^2 + 3x - 4$ geeft $f_p'(x) = -x^2 + 2px + 3$

$f_p'(x) = 0$ geeft $-x^2 + 2px + 3 = 0$

$2px = x^2 - 3$

$$\left.\begin{array}{l} \text{voor } x \neq 0 \text{ geldt } p = \dfrac{x^2 - 3}{2x} \\ y = -\frac{1}{3}x^3 + px^2 + 3x - 4 \end{array}\right\} y = -\tfrac{1}{3}x^3 + \frac{x^2 - 3}{2x} \cdot x^2 + 3x - 4$$

$$y = -\tfrac{1}{3}x^3 + \tfrac{1}{2}x^3 - 1\tfrac{1}{2}x + 3x - 4$$

$$y = \tfrac{1}{6}x^3 + 1\tfrac{1}{2}x - 4$$

De formule van de kromme waarop alle toppen liggen is $y = \frac{1}{6}x^3 + 1\frac{1}{2}x - 4$.

13 **a** $f(x) = x^3 - 3x$ geeft $f'(x) = 3x^2 - 3$

$g_p(x) = px + 16$ geeft $g_p'(x) = p$

$f(x) = g_p(x) \wedge f'(x) = g_p'(x)$

$x^3 - 3x = px + 16 \wedge 3x^2 - 3 = p$

Substitutie van $p = 3x^2 - 3$ in $x^3 - 3x = px + 16$ geeft $x^3 - 3x = (3x^2 - 3)x + 16$

$$x^3 - 3x = 3x^3 - 3x + 16$$

$$-2x^3 = 16$$

$$x^3 = -8$$

$$\left.\begin{array}{l} x = -2 \\ p = 3x^2 - 3 \end{array}\right\} p = 12 - 3 = 9$$

Dus voor $p = 9$ raken de grafieken elkaar.

b $f(x) = \sqrt{x^2 + 1}$ geeft $f'(x) = \dfrac{1}{2\sqrt{x^2 + 1}} \cdot 2x = \dfrac{x}{\sqrt{x^2 + 1}}$

$g_p(x) = px + 4$ geeft $g_p'(x) = p$

$f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = -1$

$\sqrt{x^2 + 1} = px + 4 \wedge \dfrac{x}{\sqrt{x^2 + 1}} \cdot p = -1$

$\sqrt{x^2 + 1} = px + 4 \wedge p = \dfrac{-\sqrt{x^2 + 1}}{x}$

Dit geeft $\sqrt{x^2 + 1} = \dfrac{-\sqrt{x^2 + 1}}{x} \cdot x + 4$

$$\sqrt{x^2 + 1} = -\sqrt{x^2 + 1} + 4$$

$$2\sqrt{x^2 + 1} = 4$$

$$\sqrt{x^2 + 1} = 2$$

$$x^2 + 1 = 4$$

$$x^2 = 3$$

$$x = \sqrt{3} \vee x = -\sqrt{3}$$

$x = \sqrt{3}$ geeft $p = -\dfrac{\sqrt{3 + 1}}{\sqrt{3}} = -\dfrac{2}{\sqrt{3}} = -\frac{2}{3}\sqrt{3}$

$x = -\sqrt{3}$ geeft $p = -\dfrac{\sqrt{3 + 1}}{-\sqrt{3}} = \dfrac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$

Dus voor $p = -\frac{2}{3}\sqrt{3}$ en voor $p = \frac{2}{3}\sqrt{3}$ snijden de grafieken elkaar loodrecht.

7 Meetkunde met coördinaten

Voorkennis Stelsels vergelijkingen

Bladzijde 93

1 **a** $\begin{cases} 5x - 7y = 1 \\ 4x - 3y = 6 \end{cases} \begin{vmatrix} 4 \\ 5 \end{vmatrix}$ geeft $\begin{cases} 20x - 28y = 4 \\ 20x - 15y = 30 \end{cases}$ −

$-13y = -26$

$\left.\begin{array}{l} y = 2 \\ 5x - 7y = 1 \end{array}\right\}$ $5x - 7 \cdot 2 = 1$

$5x - 14 = 1$

$5x = 15$

$x = 3$

De oplossing is $(x, y) = (3, 2)$.

b $\begin{cases} 2x + 3y = 15 \\ 10x - 9y = -5 \end{cases} \begin{vmatrix} 3 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 6x + 9y = 45 \\ 10x - 9y = -5 \end{cases}$ +

$16x = 40$

$\left.\begin{array}{l} x = 2\frac{1}{2} \\ 2x + 3y = 15 \end{array}\right\}$ $2 \cdot 2\frac{1}{2} + 3y = 15$

$5 + 3y = 15$

$3y = 10$

$y = 3\frac{1}{3}$

De oplossing is $(x, y) = (2\frac{1}{2}, 3\frac{1}{3})$.

c $\begin{cases} 2x - 5y = 16 \\ 3x + 4y = 10 \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 6x - 15y = 48 \\ 6x + 8y = 20 \end{cases}$ −

$-23y = 28$

$\left.\begin{array}{l} y = -1\frac{5}{23} \\ 2x - 5y = 16 \end{array}\right\}$ $2x - 5 \cdot -1\frac{5}{23} = 16$

$2x + 6\frac{2}{23} = 16$

$2x = 9\frac{21}{23}$

$x = 4\frac{22}{23}$

De oplossing is $(x, y) = (4\frac{22}{23}, -1\frac{5}{23})$.

2 **a** $\begin{cases} 3x + 2y = 5 \\ x - 4y = 11 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 6x + 4y = 10 \\ x - 4y = 11 \end{cases}$ +

$7x = 21$

$\left.\begin{array}{l} x = 3 \\ 3x + 2y = 5 \end{array}\right\}$ $9 + 2y = 5$

$2y = -4$

$y = -2$

Dus $P(3, -2)$.

b $\begin{cases} 4x - 5y = 6 \\ 3x - 2y = 1 \end{cases} \begin{vmatrix} 3 \\ 4 \end{vmatrix}$ geeft $\begin{cases} 12x - 15y = 18 \\ 12x - 8y = 4 \end{cases}$ −

$-7y = 14$

$\left.\begin{array}{l} y = -2 \\ 4x - 5y = 6 \end{array}\right\}$ $4x + 10 = 6$

$4x = -4$

$x = -1$

Dus $Q(-1, -2)$.

7.1 Lijnen en hoeken

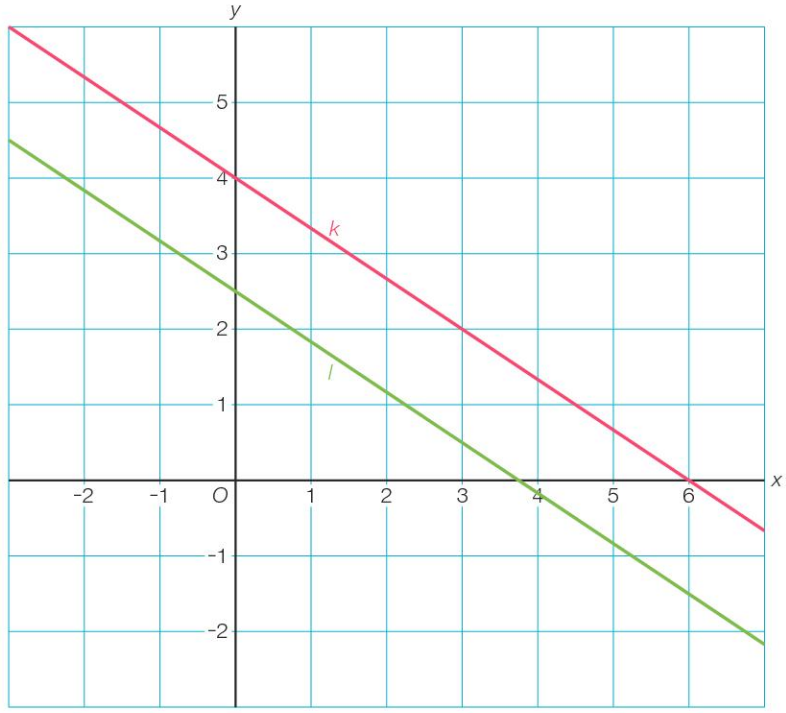
Bladzijde 94

1 **a** k:

x	0	6
y	4	0

l:

x	0	$3\frac{3}{4}$
y	$2\frac{1}{2}$	0



b De lijnen k en l zijn evenwijdig en vallen niet samen, dus hebben geen punt gemeenschappelijk.
Dus het stelsel heeft geen oplossing.

c k: $2x + 3y = 12$
$3y = -2x + 12$
$y = -\frac{2}{3}x + 4$

l: $4x + 6y = 15$
$6y = -4x + 15$
$y = -\frac{2}{3}x + 2\frac{1}{2}$

Je ziet $rc_k = rc_l$, dus $k \mathbin{/\!/} l$.

Bladzijde 95

2 **a** Voor $p = \frac{1}{3}$ en $q \neq -10$ zijn de lijnen evenwijdig.

b Voor $p \neq \frac{1}{3}$ en q elk getal van $\mathbb{R}$ hebben de lijnen een snijpunt.

c $(2, -3)$ op k_p geeft $2p - 3(p + 1) = 5$
$2p - 3p - 3 = 5$
$-p = 8$
$p = -8$
$p = -8$ geeft l_q: $-9x - 11y = q$
$(2, -3)$ op l_q geeft $q = -9 \cdot 2 - 11 \cdot -3 = 15$.
Dus voor $p = -8$ en $q = 15$ snijden de lijnen elkaar in het punt $(2, -3)$.

3 **a** Samenvallen, dus $\frac{3}{p-1}=\frac{p}{p+4}=\frac{5}{q}$.

$\frac{3}{p-1}=\frac{p}{p+4}$ geeft $p(p-1)=3(p+4)$

$p^2-p=3p+12$

$p^2-4p-12=0$

$(p+2)(p-6)=0$

$p=-2 \vee p=6$

$p=-2$ geeft $\frac{3}{-3}=\frac{-2}{2}=\frac{5}{q}$ oftewel $-1=\frac{5}{q}$, dus $q=-5$.

$p=6$ geeft $\frac{3}{5}=\frac{6}{10}=\frac{5}{q}$ oftewel $3q=25$, dus $q=8\frac{1}{3}$.

Dus voor $p=-2$ en $q=-5$ en voor $p=6$ en $q=8\frac{1}{3}$ vallen de lijnen samen.

b Voor $p=-2$ en q elk getal van $\mathbb{R}$ en voor $p=6$ en q elk getal van $\mathbb{R}$ zijn de lijnen evenwijdig.

4 Samenvallen, dus $\frac{p}{q+3}=\frac{q}{p-1}=\frac{4}{1}$.

Uit $\frac{p}{q+3}=\frac{4}{1}$ volgt $p=4(q+3)$ oftewel $p=4q+12$.

Uit $\frac{q}{p-1}=\frac{4}{1}$ volgt $q=4(p-1)$ oftewel $q=4p-4$.

Substitutie van $p=4q+12$ in $q=4p-4$ geeft $q=4(4q+12)-4$

$q=16q+48-4$

$-15q=44$

$q=-2\frac{14}{15}$

$q=-2\frac{14}{15}$ geeft $p=4\cdot -2\frac{14}{15}+12=\frac{4}{15}$

Dus voor $p=\frac{4}{15}$ en $q=-2\frac{14}{15}$ vallen de lijnen samen.

5 **a** $y=0$ geeft $2x=18$

$x=9$

Dus het snijpunt met de x-as is $(9, 0)$.

$x=0$ geeft $3y=18$

$y=6$

Dus het snijpunt met de y-as is $(0, 6)$.

b Bij $2x+3y=18$ het linker- en rechterlid delen door 18 geeft $\frac{x}{9}+\frac{y}{6}=1$.

c In de noemer van de breuk $\frac{x}{9}$ staat de x-coördinaat van het snijpunt met de x-as.

In de noemer van de breuk $\frac{y}{6}$ staat de y-coördinaat van het snijpunt met de y-as.

6 **a** $\left.\begin{array}{l}\frac{x}{a}+\frac{y}{b}=1\\ y=0\end{array}\right\}\ \frac{x}{a}+0=1$

$\frac{x}{a}=1$

$x=a$

Dus het snijpunt met de x-as is $(a, 0)$.

b $\left.\begin{array}{l}\frac{x}{a}+\frac{y}{b}=1\\ x=0\end{array}\right\}\ 0+\frac{y}{b}=1$

$\frac{y}{b}=1$

$y=b$

Dus het snijpunt met de y-as is $(0, b)$.

Bladzijde 96

7 **a** $l\colon \frac{x}{p}+\frac{y}{5}=1$

$l\colon 5x+py=5p$

b $m\colon \frac{x}{4}+\frac{y}{q}=1$

$m\colon qx+4y=4q$

c $n\colon \frac{x}{3r}+\frac{y}{r}=1$

$n\colon x+3y=3r$

8 **a** $k: \frac{x}{p} + \frac{y}{8} = 1$

$k: 8x + py = 8p$

b $\left.\begin{array}{l} k: 8x + py = 8p \\ \text{door } (1, 2) \end{array}\right\}$ $8 \cdot 1 + p \cdot 2 = 8p$

$8 + 2p = 8p$

$-6p = -8$

$p = 1\frac{1}{3}$

c $k: 8x + py = 8p$ en $l: 2x - y = -3$

$k \,//\, l$ geeft $\frac{8}{2} = \frac{p}{-1} \neq \frac{8p}{-3}$

$\frac{8}{2} = \frac{p}{-1}$ geeft $p = -4$ en $p = -4$ geeft $\frac{8}{2} = \frac{-4}{-1} \neq \frac{-32}{-3}$.

Dus voor $p = -4$ is $k \,//\, l$.

Bladzijde 97

9 **a** $k: \frac{x}{3} + \frac{y}{p} = 1$

$k: px + 3y = 3p$

$l: \frac{x}{2p} + \frac{y}{5} = 1$

$l: 5x + 2py = 10p$

b $\left.\begin{array}{l} k: px + 3y = 3p \\ \text{door } A(1, 2) \end{array}\right\}$ $p \cdot 1 + 3 \cdot 2 = 3p$

$p + 6 = 3p$

$-2p = -6$

$p = 3$

$\left.\begin{array}{l} l: 5x + 2py = 10p \\ \text{door } A(1, 2) \end{array}\right\}$ $5 \cdot 1 + 2p \cdot 2 = 10p$

$5 + 4p = 10p$

$-6p = -5$

$p = \frac{5}{6}$

c $k: px + 3y = 3p$ en $m: 4x - y = -5$

$k \,//\, m$ geeft $\frac{p}{4} = \frac{3}{-1} \neq \frac{3p}{-5}$

$\frac{p}{4} = \frac{3}{-1}$ geeft $p = -12$ en $p = -12$ geeft $\frac{-12}{4} = \frac{3}{-1} \neq \frac{-36}{-5}$.

Dus voor $p = -12$ is $k \,//\, m$.

d $l: 5x + 2py = 10p$ en $n: 2x + 3y = 10$

$l \,//\, n$ geeft $\frac{5}{2} = \frac{2p}{3} \neq \frac{10p}{10}$

$\frac{5}{4} = \frac{2p}{3}$ geeft $4p = 15$ oftewel $p = 3\frac{3}{4}$ en $p = 3\frac{3}{4}$ geeft $\frac{5}{2} = \frac{7\frac{1}{2}}{3} \neq \frac{37\frac{1}{2}}{10}$.

Dus voor $p = 3\frac{3}{4}$ is $l \,//\, n$.

10 **a** $\left.\begin{array}{l} l_p: \frac{x}{p} + \frac{y}{p+2} = 1 \\ \text{door } (3, 4) \end{array}\right\}$ $\frac{3}{p} + \frac{4}{p+2} = 1$

$3(p + 2) + 4p = p(p + 2)$

$3p + 6 + 4p = p^2 + 2p$

$p^2 - 5p - 6 = 0$

$(p + 1)(p - 6) = 0$

$p = -1 \vee p = 6$

vold. vold.

b $\frac{x}{p}+\frac{y}{p+2}=1$

$(p+2)x+py=p(p+2)$

$py=-(p+2)x+p(p+2)$

$y=-\frac{p+2}{p}\cdot x+p+2$

rc = 2 geeft $-\frac{p+2}{p}=2$

$p+2=-2p$

$3p=-2$

$p=-\frac{2}{3}$ vold.

11 **a** $\tan(\angle CAB)=\frac{BC}{AC}=\frac{2}{1}=2$

$\angle CAB = 63{,}4349...°$

Dus $\angle CAB \approx 63{,}435°$.

b De tangens van de hoek tussen l en de x-as is $\frac{1}{4}$.

Dit geeft dat de hoek gelijk is aan 14,0362...°.

c Dus de hoek tussen k en l is 63,4349...° + 14,0362...° ≈ 77,5°.

Bladzijde 99

12 Als de eenheden langs de assen verschillend zijn, dan maakt bijvoorbeeld de lijn $y = x$ geen hoek van 45° met de x-as, maar is elke hoek mogelijk.

13 **a** k: $y=3x+4$

$\tan(\alpha)=\text{rc}_k=3$ geeft $\alpha=71{,}56...°$

l: $y=2x-1$

$\tan(\beta)=\text{rc}_l=2$ geeft $\beta=63{,}43...°$

$\alpha-\beta=71{,}56...°-63{,}43...°\approx 8°$

Dus $\angle(k, l)\approx 8°$.

b m: $y=1\frac{1}{2}x+2$

$\tan(\alpha)=\text{rc}_m=1\frac{1}{2}$ geeft $\alpha=56{,}30...°$

n: $y=-\frac{1}{2}x+3$

$\tan(\beta)=\text{rc}_n=-\frac{1}{2}$ geeft $\beta=-26{,}56...°$

$\alpha-\beta=56{,}30...°--26{,}56...°\approx 83°$

Dus $\angle(m, n)\approx 83°$.

c p: $y=3\frac{1}{2}x-1$

$\tan(\alpha)=\text{rc}_p=3\frac{1}{2}$ geeft $\alpha=74{,}05...°$

q: $y=-1\frac{1}{4}x+5$

$\tan(\beta)=\text{rc}_q=-1\frac{1}{4}$ geeft $\beta=-51{,}34...°$

$\alpha-\beta=74{,}05...°--51{,}34...°\approx 125°$

Dus $\angle(p, q)\approx 180°-125°=55°$.

14 **a** k: $3x-2y=5$

$-2y=-3x+5$

$y=1\frac{1}{2}x-2\frac{1}{2}$

$\tan(\alpha)=\text{rc}_k=1\frac{1}{2}$ geeft $\alpha=56{,}30...°$

l: $4x-3y=6$

$-3y=-4x+6$

$y=1\frac{1}{3}x-2$

$\tan(\beta)=\text{rc}_l=1\frac{1}{3}$ geeft $\beta=53{,}13...°$

$\alpha-\beta=56{,}30...°-53{,}13...°\approx 3{,}2°$

Dus $\angle(k, l)\approx 3{,}2°$.

b $m: 4x + y = 1$
$y = -4x + 1$
$\tan(\alpha) = rc_m = -4$ geeft $\alpha = -75{,}96...°$
$n: 3x + 4y = 2$
$4y = -3x + 2$
$y = -\frac{3}{4}x + \frac{1}{2}$
$\tan(\beta) = rc_n = -\frac{3}{4}$ geeft $\beta = -36{,}86...°$
$\beta - \alpha = -36{,}86...° - -75{,}96...° \approx 39{,}1°$
Dus $\angle(m, n) \approx 39{,}1°$.

c $p: 5x + 3y = 4$
$3y = -5x + 4$
$y = -1\frac{2}{3}x + 1\frac{1}{3}$
$\tan(\alpha) = rc_p = -1\frac{2}{3}$ geeft $\alpha = -59{,}03...°$
$q: 6x - 5y = 1$
$-5y = -6x + 1$
$y = 1\frac{1}{5}x - \frac{1}{5}$
$\tan(\beta) = rc_q = 1\frac{1}{5}$ geeft $\beta = 50{,}19...°$
$\beta - \alpha = 50{,}19...° - -59{,}03...° \approx 109{,}2°$
Dus $\angle(p, q) \approx 180° - 109{,}2° = 70{,}8°$.

15 **a** $k: y = \frac{2}{3}x + 4$
$\tan(\alpha) = rc_k = \frac{2}{3}$ geeft $\alpha = 33{,}69...°$
$l: 6x - 5y = 3$
$-5y = -6x + 3$
$y = 1\frac{1}{5}x - \frac{3}{5}$
$\tan(\beta) = rc_l = 1\frac{1}{5}$ geeft $\beta = 50{,}19...°$
$\beta - \alpha = 50{,}19...° - 33{,}69...° \approx 16{,}5°$
Dus $\angle(k, l) \approx 16{,}5°$.

b $rc_m = \frac{0 - 5}{4 - 0} = -1\frac{1}{4}$
$\tan(\alpha) = -1\frac{1}{4}$ geeft $\alpha = -51{,}34...°$
$rc_n = \frac{1 - 0}{0 - -2} = \frac{1}{2}$
$\tan(\beta) = \frac{1}{2}$ geeft $\beta = 26{,}56...°$
$\beta - \alpha = 26{,}56...° - -51{,}34...° \approx 77{,}9°$
Dus $\angle(m, n) \approx 77{,}9°$.

c $rc_p = \frac{6 - 1}{5 - 2} = 1\frac{2}{3}$
$\tan(\alpha) = 1\frac{2}{3}$ geeft $\alpha = 59{,}03...°$
$rc_q = \frac{-6 - 1}{2 - -3} = -1\frac{2}{5}$
$\tan(\beta) = -1\frac{2}{5}$ geeft $\beta = -54{,}46...°$
$\alpha - \beta = 59{,}03...° - -54{,}46...° \approx 113{,}5°$
Dus $\angle(p, q) \approx 180° - 113{,}5° = 66{,}5°$.

16 De richtingshoek van k is α en de richtingshoek van l is β.
$\tan(\alpha) = rc_k = 2$ geeft $\alpha = 63{,}43...°$
$\beta = \alpha + 20° = 63{,}43...° + 20° = 83{,}43...°$ en dit geeft $rc_l = \tan(83{,}43...°) \approx 8{,}69$.
$\beta = \alpha - 20° = 63{,}43...° - 20° = 43{,}43...°$ en dit geeft $rc_l = \tan(43{,}43...°) \approx 0{,}95$.

17 De richtingshoek van k is α en de richtingshoek van l is β.

k: $2x - 5y = 9$

$-5y = -2x + 9$

$y = \frac{2}{5}x - 1\frac{4}{5}$

$\tan(\alpha) = \text{rc}_k = \frac{2}{5}$ geeft $\alpha = 21{,}80...°$

$\beta = 21{,}80...° + 60° = 81{,}80...°$ geeft $\text{rc}_l = \tan(81{,}80...°) = 6{,}94...$

$\beta = 21{,}80...° - 60° = -38{,}19...°$ geeft $\text{rc}_l = \tan(-38{,}19...°) = -0{,}78...$

$S(2, -1)$ op l geeft $2a - b = 3$, dus $b = 2a - 3$.

l: $ax + (2a - 3)y = 3$

$(2a - 3)y = -ax + 3$

$y = \frac{-a}{2a - 3}x + \frac{3}{2a - 3}$, dus $\text{rc}_l = \frac{-a}{2a - 3}$.

$\frac{-a}{2a - 3} = 6{,}94...$	$\frac{-a}{2a - 3} = -0{,}78...$
$6{,}94...(2a - 3) = -a$	$-0{,}78(2a - 3) = -a$
$13{,}88...a - 20{,}82... = -a$	$-1{,}57...a + 2{,}36... = -a$
$14{,}88...a = 20{,}82...$	$-0{,}57...a = -2{,}36...$
$a = 1{,}39...$	$a = 4{,}11...$

$a = 1{,}39...$ geeft $b = 2 \cdot 1{,}39... - 3 \approx -0{,}20$

$a = 4{,}11...$ geeft $b = 2 \cdot 4{,}11... - 3 \approx 5{,}23$

Dus $a \approx 1{,}40$ en $b \approx -0{,}20$ of $a \approx 4{,}11$ en $b \approx 5{,}23$.

7.2 Afstanden bij punten en lijnen

Bladzijde 101

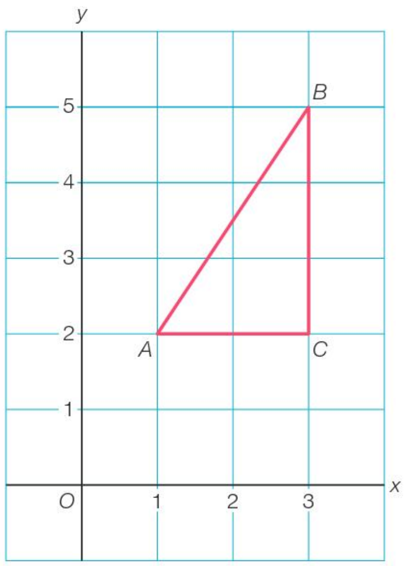
18 **a** In $\triangle ABC$ geeft de stelling van Pythagoras

$AB^2 = AC^2 + BC^2$

$AB^2 = 2^2 + 3^2$

$AB^2 = 13$

$AB = \sqrt{13}$



b $x_M = \frac{1}{2}(x_A + x_B)$ en $y_M = \frac{1}{2}(y_A + y_B)$

$M(2, 3\frac{1}{2})$

c $x_N = \frac{1}{2}(83 + 89) = 86$ en $y_N = \frac{1}{2}(61 + 69) = 65$, dus $N(86, 65)$.

Bladzijde 102

19 **a** $d(A, B) = \sqrt{(12 - 0)^2 + (7 - 12)^2} = \sqrt{144 + 25} = \sqrt{169} = 13$

b $M_1(\frac{1}{2}(0 + -2), \frac{1}{2}(-2 + 7)) = M_1(-1, 2\frac{1}{2})$

c $d(A, B) = \sqrt{(4 - 0)^2 + (7 - 4)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$d(A, M_2) = \frac{1}{2} \cdot 5 = 2\frac{1}{2}$

d $M_3(\frac{1}{2}(0 + p), \frac{1}{2}(p + 7)) = M_3(\frac{1}{2}p, \frac{1}{2}p + 3\frac{1}{2})$

e $d(A, B) = \sqrt{(p - 0)^2 + (7 - p)^2} = \sqrt{p^2 + 49 - 14p + p^2} = \sqrt{2p^2 - 14p + 49}$

20 **a** $M(\frac{1}{2}(p + p + 2), \frac{1}{2}(3 + 2p)) = M(p + 1, p + 1\frac{1}{2})$

b $d(A, B) = \sqrt{(p + 2 - p)^2 + (2p - 3)^2} = \sqrt{2^2 + 4p^2 - 12p + 9} = \sqrt{4p^2 - 12p + 13}$

c Voer in $y_1 = \sqrt{4x^2 - 12x + 13}$ en $y_2 = 5$.

De optie snijpunt geeft $x \approx 3{,}79$

Dus $p \approx 3{,}79$.

21 **a** $x_M = 10$

$\frac{1}{2}(3 + p + 5) = 10$

$\frac{1}{2}p + 4 = 10$

$\frac{1}{2}p = 6$

$p = 12$

$y_M = \frac{1}{2}(4 + 14) = 9$

b $M(\frac{1}{2}(3 + p + 5), \frac{1}{2}(4 + p + 2))$

$\left.\begin{array}{l} M(\frac{1}{2}p + 4, \frac{1}{2}p + 3) \\ M \text{ op } k\text{: } y = 2x - 3 \end{array}\right\} 2(\frac{1}{2}p + 4) - 3 = \frac{1}{2}p + 3$

$p + 8 - 3 = \frac{1}{2}p + 3$

$\frac{1}{2}p = -2$

$p = -4$

c $d = \sqrt{(p + 5 - 3)^2 + (p + 2 - 4)^2} = \sqrt{(p + 2)^2 + (p - 2)^2} = \sqrt{p^2 + 4p + 4 + p^2 - 4p + 4} = \sqrt{2p^2 + 8}$

Dus $a = 2$ en $b = 8$.

22 **a** k: $2x - y = 2$

$-y = -2x + 2$

$y = 2x - 2$

$\text{rc}_k = 2$

l: $x + 2y = 3$

$2y = -x + 3$

$y = -\frac{1}{2}x + 1\frac{1}{2}$

$\text{rc}_l = -\frac{1}{2}$

b Er geldt $\text{rc}_k \cdot \text{rc}_l = 2 \cdot -\frac{1}{2} = -1$, dus $k \perp l$.

Bladzijde 104

23 In de theorie gaat het over de breuken $-\frac{a}{b}$ en $\frac{b}{a}$, dus $b \neq 0$ en $a \neq 0$.

Als in de kernzin

- $b = 0$, dan is k een verticale lijn en l een horizontale lijn, dus staan k en l loodrecht op elkaar;
- $a = 0$, dan is k een horizontale lijn en l een verticale lijn, dus staan k en l loodrecht op elkaar.

Dus in de kernzin zijn de voorwaarden $b \neq 0$ en $a \neq 0$ niet nodig.

24 **a** $\left.\begin{array}{l} k \perp l\text{, dus } k\text{: } 3x + 2y = c \\ \text{door } A(4, 1) \end{array}\right\} c = 3 \cdot 4 + 2 \cdot 1 = 14$

Dus k: $3x + 2y = 14$.

b $\left.\begin{array}{l} m \perp n\text{, dus } m\text{: } 5x - 4y = c \\ \text{door } B(3, -1) \end{array}\right\} c = 5 \cdot 3 - 4 \cdot -1 = 19$

Dus m: $5x - 4y = 19$.

c $\left.\begin{array}{l} p \perp q\text{, dus } p\text{: } x + 2y = c \\ \text{door } C(-4, 3) \end{array}\right\} c = -4 + 2 \cdot 3 = 2$

$x + 2y = 2$

$2y = -x + 2$

$y = -\frac{1}{2}x + 1$

Dus p: $y = -\frac{1}{2}x + 1$.

25 **a** $rc_k = \frac{5-3}{6-2} = \frac{2}{4} = \frac{1}{2}$

$l \perp k$, dus $rc_k \cdot rc_l = -1$

$\frac{1}{2} \cdot rc_l = -1$

$rc_l = -2$

$\left.\begin{array}{l} l\colon y = -2x + b \\ \text{door } C(4, 6) \end{array}\right\} \begin{array}{l} -2 \cdot 4 + b = 6 \\ -8 + b = 6 \\ b = 14 \end{array}$

Dus $l\colon y = -2x + 14$.

b $rc_m = \frac{-6-4}{2--3} = \frac{-10}{5} = -2$

$n \perp m$, dus $rc_m \cdot rc_n = -1$.

$-2 \cdot rc_n = -1$

$rc_n = \frac{1}{2}$

$\left.\begin{array}{l} n\colon y = \frac{1}{2}x + b \\ \text{door } F(3, 7) \end{array}\right\} \begin{array}{l} \frac{1}{2} \cdot 3 + b = 7 \\ 1\frac{1}{2} + b = 7 \\ b = 5\frac{1}{2} \end{array}$

$y = \frac{1}{2}x + 5\frac{1}{2}$

$2y = x + 11$

$-x + 2y = 11$

Dus $n\colon x - 2y = -11$.

26 **a** De lijn p gaat door A en staat loodrecht op k.

$\left.\begin{array}{l} p\colon x - 2y = c \\ \text{door } A(6, 0) \end{array}\right\} c = 6 - 2 \cdot 0 = 6$

Dus $p\colon x - 2y = 6$.

k en p snijden geeft het punt A'.

$\begin{cases} 2x + y = 2 \\ x - 2y = 6 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 4x + 2y = 4 \\ x - 2y = 6 \end{cases}$ +

$5x = 10$

$\left.\begin{array}{l} x = 2 \\ 2x + y = 2 \end{array}\right\} \begin{array}{l} 2 \cdot 2 + y = 2 \\ 4 + y = 2 \\ y = -2 \end{array}$

Dus $A'(2, -2)$.

$d(A, k) = d(A, A') = \sqrt{(2-6)^2 + (-2-0)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

b De lijn q gaat door B en staat loodrecht op l.

$rc_l \cdot rc_q = -1$ geeft $\frac{1}{2} \cdot rc_q = -1$, dus $rc_q = -2$.

$\left.\begin{array}{l} q\colon y = -2x + b \\ \text{door } B(3, 0) \end{array}\right\} \begin{array}{l} -2 \cdot 3 + b = 0 \\ -6 + b = 0 \\ b = 6 \end{array}$

Dus $q\colon y = -2x + 6$.

l en q snijden geeft het punt B'.

$\frac{1}{2}x + 1 = -2x + 6$

$2\frac{1}{2}x = 5$

$\left.\begin{array}{l} x = 2 \\ y = \frac{1}{2}x + 1 \end{array}\right\} y = \frac{1}{2} \cdot 2 + 1 = 2$

Dus $B'(2, 2)$.

$d(B, l) = d(B, B') = \sqrt{(2-3)^2 + (2-0)^2} = \sqrt{1+4} = \sqrt{5}$

c De lijn r gaat door O en staat loodrecht op m.

$\left.\begin{array}{l} r\colon x - 3y = c \\ \text{door } O(0, 0) \end{array}\right\} c = 0$

Dus $r\colon x - 3y = 0$ oftewel $r\colon x = 3y$.

m en r snijden geeft het punt C.

Substitutie van $x = 3y$ in $3x + y = 10$ geeft $3 \cdot 3y + y = 10$

$10y = 10$

$y = 1$

$y = 1$ geeft $x = 3$

Dus $C(3, 1)$.

$d(O, m) = d(O, C) = \sqrt{(3-0)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$

Bladzijde 105

27 De lijn k door de punten $A(-5, -3)$ en $B(-2, 6)$ heeft richtingscoëfficiënt $\frac{6 - -3}{-2 - -5} = 3$.

$\left.\begin{array}{l} k\colon y = 3x + b \\ \text{door } B(-2, 6) \end{array}\right\} 3 \cdot -2 + b = 6$

$-6 + b = 6$

$b = 12$

Dus $k\colon y = 3x + 12$.

De lijn l gaat door C en staat loodrecht op k.

$rc_k \cdot rc_l = -1$ geeft $3 \cdot rc_l = -1$ dus $rc_l = -\frac{1}{3}$.

$\left.\begin{array}{l} l\colon y = -\frac{1}{3}x + b \\ \text{door } C(2, -12) \end{array}\right\} -\frac{1}{3} \cdot 2 + b = -12$

$-\frac{2}{3} + b = -12$

$b = -11\frac{1}{3}$

Dus $l\colon y = -\frac{1}{3}x - 11\frac{1}{3}$.

k en l snijden geeft het punt D.

$3x + 12 = -\frac{1}{3}x - 11\frac{1}{3}$

$3\frac{1}{3}x = -23\frac{1}{3}$

$10x = -70$

$\left.\begin{array}{l} x = -7 \\ y = 3x + 12 \end{array}\right\} y = 3 \cdot -7 + 12 = -9$

Dus $D(-7, -9)$.

$d(C, k) = d(C, D) = \sqrt{(-7-2)^2 + (-9 - -12)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$

28 **a** De lijn k door $A(1, 0)$ en $B(7, 4)$ heeft richtingscoëfficiënt $\frac{4-0}{7-1} = \frac{2}{3}$.

$\left.\begin{array}{l} k\colon y = \frac{2}{3}x + b \\ \text{door } A(1, 0) \end{array}\right\} \frac{2}{3} \cdot 1 + b = 0$

$b = -\frac{2}{3}$

Dus $k\colon y = \frac{2}{3}x - \frac{2}{3}$.

De lijn l gaat door C en staat loodrecht op k.

$rc_k \cdot rc_l = -1$ geeft $\frac{2}{3} \cdot rc_l = -1$, dus $rc_l = -\frac{3}{2}$.

$\left.\begin{array}{l} l\colon y = -\frac{3}{2}x + b \\ \text{door } C(3\frac{1}{2}, 6) \end{array}\right\} -\frac{3}{2} \cdot \frac{7}{2} + b = 6$

$-\frac{21}{4} + b = 6$

$b = 11\frac{1}{4}$

Dus $l\colon y = -\frac{3}{2}x + 11\frac{1}{4}$.

k en l snijden geeft het punt D.

$\frac{2}{3}x - \frac{2}{3} = -\frac{3}{2}x + 11\frac{1}{4}$

$8x - 8 = -18x + 135$

$26x = 143$

$\left.\begin{array}{l} x = 5\frac{1}{2} \\ y = \frac{2}{3}x - \frac{2}{3} \end{array}\right\} y = \frac{2}{3} \cdot \frac{11}{2} - \frac{2}{3} = 3$

Dus $D(5\frac{1}{2}, 3)$.

$d(C, k) = d(C, D) = \sqrt{(5\frac{1}{2} - 3\frac{1}{2})^2 + (3 - 6)^2} = \sqrt{4 + 9} = \sqrt{13}$

b $d(A, B) = \sqrt{(7 - 1)^2 + (4 - 0)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$

$O(\triangle ABC) = \frac{1}{2} \cdot d(A, B) \cdot d(C, AB) = \frac{1}{2} \cdot 2\sqrt{13} \cdot \sqrt{13} = 13$

29 De lijnen k en l zijn evenwijdig, dus de afstand tussen k en l is gelijk aan de afstand van een punt van k tot l.

Neem het punt $A(1, 0)$ op k.

De lijn m gaat door A en staat loodrecht op k.

$\left.\begin{array}{l} m \perp k, \text{ dus } m\text{: } 3x + y = c \\ \text{door } A(1, 0) \end{array}\right\} c = 3 + 0 = 3$

Dus m: $3x + y = 3$.

l en m snijden geeft het punt B.

$\begin{cases} x - 3y = 6 \\ 3x + y = 3 \end{cases} \begin{vmatrix} 1 \\ 3 \end{vmatrix}$ geeft $\begin{cases} x - 3y = 6 \\ 9x + 3y = 9 \end{cases}$ +

$10x = 15$

$\left.\begin{array}{l} x = 1\frac{1}{2} \\ 3x + y = 3 \end{array}\right\} 3 \cdot 1\frac{1}{2} + y = 3$

$4\frac{1}{2} + y = 3$

$y = -1\frac{1}{2}$

Dus $B(1\frac{1}{2}, -1\frac{1}{2})$.

$d(k, l) = d(A, l) = d(A, B) = \sqrt{(1\frac{1}{2} - 1)^2 + (-1\frac{1}{2} - 0)^2} = \sqrt{\frac{1}{4} + \frac{9}{4}} = \sqrt{\frac{10}{4}} = \frac{\sqrt{10}}{\sqrt{4}} = \frac{1}{2}\sqrt{10}$

30 **a** $m \perp k$: $ax + by = c$, dus m: $bx - ay = c$.

$\left.\begin{array}{l} m\text{: } bx - ay = c \\ \text{door } O(0, 0) \end{array}\right\} c = 0$

Dus m: $bx - ay = 0$

$-ay = -bx$

$y = \frac{b}{a}x$

b k: $ax + by = c$

$by = -ax + c$

$y = -\frac{a}{b}x + \frac{c}{b}$

m snijden met k geeft $\frac{b}{a}x = -\frac{a}{b}x + \frac{c}{b}$

$b^2x = -a^2x + ac$

$(a^2 + b^2)x = ac$

$x = \frac{ac}{a^2 + b^2}$

$x = \frac{ac}{a^2 + b^2}$ geeft $y = \frac{b}{a} \cdot \frac{ac}{a^2 + b^2} = \frac{bc}{a^2 + b^2}$

Dus $R\left(\frac{ac}{a^2 + b^2}, \frac{bc}{a^2 + b^2}\right)$.

In l: $ax + by = ax_P + by_P$ is $c = ax_P + by_P$.

Gebruik het resultaat hierboven.

Zo krijg je $S\left(\frac{a(ax_P + by_P)}{a^2 + b^2}, \frac{b(ax_P + by_P)}{a^2 + b^2}\right) = S\left(\frac{a^2x_P + aby_P}{a^2 + b^2}, \frac{abx_P + b^2y_P}{a^2 + b^2}\right)$.

c $d(S, R) = \sqrt{(x_S - x_R)^2 + (y_S - y_R)^2}$ en dit geeft

$$d(S, R) = \sqrt{\left(\frac{a^2x_P + aby_P - ac}{a^2 + b^2}\right)^2 + \left(\frac{abx_P + b^2y_P - bc}{a^2 + b^2}\right)^2}.$$

Herleiden geeft $d(S, R) = \sqrt{\frac{(a^2x_P + aby_P - ac)^2}{(a^2 + b^2)^2} + \frac{(abx_P + b^2y_P - bc)^2}{(a^2 + b^2)^2}}$

$$= \frac{\sqrt{(a^2x_P + aby_P - ac)^2 + (abx_P + b^2y_P - bc)^2}}{\sqrt{(a^2 + b^2)^2}}$$

$$= \frac{\sqrt{(a(ax_P + by_P - c))^2 + (b(ax_P + by_P - c))^2}}{a^2 + b^2}$$

d $d(S, R) = \frac{\sqrt{(aE)^2 + (bE)^2}}{a^2 + b^2} = \frac{\sqrt{a^2E^2 + b^2E^2}}{a^2 + b^2} = \frac{\sqrt{(a^2 + b^2)E^2}}{a^2 + b^2}$

e $d(S, R) = \frac{\sqrt{(a^2 + b^2)E^2}}{a^2 + b^2} = \frac{\sqrt{a^2 + b^2} \cdot \sqrt{E^2}}{\sqrt{a^2 + b^2} \cdot \sqrt{a^2 + b^2}} = \frac{\sqrt{E^2}}{\sqrt{a^2 + b^2}} = \frac{|E|}{\sqrt{a^2 + b^2}}$

$$\left.\begin{array}{l} d(P, k) = d(S, R) \\ d(S, R) = \dfrac{|E|}{\sqrt{a^2 + b^2}} \\ E = ax_P + by_P - c \end{array}\right\} d(P, k) = \frac{|ax_P + by_P - c|}{\sqrt{a^2 + b^2}}$$

Bladzijde 107

31 **a** k: $ax + by = c$ door $B(5, 0)$ geeft $5a = c$, dus k: $ax + by = 5a$ oftewel k: $ax + by - 5a = 0$.
Je krijgt één vergelijking met twee onbekenden.

b k: $x + by = c$ door $B(5, 0)$ geeft $5 = c$, dus k: $x + by = 5$ oftewel k: $x + by - 5 = 0$.
Je had dus ook uit kunnen gaan van k: $x + by = c$.
k: $ax + by = 1$ door $B(5, 0)$ geeft $5a = 1$ oftewel $a = \frac{1}{5}$, dus k: $\frac{1}{5}x + by = 1$ oftewel
k: $\frac{1}{5}x + by - 1 = 0$.
Je had dus ook uit kunnen gaan van k: $ax + by = 1$.

32 $d(A, k) = \frac{|1 - 4 \cdot -1 + 4|}{\sqrt{1^2 + (-4)^2}} = \frac{|9|}{\sqrt{17}} = \frac{9}{\sqrt{17}} = \frac{9}{17}\sqrt{17}$

$d(B, k) = \frac{|2 - 4 \cdot 3\frac{1}{2} + 4|}{\sqrt{1^2 + (-4)^2}} = \frac{|-8|}{\sqrt{17}} = \frac{8}{\sqrt{17}} = \frac{8}{17}\sqrt{17}$

$d(C, k) = \frac{|6 - 4 \cdot 5 + 4|}{\sqrt{1^2 + (-4)^2}} = \frac{|-10|}{\sqrt{17}} = \frac{10}{\sqrt{17}} = \frac{10}{17}\sqrt{17}$

Het punt B ligt het dichtst bij de lijn k.

33 **a** $d(A, k) = \frac{|3 \cdot 2 + 4 \cdot 5 - 10|}{\sqrt{3^2 + 4^2}} = \frac{|16|}{5} = \frac{16}{5} = 3\frac{1}{5}$

b l: $y = -2x + 5$ oftewel l: $2x + y - 5 = 0$ geeft $d(B, l) = \frac{|2 \cdot 4 + -1 - 5|}{\sqrt{2^2 + 1^2}} = \frac{|2|}{\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2}{5}\sqrt{5}$.

c m: $\frac{x}{6} + \frac{y}{2} = 1$ oftewel m: $x + 3y - 6 = 0$ geeft $d(C, m) = \frac{|5 + 3 \cdot 3 - 6|}{\sqrt{1^2 + 3^2}} = \frac{|8|}{\sqrt{10}} = \frac{8}{\sqrt{10}} = \frac{8}{10}\sqrt{10} = \frac{4}{5}\sqrt{10}$.

34 **a** Stel k: $y = ax + b$.
k door (0, 4) geeft $b = 4$, dus k: $y = ax + 4$ oftewel k: $ax - y + 4 = 0$.

$d(B, k) = 5$ geeft $\dfrac{|5\frac{1}{2}a - 5 + 4|}{\sqrt{a^2 + 1}} = 5$

$|5\frac{1}{2}a - 1| = 5\sqrt{a^2 + 1}$
$30\frac{1}{4}a^2 - 11a + 1 = 25(a^2 + 1)$
$30\frac{1}{4}a^2 - 11a + 1 = 25a^2 + 25$
$5\frac{1}{4}a^2 - 11a - 24 = 0$
$D = (-11)^2 - 4 \cdot 5\frac{1}{4} \cdot -24 = 625$
$a = \dfrac{11 + 25}{10\frac{1}{2}} = 3\frac{3}{7} \vee a = \dfrac{11 - 25}{10\frac{1}{2}} = -1\frac{1}{3}$

Dus k_1: $y = 3\frac{3}{7}x + 4$ en k_2: $y = -1\frac{1}{3}x + 4$.

b Stel l: $y = ax + b$.
(3, 0) op l geeft $3a + b = 0$, dus $b = -3a$.
k: $y = ax - 3a$ oftewel $ax - y - 3a = 0$

$d(D, l) = \sqrt{5}$ geeft $\dfrac{|6a - 4 - 3a|}{\sqrt{a^2 + 1}} = \sqrt{5}$

$|3a - 4| = \sqrt{5a^2 + 5}$
$9a^2 - 24a + 16 = 5a^2 + 5$
$4a^2 - 24a + 11 = 0$
$D = (-24)^2 - 4 \cdot 4 \cdot 11 = 400$
$a = \dfrac{24 + 20}{8} = 5\frac{1}{2} \vee a = \dfrac{24 - 20}{8} = \frac{1}{2}$

$a = 5\frac{1}{2}$ geeft $b = -16\frac{1}{2}$, dus l_1: $y = 5\frac{1}{2}x - 16\frac{1}{2}$.
$a = \frac{1}{2}$ geeft $b = -1\frac{1}{2}$, dus l_2: $y = \frac{1}{2}x - 1\frac{1}{2}$.

35 $P(-2, 3)$ is een punt op k.

$d(P, l) = 3$ geeft $\dfrac{|5 \cdot -2 + 12 \cdot 3 - c|}{\sqrt{5^2 + 12^2}} = 3$

$\dfrac{|26 - c|}{\sqrt{169}} = 3$

$\dfrac{|26 - c|}{13} = 3$

$|26 - c| = 39$
$26 - c = 39 \vee 26 - c = -39$
$c = -13 \vee c = 65$

Dus l_1: $5x + 12y = -13$ en l_2: $5x + 12y = 65$.

36 **a** Stel l: $3x + 4y = c$.
$P(4, 0)$ is een punt op k.

$d(P, l) = 2$ geeft $\dfrac{|3 \cdot 4 + 4 \cdot 0 - c|}{\sqrt{3^2 + 4^2}} = 2$

$\dfrac{|12 - c|}{\sqrt{25}} = 2$

$\dfrac{|12 - c|}{5} = 2$

$|12 - c| = 10$
$12 - c = 10 \vee 12 - c = -10$
$c = 2 \vee c = 22$

Dus l_1: $3x + 4y = 2$ en l_2: $3x + 4y = 22$.

b Stel $P(p, 0)$.

$d(P, k) = 3$ geeft $\dfrac{|3 \cdot p + 4 \cdot 0 - 12|}{\sqrt{3^2 + 4^2}} = 3$

$\dfrac{|3p - 12|}{\sqrt{25}} = 3$

$\dfrac{|3p - 12|}{5} = 3$

$|3p - 12| = 15$

$3p - 12 = 15 \vee 3p - 12 = -15$

$3p = 27 \vee 3p = -3$

$p = 9 \vee p = -1$

Dus $P_1(9, 0)$ en $P_2(-1, 0)$.

37 Stel een punt op de parabool is $P(p, 2p^2)$.

$d(P, k) = \sqrt{5}$ geeft $\dfrac{|p - 2 \cdot 2p^2 - 2|}{\sqrt{1^2 + (-2)^2}} = \sqrt{5}$

$\dfrac{|p - 4p^2 - 2|}{\sqrt{5}} = \sqrt{5}$

$|-4p^2 + p - 2| = 5$

$-4p^2 + p - 2 = 5 \qquad \vee -4p^2 + p - 2 = -5$

$-4p^2 + p - 7 = 0 \qquad \vee -4p^2 + p + 3 = 0$

$D = 1^2 - 4 \cdot -4 \cdot -7 = -111 \qquad D = 1^2 - 4 \cdot -4 \cdot 3 = 49$

geen opl. $\qquad p = \dfrac{-1 + 7}{-8} = -\frac{3}{4} \vee p = \dfrac{-1 - 7}{-8} = 1$

Dus $P_1(-\frac{3}{4}, 1\frac{1}{8})$ en $P_2(1, 2)$.

38 Stel $k: ax + by = 1$.

$d(A, k) = \sqrt{2} \wedge d(B, k) = 2\sqrt{2}$ geeft $\dfrac{|4a + 0 - 1|}{\sqrt{a^2 + b^2}} = \sqrt{2} \wedge \dfrac{|6a + 0 - 1|}{\sqrt{a^2 + b^2}} = 2\sqrt{2}$

$|4a - 1| = \sqrt{2} \cdot \sqrt{a^2 + b^2} \wedge |6a - 1| = 2\sqrt{2} \cdot \sqrt{a^2 + b^2}$

Substitutie van $\sqrt{2} \cdot \sqrt{a^2 + b^2} = |4a - 1|$ in $|6a - 1| = 2\sqrt{2} \cdot \sqrt{a^2 + b^2}$ geeft

$|6a - 1| = 2 \cdot |4a - 1|$

$|6a - 1| = |8a - 2|$

$6a - 1 = 8a - 2 \vee 6a - 1 = -8a + 2$

$-2a = -1 \vee 14a = 3$

$a = \frac{1}{2} \vee a = \frac{3}{14}$

Substitutie van $a = \frac{1}{2}$ in $\sqrt{2} \cdot \sqrt{a^2 + b^2} = |4a - 1|$ geeft $\sqrt{2} \cdot \sqrt{\frac{1}{4} + b^2} = |2 - 1|$

$\sqrt{\frac{1}{2} + 2b^2} = 1$

$\frac{1}{2} + 2b^2 = 1$

$2b^2 = \frac{1}{2}$

$b^2 = \frac{1}{4}$

$b = \frac{1}{2} \vee b = -\frac{1}{2}$

Substitutie van $a = \frac{3}{14}$ in $\sqrt{2} \cdot \sqrt{a^2 + b^2} = |4a - 1|$ geeft $\sqrt{2} \cdot \sqrt{\frac{9}{196} + b^2} = |\frac{6}{7} - 1|$

$\sqrt{\frac{9}{98} + 2b^2} = \frac{1}{7}$

$\frac{9}{98} + 2b^2 = \frac{1}{49}$

$2b^2 = -\frac{7}{98}$

$b^2 = -\frac{7}{196}$

geen opl.

Dus $k_1: \frac{1}{2}x + \frac{1}{2}y = 1$ en $k_2: \frac{1}{2}x - \frac{1}{2}y = 1$ oftewel $k_1: x + y = 2$ en $k_2: x - y = 2$.

7.3 Cirkelvergelijkingen

Bladzijde 109

39 **a** Dit volgt rechtstreeks uit de formule van de afstand tussen twee punten.

b Als de afstand van $P(x, y)$ tot $M(1, 4)$ gelijk is aan 5, dan ligt P op de cirkel met middelpunt M en straal 5.

c Een vergelijking van de cirkel met middelpunt $M(1, 4)$ en straal 10 is $(x-1)^2 + (y-4)^2 = 10^2$, oftewel $(x-1)^2 + (y-4)^2 = 100$.

d Middelpunt $(-2, 3)$ en straal $\sqrt{16} = 4$.

Bladzijde 110

40 **a** c_1: $(x-2)^2 + (y+5)^2 = 9$

b $\left.\begin{array}{l} c_2: (x-3)^2 + (y-4)^2 = r^2 \\ \text{door } O(0, 0) \end{array}\right\} r^2 = (0-3)^2 + (0-4)^2 = 9 + 16 = 25$

Dus c_2: $(x-3)^2 + (y-4)^2 = 25$.

c $\left.\begin{array}{l} c_3: (x+3)^2 + (y-5)^2 = r^2 \\ \text{door } A(-1, 2) \end{array}\right\} r^2 = (-1+3)^2 + (2-5)^2 = 4 + 9 = 13$

Dus c_3: $(x+3)^2 + (y-5)^2 = 13$.

d c_4 raakt de x-as, dus $r = d(M_4, x\text{-as}) = 1$.

c_4: $(x-3)^2 + (y-1)^2 = 1$

e c_5 raakt de y-as, dus $r = d(M_5, y\text{-as}) = 4$.

c_5: $(x+4)^2 + (y-2)^2 = 16$

41 De cirkels raken de x-as en de straal is 2, dus $y_M = -2$ of $y_M = 2$.
M op k, dus $M_1(-6, -2)$ en $M_2(6, 2)$.
c_1: $(x+6)^2 + (y+2)^2 = 4$ en c_2: $(x-6)^2 + (y-2)^2 = 4$

Bladzijde 111

42 De cirkels raken de y-as en de straal is 10, dus $x_M = -10$ of $x_M = 10$.
M op k, dus $M_1(-10, 6)$ en $M_2(10, -6)$.
c_1: $(x+10)^2 + (y-6)^2 = 100$ en c_2: $(x-10)^2 + (y+6)^2 = 100$

43 **a** Middelpunt A en c_1 raakt de x-as, dus $r = d(A, x\text{-as}) = 7$.

c_1: $(x-3)^2 + (y-7)^2 = 49$

b Middelpunt B en c_2 door A, dus straal is $d(A, B) = \sqrt{(9-3)^2 + (1-7)^2} = \sqrt{36+36} = \sqrt{72}$.

c_2: $(x-9)^2 + (y-1)^2 = 72$

c Middellijn AB, dus middelpunt is het midden van AB, dus $M_3(6, 4)$.

De straal is $d(A, M_3) = \sqrt{(6-3)^2 + (4-7)^2} = \sqrt{9+9} = \sqrt{18}$.

c_3: $(x-6)^2 + (y-4)^2 = 18$

d De cirkels raken de y-as en de straal is 2, dus $x_M = -2$ of $x_M = 2$.

$x_M = -2$ en M op k geeft $5 \cdot -2 + 2y_M = 12$

$$-10 + 2y_M = 12$$
$$2y_M = 22$$
$$y_M = 11$$

Dus $M_4(-2, 11)$ en c_4: $(x+2)^2 + (y-11)^2 = 4$.

$x_M = 2$ en M op k geeft $5 \cdot 2 + 2y_M = 12$

$$10 + 2y_M = 12$$
$$2y_M = 2$$
$$y_M = 1$$

Dus $M_5(2, 1)$ en c_5: $(x-2)^2 + (y-1)^2 = 4$.

44 **a** Middellijn QR, dus middelpunt is het midden van QR, dus $M(1\frac{1}{2}, 6\frac{1}{2})$.
De straal is $d(Q, M) = \sqrt{(1\frac{1}{2} - -2)^2 + (6\frac{1}{2} - 6)^2} = \sqrt{12\frac{1}{4} + \frac{1}{4}} = \sqrt{12\frac{1}{2}}$.
Dus c: $(x - 1\frac{1}{2})^2 + (y - 6\frac{1}{2})^2 = 12\frac{1}{2}$.
c door $P(-1, p)$ geeft $(-2\frac{1}{2})^2 + (p - 6\frac{1}{2})^2 = 12\frac{1}{2}$

$$6\frac{1}{4} + (p - 6\frac{1}{2})^2 = 12\frac{1}{2}$$
$$(p - 6\frac{1}{2})^2 = 6\frac{1}{4}$$
$$p - 6\frac{1}{2} = 2\frac{1}{2} \vee p - 6\frac{1}{2} = -2\frac{1}{2}$$
$$p = 9 \vee p = 4$$

b Middellijn PR, dus middelpunt is het midden van PR, dus $M(2, \frac{1}{2}p + 3\frac{1}{2})$.
De straal is
$d(R, M) = \sqrt{(2 - 5)^2 + (\frac{1}{2}p + 3\frac{1}{2} - 7)^2} = \sqrt{9 + (\frac{1}{2}p - 3\frac{1}{2})^2} = \sqrt{9 + \frac{1}{4}p^2 - 3\frac{1}{2}p + 12\frac{1}{4}} = \sqrt{\frac{1}{4}p^2 - 3\frac{1}{2}p + 21\frac{1}{4}}$.
Dus c: $(x - 2)^2 + (y - \frac{1}{2}p - 3\frac{1}{2})^2 = \frac{1}{4}p^2 - 3\frac{1}{2}p + 21\frac{1}{4}$.
c door $Q(-2, 6)$ geeft $(-4)^2 + (2\frac{1}{2} - \frac{1}{2}p)^2 = \frac{1}{4}p^2 - 3\frac{1}{2}p + 21\frac{1}{4}$

$$16 + 6\frac{1}{4} - 2\frac{1}{2}p + \frac{1}{4}p^2 = \frac{1}{4}p^2 - 3\frac{1}{2}p + 21\frac{1}{4}$$
$$p = -1$$

45 c: $(x - p)^2 + (y - q)^2 = 13$
$A(4, -1)$ op c geeft $(4 - p)^2 + (-1 - q)^2 = 13$.
$B(-1, -2)$ op c geeft $(-1 - p)^2 + (-2 - q)^2 = 13$.
Hieruit volgt $(4 - p)^2 + (-1 - q)^2 = (-1 - p)^2 + (-2 - q)^2$

$$16 - 8p + p^2 + 1 + 2q + q^2 = 1 + 2p + p^2 + 4 + 4q + q^2$$
$$-2q = 10p - 12$$
$$q = -5p + 6$$

$q = -5p + 6$ substitueren in $(4 - p)^2 + (-1 - q)^2 = 13$ geeft $(4 - p)^2 + (5p - 7)^2 = 13$

$$16 - 8p + p^2 + 25p^2 - 70p + 49 - 13 = 0$$
$$26p^2 - 78p + 52 = 0$$
$$p^2 - 3p + 2 = 0$$
$$(p - 1)(p - 2) = 0$$
$$p = 1 \vee p = 2$$

$p = 1$ geeft $q = -5 \cdot 1 + 6 = 1$ en $p = 2$ geeft $q = -5 \cdot 2 + 6 = -4$.
Dus $(p = 1 \wedge q = 1) \vee (p = 2 \wedge q = -4)$.

46 **a** Omdat een raaklijn van een cirkel loodrecht staat op de straal naar het raakpunt, is de straal van de cirkel gelijk aan de afstand van M tot k, dus $r = d(M, k)$.

b $d(M, k) = \dfrac{|3 - 2 \cdot 4|}{\sqrt{1^2 + (-2)^2}} = \dfrac{|-5|}{\sqrt{5}} = \dfrac{5}{\sqrt{5}} = \sqrt{5}$

Dus c: $(x - 3)^2 + (y - 4)^2 = 5$.

47 **a** k: $y = x$ oftewel k: $x - y = 0$

$r = d(M_1, k) = \dfrac{|5 - 3|}{\sqrt{1^2 + (-1)^2}} = \dfrac{|2|}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

Dus c_1: $(x - 5)^2 + (y - 3)^2 = 2$.

b $r = d(M_2, m) = \dfrac{|-1 + 2 \cdot 3|}{\sqrt{1^2 + 2^2}} = \dfrac{|5|}{\sqrt{5}} = \dfrac{5}{\sqrt{5}} = \sqrt{5}$

Dus c_2: $(x + 1)^2 + (y - 3)^2 = 5$.

48 **a** $r = d(M_1, k) = \dfrac{|2 \cdot 8 + 3 \cdot 3 - 12|}{\sqrt{2^2 + 3^2}} = \dfrac{|13|}{\sqrt{13}} = \dfrac{13}{\sqrt{13}} = \sqrt{13}$

Dus c_1: $(x - 8)^2 + (y - 3)^2 = 13$.

b De cirkels raken de x-as en straal = 8, dus $y_M = -8$ of $y_M = 8$.

$y_M = -8$ en M op k geeft $2x_M + 3 \cdot -8 = 12$

$2x_M - 24 = 12$

$2x_M = 36$

$x_M = 18$

Dus $M_2(18, -8)$ en c_2: $(x - 18)^2 + (y + 8)^2 = 64$.

$y_M = 8$ en M op k geeft $2x_M + 3 \cdot 8 = 12$

$2x_M + 24 = 12$

$2x_M = -12$

$x_M = -6$

Dus $M_3(-6, 8)$ en c_3: $(x + 6)^2 + (y - 8)^2 = 64$.

c k snijden met de x-as geeft $A(6, 0)$.

k snijden met de y-as geeft $B(0, 4)$.

M_4 is het midden van AB, dus $M_4(3, 2)$.

$r = d(A, M_4) = \sqrt{(3 - 6)^2 + (2 - 0)^2} = \sqrt{9 + 4} = \sqrt{13}$

Dus c_4: $(x - 3)^2 + (y - 2)^2 = 13$.

49 **a** $r_1 = d(M, k) = \dfrac{|-1 - 2 \cdot 2 - 10|}{\sqrt{1^2 + (-2)^2}} = \dfrac{|-15|}{\sqrt{5}} = \dfrac{15}{\sqrt{5}} = 3\sqrt{5}$

Dus c_1: $(x + 1)^2 + (y - 2)^2 = (3\sqrt{5})^2$ oftewel c_1: $(x + 1)^2 + (y - 2)^2 = 45$.

k snijden met de x-as geeft $A(10, 0)$.

k raakt c_1, dus $MB \perp k$.

$\left.\begin{array}{l} MB\text{: } 2x + y = c \\ \text{door } M(-1, 2) \end{array}\right\} c = 2 \cdot -1 + 2 = 0$

Dus MB: $2x + y = 0$ oftewel MB: $y = -2x$.

MB snijden met k geeft $x - 2 \cdot -2x = 10$

$x + 4x = 10$

$5x = 10$

$\left.\begin{array}{l} x = 2 \\ y = -2x \end{array}\right\} y = -4$

Dus $B(2, -4)$.

$r_2 = d(A, B) = \sqrt{(2 - 10)^2 + (-4 - 0)^2} = \sqrt{64 + 16} = \sqrt{80}$

Dus c_2: $(x - 10)^2 + y^2 = 80$.

b Het midden van AM is $M_3(4\frac{1}{2}, 1)$.

$r_3 = d(A, M_3) = \sqrt{(4\frac{1}{2} - 10)^2 + (1 - 0)^2} = \sqrt{30\frac{1}{4} + 1} = \sqrt{31\frac{1}{4}}$

Dus c_3: $(x - 4\frac{1}{2})^2 + (y - 1)^2 = 31\frac{1}{4}$.

$x = 2$ en $y = -4$ invullen in de vergelijking van c_3 geeft $(-2\frac{1}{2})^2 + (-5)^2 = 31\frac{1}{4}$

$6\frac{1}{4} + 25 = 31\frac{1}{4}$

Dit klopt, dus c_3 gaat door B.

50 Lijnen l evenwijdig met k op afstand 5 van k.
Stel l: $3x - 4y = c$.
$P(0, -2\frac{1}{2})$ is een punt op k.

$$d(P, l) = 5 \text{ geeft } \frac{|3 \cdot 0 - 4 \cdot -2\frac{1}{2} - c|}{\sqrt{3^2 + (-4)^2}} = 5$$
$$\frac{|10 - c|}{\sqrt{25}} = 5$$
$$\frac{|10 - c|}{5} = 5$$
$$|10 - c| = 25$$
$$10 - c = 25 \vee 10 - c = -25$$
$$c = -15 \vee c = 35$$

Dus l_1: $3x - 4y = -15$ en l_2: $3x - 4y = 35$.
De middelpunten van de cirkels zijn de snijpunten van l_1 en l_2 met de lijn m door A die loodrecht staat op k.

$$\left.\begin{array}{l} m \perp k \text{, dus } m\text{: } 4x + 3y = c \\ \text{door } A(2, -1) \end{array}\right\} c = 4 \cdot 2 + 3 \cdot -1 = 5$$

Dus m: $4x + 3y = 5$.

$$l_1 \text{ en } m \text{ snijden geeft } \begin{cases} 3x - 4y = -15 \\ 4x + 3y = 5 \end{cases} \begin{vmatrix} 3 \\ 4 \end{vmatrix} \text{ geeft } \begin{cases} 9x - 12y = -45 \\ 16x + 12y = 20 \end{cases} +$$
$$25x = -25$$
$$\left.\begin{array}{l} x = -1 \\ 4x + 3y = 5 \end{array}\right\} \begin{array}{l} 4 \cdot -1 + 3y = 5 \\ y = 3 \end{array}$$

Dus snijpunt $(-1, 3)$.

$$l_2 \text{ en } m \text{ snijden geeft } \begin{cases} 3x - 4y = 35 \\ 4x + 3y = 5 \end{cases} \begin{vmatrix} 3 \\ 4 \end{vmatrix} \text{ geeft } \begin{cases} 9x - 12y = 105 \\ 16x + 12y = 20 \end{cases} +$$
$$25x = 125$$
$$\left.\begin{array}{l} x = 5 \\ 4x + 3y = 5 \end{array}\right\} \begin{array}{l} 4 \cdot 5 + 3y = 5 \\ y = -5 \end{array}$$

Dus snijpunt $(5, -5)$.
Dus de vergelijkingen van deze cirkels zijn $(x + 1)^2 + (y - 3)^2 = 25$ en $(x - 5)^2 + (y + 5)^2 = 25$.

51 $(x - 4)^2 + (y + 1)^2 = 36$
$x^2 - 8x + 16 + y^2 + 2y + 1 = 36$
$x^2 + y^2 - 8x + 2y - 19 = 0$

Bladzijde 113

52 **a** $x^2 + y^2 + 6x - 4y + 4 = 0$
$x^2 + 6x + y^2 - 4y + 4 = 0$
$(x + 3)^2 - 9 + (y - 2)^2 - 4 + 4 = 0$
$(x + 3)^2 + (y - 2)^2 = 9$
Dus van c_1 is het middelpunt $(-3, 2)$ en de straal 3.

b $x^2 + y^2 - 8x + 10y + 31 = 0$
$x^2 - 8x + y^2 + 10y + 31 = 0$
$(x - 4)^2 - 16 + (y + 5)^2 - 25 + 31 = 0$
$(x - 4)^2 + (y + 5)^2 = 10$
Dus van c_2 is het middelpunt $(4, -5)$ en de straal $\sqrt{10}$.

c $x^2 + y^2 + 5x + 3y + 3 = 0$
$x^2 + 5x + y^2 + 3y + 3 = 0$
$(x + 2\frac{1}{2})^2 - 6\frac{1}{4} + (y + 1\frac{1}{2})^2 - 2\frac{1}{4} + 3 = 0$
$(x + 2\frac{1}{2})^2 + (y + 1\frac{1}{2})^2 = 5\frac{1}{2}$
Dus van c_3 is het middelpunt $(-2\frac{1}{2}, -1\frac{1}{2})$ en de straal $\sqrt{5\frac{1}{2}}$.

d $x^2 + y^2 - 7x + 8y = 0$
$x^2 - 7x + y^2 + 8y = 0$
$(x - 3\frac{1}{2})^2 - 12\frac{1}{4} + (y + 4)^2 - 16 = 0$
$(x - 3\frac{1}{2})^2 + (y + 4)^2 = 28\frac{1}{4}$
Dus van c_4 is het middelpunt $(3\frac{1}{2}, -4)$ en de straal $\sqrt{28\frac{1}{4}}$.

53 **a** $x^2 + y^2 + 6x - 8y + 15 = 0$
$x^2 + 6x + y^2 - 8y + 15 = 0$
$(x + 3)^2 - 9 + (y - 4)^2 - 16 + 15 = 0$
$(x + 3)^2 + (y - 4)^2 = 10$
Dus $M(-3, 4)$ en $r = \sqrt{10}$.

b $d(A, M) = \sqrt{(-3 - 0)^2 + (4 - 4)^2} = \sqrt{9 + 0} = 3$
A ligt binnen c want $d(A, M) < r$.

c $d(B, M) = \sqrt{(-3 - -3)^2 + (4 - 0)^2} = \sqrt{0 + 16} = 4$
B ligt buiten c want $d(B, M) > r$.

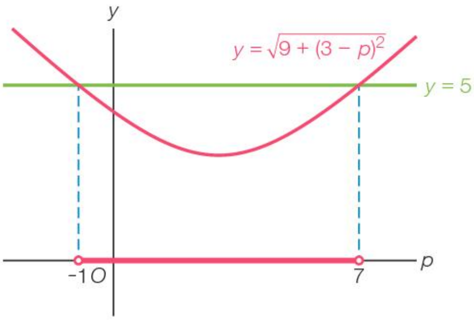
d $d(C, M) = \sqrt{(-3 - -2)^2 + (4 - 1)^2} = \sqrt{1 + 9} = \sqrt{10}$
C ligt op c want $d(C, M) = r$.

54 **a** $x^2 + y^2 - 6x - 8y = 0$
$x^2 - 6x + y^2 - 8y = 0$
$(x - 3)^2 - 9 + (y - 4)^2 - 16 = 0$
$(x - 3)^2 + (y - 4)^2 = 25$
Dus $r = 5$ en $M(3, 4)$.
$d(A, M) = \sqrt{(3 - -1)^2 + (4 - 2)^2} = \sqrt{16 + 4} = \sqrt{20}$
$d(A, M) < r$, dus A ligt binnen c_1.

b $x^2 + y^2 - 8x - 4y + 2 = 0$
$x^2 - 8x + y^2 - 4y + 2 = 0$
$(x - 4)^2 - 16 + (y - 2)^2 - 4 + 2 = 0$
$(x - 4)^2 + (y - 2)^2 = 18$
Dus $r = \sqrt{18}$ en $M(4, 2)$.
$d(O, M) = \sqrt{4^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20}$
$d(O, M) > r$, dus O ligt buiten c_2.

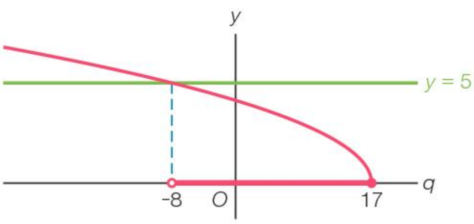
c $x^2 + y^2 + 4x - 6y + 3 = 0$
$x^2 + 4x + y^2 - 6y + 3 = 0$
$(x + 2)^2 - 4 + (y - 3)^2 - 9 + 3 = 0$
$(x + 2)^2 + (y - 3)^2 = 10$
Dus $r = \sqrt{10}$ en $M(-2, 3)$.
$d(B, M) = \sqrt{(-2 - 1)^2 + (3 - 4)^2} = \sqrt{9 + 1} = \sqrt{10}$
$d(B, M) = r$, dus B ligt op c_3.

55 **a** $x^2 + y^2 + 4x - 6y - 12 = 0$
$x^2 + 4x + y^2 - 6y - 12 = 0$
$(x + 2)^2 - 4 + (y - 3)^2 - 9 - 12 = 0$
$(x + 2)^2 + (y - 3)^2 = 25$
Dus $r = 5$ en $M(-2, 3)$.
$d(P, M) = \sqrt{(-2 - 1)^2 + (3 - p)^2} = \sqrt{9 + (p - 3)^2}$
$d(P, M) = r$ geeft $\sqrt{9 + (3 - p)^2} = 5$
$\sqrt{9 + 9 - 6p + p^2} = 5$
$\sqrt{p^2 - 6p + 18} = 5$
$p^2 - 6p + 18 = 25$
$p^2 - 6p - 7 = 0$
$(p + 1)(p - 7) = 0$
$p = -1 \vee p = 7$



$d(P, M) < 5$ geeft $-1 < p < 7$
Dus voor $-1 < p < 7$ ligt P binnen c_1.

b $x^2 + y^2 - 8x - 2y + q = 0$
$x^2 - 8x + y^2 - 2y + q = 0$
$(x - 4)^2 - 16 + (y - 1)^2 - 1 + q = 0$
$(x - 4)^2 + (y - 1)^2 = 17 - q$
Dus $r = \sqrt{17 - q}$ en $M(4, 1)$.
$d(Q, M) = \sqrt{(4 - 1)^2 + (1 - 5)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$
$d(Q, M) = r$ geeft $5 = \sqrt{17 - q}$
$25 = 17 - q$
$q = -8$



$d(Q, M) > r$ oftewel $5 > \sqrt{17 - q}$ geeft $-8 < q \leq 17$.
Dus voor $-8 < q \leq 17$ ligt Q buiten c_2.

56 $x^2 + y^2 + ax + by = 71$

$x^2 + ax + y^2 + by = 71$

$(x + \frac{1}{2}a)^2 - \frac{1}{4}a^2 + (y + \frac{1}{2}b)^2 - \frac{1}{4}b^2 = 71$

$(x + \frac{1}{2}a)^2 + (y + \frac{1}{2}b)^2 = \frac{1}{4}a^2 + \frac{1}{4}b^2 + 71$

Van c is het middelpunt $M(-\frac{1}{2}a, -\frac{1}{2}b)$ en de straal $r = \sqrt{\frac{1}{4}a^2 + \frac{1}{4}b^2 + 71}$.

$M(-\frac{1}{2}a, -\frac{1}{2}b)$ op k: $x + 2y = 1$ geeft $-\frac{1}{2}a - b = 1$ oftewel $b = -\frac{1}{2}a - 1$.

De straal is 10 geeft $\sqrt{\frac{1}{4}a^2 + \frac{1}{4}b^2 + 71} = 10$ oftewel $\frac{1}{4}a^2 + \frac{1}{4}b^2 + 71 = 100$.

$b = -\frac{1}{2}a - 1$ substitueren in $\frac{1}{4}a^2 + \frac{1}{4}b^2 + 71 = 100$ geeft $\frac{1}{4}a^2 + \frac{1}{4}(-\frac{1}{2}a - 1)^2 + 71 = 100$

$\frac{1}{4}a^2 + \frac{1}{4}(\frac{1}{4}a^2 + a + 1) + 71 = 100$

$\frac{1}{4}a^2 + \frac{1}{16}a^2 + \frac{1}{4}a + \frac{1}{4} + 71 = 100$

$\frac{5}{16}a^2 + \frac{1}{4}a - 28\frac{3}{4} = 0$

$5a^2 + 4a - 460 = 0$

$D = 4^2 - 4 \cdot 5 \cdot -460 = 9216$

$a = \frac{-4 + 96}{10} = 9\frac{1}{5} \vee a = \frac{-4 - 96}{10} = -10$

$a = 9\frac{1}{5}$ geeft $b = -\frac{1}{2} \cdot 9\frac{1}{5} - 1 = -5\frac{3}{5}$ en $a = -10$ geeft $b = -\frac{1}{2} \cdot -10 - 1 = 4$.

Dus $(a = 9\frac{1}{5} \wedge b = -5\frac{3}{5}) \vee (a = -10 \wedge b = 4)$.

7.4 Afstanden en raaklijnen bij cirkels

Bladzijde 115

57 **a** $x^2 + y^2 - 10x - 6y + 18 = 0$

$x^2 - 10x + y^2 - 6y + 18 = 0$

$(x - 5)^2 - 25 + (y - 3)^2 - 9 + 18 = 0$

$(x - 5)^2 + (y - 3)^2 = 16$

Dus $M(5, 3)$ en $r = \sqrt{16} = 4$.

b $d(P, M) = \sqrt{(5 - 1)^2 + (3 - 6)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$d(P, M) > r$, dus P ligt buiten c.

$d(Q, M) = \sqrt{(5 - 6)^2 + (3 - -2)^2} = \sqrt{1 + 25} = \sqrt{26}$

$d(Q, M) > r$, dus Q ligt buiten c.

c $5 < \sqrt{26}$, dus P ligt dichter bij het middelpunt M dan Q.

Dus $d(P, c) < d(Q, c)$.

Bladzijde 116

58 **a** $x^2 + y^2 - 6x - 4y + 3 = 0$

$x^2 - 6x + y^2 - 4y + 3 = 0$

$(x - 3)^2 - 9 + (y - 2)^2 - 4 + 3 = 0$

$(x - 3)^2 + (y - 2)^2 = 10$

Dus $M(3, 2)$ en $r = \sqrt{10}$.

$d(A, M) = \sqrt{(3 - 2)^2 + (2 - 1)^2} = \sqrt{1 + 1} = \sqrt{2}$

$d(A, c) = r - d(A, M) = \sqrt{10} - \sqrt{2}$

b $d(B, M) = \sqrt{(3 - -1)^2 + (2 - 5)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$d(B, c) = d(B, M) - r = 5 - \sqrt{10}$

c $d(C, M) = \sqrt{(3 - 9)^2 + (2 - 4)^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$

$d(C, c) = d(C, M) - r = 2\sqrt{10} - \sqrt{10} = \sqrt{10}$

59 M op k, dus $2 + 4y_M = 16$ en dit geeft $y_M = 3\frac{1}{2}$.
Dus de straal r_1 van c_1 is $3\frac{1}{2}$ en $M(2, 3\frac{1}{2})$.
N op k, dus $8 + 4y_M = 16$ en dit geeft $y_M = 2$.
Dus de straal r_2 van c_2 is 2 en $N(8, 2)$.
$d(M, N) = \sqrt{(8-2)^2 + (2 - 3\frac{1}{2})^2} = \sqrt{36 + 2\frac{1}{4}} = \sqrt{38\frac{1}{4}} = 1\frac{1}{2}\sqrt{17}$
Dus $d(c_1, c_2) = d(M, N) - r_1 - r_2 = 1\frac{1}{2}\sqrt{17} - 3\frac{1}{2} - 2 = 1\frac{1}{2}\sqrt{17} - 5\frac{1}{2}$.

Bladzijde 117

60 **a** $x^2 + y^2 - 6x - 2y + 9 = 0$
$x^2 - 6x + y^2 - 2y + 9 = 0$
$(x-3)^2 - 9 + (y-1)^2 - 1 + 9 = 0$
$(x-3)^2 + (y-1)^2 = 1$
Dus $M(3, 1)$ en $r_1 = 1$.
$x^2 + y^2 - 12x - 4y + 36 = 0$
$x^2 - 12x + y^2 - 4y + 36 = 0$
$(x-6)^2 - 36 + (y-2)^2 - 4 + 36 = 0$
$(x-6)^2 + (y-2)^2 = 4$
Dus $N(6, 2)$ en $r_2 = 2$.
$d(M, N) = \sqrt{(6-3)^2 + (2-1)^2} = \sqrt{9+1} = \sqrt{10}$
Dus $d(c_1, c_2) = d(M, N) - r_1 - r_2 = \sqrt{10} - 1 - 2 = \sqrt{10} - 3$.

b $d(M, l) = \dfrac{|3 \cdot 3 - 4 \cdot 1|}{\sqrt{3^2 + (-4)^2}} = \dfrac{|5|}{\sqrt{25}} = \dfrac{5}{5} = 1$, dus $d(M, l) = r_1$, dus l raakt c_1.

$d(N, l) = \dfrac{|3 \cdot 6 - 4 \cdot 2|}{\sqrt{3^2 + (-4)^2}} = \dfrac{|10|}{\sqrt{25}} = \dfrac{10}{5} = 2$, dus $d(N, l) = r_2$, dus l raakt c_2.

c $r_3 = d(P, l) = \dfrac{|3 \cdot 2 - 4 \cdot 5|}{\sqrt{3^2 + (-4)^2}} = \dfrac{|-14|}{\sqrt{25}} = \dfrac{14}{5} = 2\frac{4}{5}$

Dus c_3: $(x-2)^2 + (y-5)^2 = (2\frac{4}{5})^2$ oftewel c_3: $(x-2)^2 + (y-5)^2 = 7\frac{21}{25}$.

61 $d(M, N) = \sqrt{(9-4)^2 + (-10-2)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

De straal van c_1 heeft lengte $d(M, k) = \dfrac{|4 \cdot 4 - 3 \cdot 2|}{\sqrt{4^2 + (-3)^2}} = \dfrac{|10|}{\sqrt{25}} = \dfrac{10}{5} = 2$.

Er geldt $d(M, N) = 2 + 2r + r$, dus $2 + 2r + r = 13$
$3r = 11$
$r = 3\frac{2}{3}$
Dus c_2: $(x-9)^2 + (y+10)^2 = (3\frac{2}{3})^2$ oftewel c_2: $(x-9)^2 + (y+10)^2 = 13\frac{4}{9}$.

62 $x^2 + y^2 + 4x - 2y - 4 = 0$
$x^2 + 4x + y^2 - 2y - 4 = 0$
$(x+2)^2 - 4 + (y-1)^2 - 1 - 4 = 0$
$(x+2)^2 + (y-1)^2 = 9$
Dus $r = 3$ en $M(-2, 1)$.
$d(M, k) = \dfrac{|2 \cdot -2 + 1 - 7|}{\sqrt{2^2 + 1^2}} = \dfrac{|-10|}{\sqrt{5}} = \dfrac{10}{\sqrt{5}} = 2\sqrt{5}$
Dus $d(k, c) = d(M, k) - r = 2\sqrt{5} - 3$.

63 $x^2 + y^2 + 8x + 2y - 8 = 0$
$x^2 + 8x + y^2 + 2y - 8 = 0$
$(x+4)^2 - 16 + (y+1)^2 - 1 - 8 = 0$
$(x+4)^2 + (y+1)^2 = 25$
Dus $r = 5$ en $M(-4, -1)$.
l: $y = -3x + 17$ oftewel l: $3x + y - 17 = 0$

$$d(M, l) = \frac{|3 \cdot -4 - 1 - 17|}{\sqrt{3^2 + 1^2}} = \frac{|-30|}{\sqrt{10}} = \frac{30}{\sqrt{10}} = 3\sqrt{10}$$

$d(l, c) = d(M, l) - r = 3\sqrt{10} - 5$
Is $3\sqrt{10} - 5 < 5$ oftewel $\sqrt{90} < 10$?
Ja, dit klopt, dus $d(l, c) < r$.

Bladzijde 118

64 Stel $r_1 = x$, dan is $r_2 = 2x$, $r_3 = 3\frac{1}{2}x$ en $d(c_1, c_2) = 2r_3 - 2r_1 - 2r_2 = 7x - 2x - 4x = x$.
opp $c_1 = \pi x^2$
opp $c_2 = \pi(2x)^2 = 4\pi x^2$
opp $c_3 = \pi(3\frac{1}{2}x)^2 = 12\frac{1}{4}\pi x^2$
opp gekleurde gebied = opp c_3 − opp c_1 − opp c_2 $= 12\frac{1}{4}\pi x^2 - \pi x^2 - 4\pi x^2 = 7\frac{1}{4}\pi x^2$
opp gekleurde gebied = π geeft $7\frac{1}{4}\pi x^2 = \pi$

$$x^2 = \frac{1}{7\frac{1}{4}} = \frac{4}{29}$$

$$x = \sqrt{\frac{4}{29}} = \frac{2}{29}\sqrt{29}$$

Dus $d(c_1, c_2) = \frac{2}{29}\sqrt{29}$.

65 **a** $\left.\begin{array}{l}(x-2)^2 + (y-1)^2 = 10\\ A(5, 2)\end{array}\right\}$ $(5-2)^2 + (2-1)^2 = 10$
$3^2 + 1^2 = 10$
$10 = 10$
klopt
Dus A op c.

b $M(2, 1)$ en $A(5, 2)$, dus $\text{rc}_l = \frac{2-1}{5-2} = \frac{1}{3}$.

c k staat loodrecht op l geeft $\frac{1}{3} \cdot \text{rc}_k = -1$, dus $\text{rc}_k = -3$.
$\left.\begin{array}{l}k\text{: } y = -3x + b\\ \text{door } A(5, 2)\end{array}\right\}$ $-3 \cdot 5 + b = 2$
$-15 + b = 2$
$b = 17$
Dus k: $y = -3x + 17$.

Bladzijde 119

66 **a** Stel l: $y = ax + b$.
Lijn m door M en B heeft $\text{rc}_m = \frac{1 - -1}{3 - 2} = 2$, dus $a = \text{rc}_l = -\frac{1}{2}$.
$\left.\begin{array}{l}l\text{: } y = -\frac{1}{2}x + b\\ \text{door } B(2, -1)\end{array}\right\}$ $-\frac{1}{2} \cdot 2 + b = -1$
$-1 + b = -1$
$b = 0$
Dus l: $y = -\frac{1}{2}x$.

b De straal van c is $r = \sqrt{5}$.
n: $y = 2x$ oftewel n: $2x - y = 0$

$$d(M, n) = \frac{|2 \cdot 3 - 1|}{\sqrt{2^2 + (-1)^2}} = \frac{|5|}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$d(M, n) = r$, dus n: $y = 2x$ raakt aan c.

c De x-as snijden, dus $y = 0$ geeft $(x-3)^2 + (-1)^2 = 5$

$$(x-3)^2 + 1 = 5$$
$$(x-3)^2 = 4$$
$$x - 3 = 2 \vee x - 3 = -2$$
$$x = 5 \vee x = 1$$

Dus $x_C = 1$.
Stel p: $y = ax + b$.

Lijn m door M en C heeft $\text{rc}_m = \frac{1-0}{3-1} = \frac{1}{2}$, dus $a = \text{rc}_p = -2$.

$$\left.\begin{array}{l} p\text{: } y = -2x + b \\ \text{door } C(1, 0) \end{array}\right\} \begin{array}{l} -2 \cdot 1 + b = 0 \\ b = 2 \end{array}$$

Dus p: $y = -2x + 2$.

Bladzijde 120

67 **a** Substitutie van $x = 3$ in de vergelijking van c geeft $9 + y^2 - 36 + 11 = 0$

$$y^2 = 16$$
$$y = 4 \vee y = -4$$

$y_A > y_B$, dus $A(3, 4)$ en $B(3, -4)$.
$x^2 + y^2 - 12x + 11 = 0$
$x^2 - 12x + y^2 + 11 = 0$
$(x-6)^2 - 36 + y^2 + 11 = 0$
$(x-6)^2 + y^2 = 25$
Dus $M(6, 0)$ en $r = 5$.

Lijn m door M en A heeft $\text{rc}_m = \frac{0-4}{6-3} = -\frac{4}{3}$, dus $\text{rc}_k = \frac{3}{4}$.

$$\left.\begin{array}{l} k\text{: } y = \frac{3}{4}x + b \\ \text{door } A(3, 4) \end{array}\right\} \begin{array}{l} \frac{3}{4} \cdot 3 + b = 4 \\ 2\frac{1}{4} + b = 4 \\ b = 1\frac{3}{4} \end{array}$$

Dus k: $y = \frac{3}{4}x + 1\frac{3}{4}$.

Lijn n door M en B heeft $\text{rc}_n = \frac{0--4}{6-3} = \frac{4}{3}$, dus $\text{rc}_l = -\frac{3}{4}$.

$$\left.\begin{array}{l} l\text{: } y = -\frac{3}{4}x + b \\ \text{door } B(3, -4) \end{array}\right\} \begin{array}{l} -\frac{3}{4} \cdot 3 + b = -4 \\ -2\frac{1}{4} + b = -4 \\ b = -1\frac{3}{4} \end{array}$$

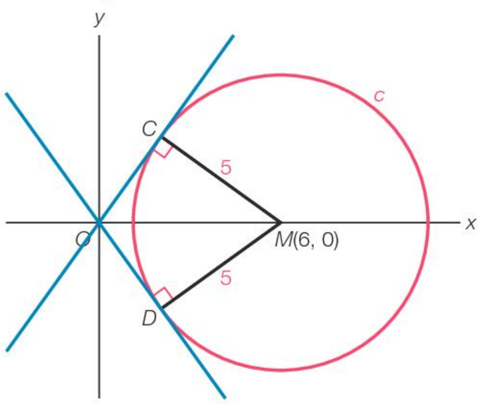
Dus l: $y = -\frac{3}{4}x - 1\frac{3}{4}$.

b $\tan(\alpha) = \text{rc}_k = \frac{3}{4}$ geeft $\alpha = 36{,}86...°$

$\tan(\beta) = \text{rc}_l = -\frac{3}{4}$ geeft $\beta = -36{,}86...°$

Dus $\angle(k, l) = 36{,}86...° - -36{,}86...° \approx 74°$.

c De raaklijnen door O raken c in de punten C en D.



In $\triangle OMC$ is $\angle C = 90°$, $OM = 6$ en $CM = 5$.
$\sin(\angle MOC) = \frac{5}{6}$ geeft $\angle MOC = 56{,}4...°$
$\angle DOC = 2 \cdot \angle MOC = 2 \cdot 56{,}4...° \approx 113°$
De hoek tussen de raaklijnen door O is ongeveer $180° - 113° = 67°$.

68 **a** $x^2 + y^2 + 6x - 6y - 7 = 0$
$x^2 + 6x + y^2 - 6y - 7 = 0$
$(x+3)^2 - 9 + (y-3)^2 - 9 - 7 = 0$
$(x+3)^2 + (y-3)^2 = 25$
Dus $M(-3, 3)$ en $r = 5$.
$y = 0$ geeft $x^2 + 6x - 7 = 0$
$(x-1)(x+7) = 0$
$x = 1 \vee x = -7$
Dus $A(-7, 0)$ en $B(1, 0)$.
$\text{rc}_{MA} = \dfrac{3-0}{-3--7} = \frac{3}{4}$
$k \perp MA$, dus $\text{rc}_k = -\frac{4}{3}$.
$\left.\begin{array}{l} k\text{: } y = -\frac{4}{3}x + b \\ \text{door } A(-7, 0) \end{array}\right\} -\frac{4}{3} \cdot -7 + b = 0$
$\frac{28}{3} + b = 0$
$b = -9\frac{1}{3}$
Dus k: $y = -1\frac{1}{3}x - 9\frac{1}{3}$.
$\text{rc}_{MB} = \dfrac{0-3}{1--3} = -\frac{3}{4}$
$l \perp MB$, dus $\text{rc}_l = \frac{4}{3}$.
$\left.\begin{array}{l} l\text{: } y = \frac{4}{3}x + b \\ \text{door } B(1, 0) \end{array}\right\} \frac{4}{3} \cdot 1 + b = 0$
$b = -1\frac{1}{3}$
Dus l: $y = 1\frac{1}{3}x - 1\frac{1}{3}$.

b $x = 0$ geeft $y^2 - 6y - 7 = 0$
$(y+1)(y-7) = 0$
$y = -1 \vee y = 7$
Dus $C(0, -1)$ en $D(0, 7)$.
$\text{rc}_{MC} = \dfrac{-1-3}{0--3} = -\frac{4}{3}$
$p \perp MC$, dus $\text{rc}_p = \frac{3}{4}$.
p: $y = \frac{3}{4}x + b$ door $C(0, -1)$, dus p: $y = \frac{3}{4}x - 1$.
$\text{rc}_{MD} = \dfrac{7-3}{0--3} = \frac{4}{3}$
$q \perp MD$, dus $\text{rc}_q = -\frac{3}{4}$.
q: $y = -\frac{3}{4}x + b$ door $D(0, 7)$, dus q: $y = -\frac{3}{4}x + 7$.

c $\text{rc}_k = -1\frac{1}{3}$ geeft richtingshoek $\alpha_k = -53{,}13...°$
$\text{rc}_l = 1\frac{1}{3}$ geeft richtingshoek $\alpha_l = 53{,}13...°$
$\alpha_l - \alpha_k = 53{,}13...° - -53{,}13...° = 106{,}26...°$
$\angle(k, l) = 180° - 106{,}26...° = 73{,}73...°$
$\text{rc}_p = \frac{3}{4}$ geeft richtingshoek $\alpha_p = 36{,}86...°$
$\text{rc}_q = -\frac{3}{4}$ geeft richtingshoek $\alpha_q = -36{,}86...°$
$\alpha_p - \alpha_q = 36{,}86...° - -36{,}86...° = 73{,}73...°$
$\angle(p, q) = 73{,}73...°$
Dus de hoek tussen k en l is gelijk aan de hoek tussen p en q.

69 **a** $x^2 + y^2 - 4x + 2y - 12 = 0$
$x^2 - 4x + y^2 + 2y - 12 = 0$
$(x-2)^2 - 4 + (y+1)^2 - 1 - 12 = 0$
$(x-2)^2 + (y+1)^2 = 17$
Dus $M(2, -1)$ en $r = \sqrt{17}$.
$y = 0$ geeft $x^2 - 4x - 12 = 0$
$(x+2)(x-6) = 0$
$x = -2 \vee x = 6$
Dus $A(-2, 0)$ en $B(6, 0)$.
$\text{rc}_{MA} = \dfrac{-1-0}{2--2} = -\frac{1}{4}$
$k \perp MA$, dus $\text{rc}_k = 4$ en dit geeft richtingshoek $\alpha = 75{,}96...°$
$\text{rc}_{MC} = \dfrac{-1-3}{2-1} = -4$
$m \perp MC$, dus $\text{rc}_m = \frac{1}{4}$ en dit geeft richtingshoek $\beta = 14{,}03...°$
$\alpha - \beta = 75{,}96...° - 14{,}03...° \approx 62°$
Dus $\angle(k, m) \approx 62°$.

b $\text{rc}_{MA} = -\frac{1}{4}$ geeft $\angle BAM = 14{,}03...°$
Ook $\angle ABM = 14{,}03...°$.
Dus $\angle AMB = 180° - 2 \cdot 14{,}03...° \approx 152°$.

c $\left.\begin{array}{l} m\colon y = \frac{1}{4}x + b \\ \text{door } C(1, 3) \end{array}\right\} \begin{array}{l} \frac{1}{4} + b = 3 \\ b = 2\frac{3}{4} \end{array}$
Dus $m\colon y = \frac{1}{4}x + 2\frac{3}{4}$.
$\text{rc}_{MB} = \dfrac{0--1}{6-2} = \frac{1}{4}$
$l \perp MB$, dus $\text{rc}_l = -4$.
$\left.\begin{array}{l} l\colon y = -4x + b \\ \text{door } B(6, 0) \end{array}\right\} \begin{array}{l} -4 \cdot 6 + b = 0 \\ b = 24 \end{array}$
Dus $l\colon y = -4x + 24$.
l en m snijden geeft $-4x + 24 = \frac{1}{4}x + 2\frac{3}{4}$
$-4\frac{1}{4}x = -21\frac{1}{4}$
$\left.\begin{array}{l} x = 5 \\ y = -4x + 24 \end{array}\right\} y = -4 \cdot 5 + 24 = 4$
Dus $D(5, 4)$.
$d(D, M) = \sqrt{(2-5)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$
$d(D, c) = d(D, M) - r = \sqrt{34} - \sqrt{17}$

70 **a** De straal van c is $r = \sqrt{5}$.
k raakt c, dus geldt $d(M, k) = r = \sqrt{5}$.

b $k\colon y = \frac{1}{2}x + b$
$-\frac{1}{2}x + y - b = 0$
$x - 2y + 2b = 0$

c Het middelpunt van c is $M(2, 3)$.
$d(M, k) = \dfrac{|2 - 2 \cdot 3 + 2b|}{\sqrt{1^2 + (-2)^2}} = \dfrac{|2b-4|}{\sqrt{5}}$
$d(M, k) = \sqrt{5}$ geeft $\dfrac{|2b-4|}{\sqrt{5}} = \sqrt{5}$ en hieruit volgt $|2b-4| = 5$.

d $|2b-4| = 5$ geeft $2b - 4 = 5 \vee 2b - 4 = -5$
$2b = 9 \vee 2b = -1$
$b = 4\frac{1}{2} \vee b = -\frac{1}{2}$
Dus $k_1\colon y = \frac{1}{2}x + 4\frac{1}{2}$ en $k_2\colon y = \frac{1}{2}x - \frac{1}{2}$.

Bladzijde 122

71 **a** Van c is $M(0, 0)$ en $r = \sqrt{10}$.
Stel l: $y = 3x + b$ oftewel l: $3x - y + b = 0$.

$d(M, l) = r$ geeft $\dfrac{|3 \cdot 0 - 0 + b|}{\sqrt{10}} = \sqrt{10}$

$|b| = 10$
$b = 10 \vee b = -10$

Dus l_1: $y = 3x + 10$ en l_2: $y = 3x - 10$.

b Stel m: $y = ax + b$.
$A(10, 0)$ op m geeft $10a + b = 0$, dus $b = -10a$.
m: $y = ax - 10a$ oftewel m: $ax - y - 10a = 0$

$d(M, m) = r$ geeft $\dfrac{|a \cdot 0 - 0 - 10a|}{\sqrt{a^2 + 1}} = \sqrt{10}$

$|-10a| = \sqrt{10a^2 + 10}$
$100a^2 = 10a^2 + 10$
$90a^2 = 10$
$a^2 = \frac{1}{9}$
$a = \frac{1}{3} \vee a = -\frac{1}{3}$

$a = \frac{1}{3}$ geeft $b = -10 \cdot \frac{1}{3} = -3\frac{1}{3}$, dus m_1: $y = \frac{1}{3}x - 3\frac{1}{3}$.
$a = -\frac{1}{3}$ geeft $b = -10 \cdot -\frac{1}{3} = 3\frac{1}{3}$, dus m_2: $y = -\frac{1}{3}x + 3\frac{1}{3}$.

72 **a** Stel k: $y = ax + b$.
$O(0, 0)$ is het middelpunt van c.
Lijn p door O en A heeft $\text{rc}_p = \dfrac{0 - -1}{0 - -4} = \frac{1}{4}$, dus $a = \text{rc}_k = -4$.

$\left.\begin{array}{l} k: y = -4x + b \\ \text{door } A(-4, -1) \end{array}\right\} -4 \cdot -4 + b = -1$
$16 + b = -1$
$b = -17$

Dus k: $y = -4x - 17$.

b Loodrecht op l: $4x - y = 3$, dus m: $x + 4y = c$.

$d(O, m) = r$ geeft $\dfrac{|0 + 4 \cdot 0 - c|}{\sqrt{1^2 + 4^2}} = \sqrt{17}$

$\dfrac{|-c|}{\sqrt{17}} = \sqrt{17}$
$|-c| = 17$
$-c = 17 \vee -c = -17$
$c = -17 \vee c = 17$

Dus m_1: $x + 4y = -17$ en m_2: $x + 4y = 17$.

c Door $(0, 17)$, dus n: $y = ax + 17$ oftewel n: $ax - y + 17 = 0$.

$d(M, n) = r$ geeft $\dfrac{|a \cdot 0 - 0 + 17|}{\sqrt{a^2 + 1}} = \sqrt{17}$

$\dfrac{17}{\sqrt{a^2 + 1}} = \sqrt{17}$
$\sqrt{17a^2 + 17} = 17$
$17a^2 + 17 = 289$
$17a^2 = 272$
$a^2 = 16$
$a = 4 \vee a = -4$

Dus n_1: $y = 4x + 17$ en n_2: $y = -4x + 17$.

73 **a** $x^2 + y^2 - 10x - 4y + 19 = 0$
$x^2 - 10x + y^2 - 4y + 19 = 0$
$(x-5)^2 - 25 + (y-2)^2 - 4 + 19 = 0$
$(x-5)^2 + (y-2)^2 = 10$
$M(5, 2)$ is het middelpunt van c.
Stel k: $y = ax + b$.
Lijn n door M en A heeft $\text{rc}_n = \frac{2-5}{5-4} = -3$, dus $a = \text{rc}_k = \frac{1}{3}$.
$\left.\begin{array}{l} k: y = \frac{1}{3}x + b \\ \text{door } A(4, 5) \end{array}\right\} \frac{1}{3} \cdot 4 + b = 5$
$1\frac{1}{3} + b = 5$
$b = 3\frac{2}{3}$
Dus k: $y = \frac{1}{3}x + 3\frac{2}{3}$.

b Stel l: $y = 3x + b$ oftewel l: $3x - y + b = 0$.
$d(M, l) = r$ geeft $\frac{|3 \cdot 5 - 2 + b|}{\sqrt{10}} = \sqrt{10}$
$|13 + b| = 10$
$13 + b = 10 \vee 13 + b = -10$
$b = -3 \vee b = -23$
Dus l_1: $y = 3x - 3$ en l_2: $y = 3x - 23$.

c Stel m: $y = ax + b$.
$B(9, 0)$ op m geeft $9a + b = 0$, dus $b = -9a$.
m: $y = ax - 9a$ oftewel m: $ax - y - 9a = 0$
$d(M, m) = r$ geeft $\frac{|5a - 2 - 9a|}{\sqrt{a^2 + 1}} = \sqrt{10}$
$|-4a - 2| = \sqrt{10a^2 + 10}$
$16a^2 + 16a + 4 = 10a^2 + 10$
$6a^2 + 16a - 6 = 0$
$3a^2 + 8a - 3 = 0$
$D = 8^2 - 4 \cdot 3 \cdot -3 = 100$
$a = \frac{-8 + 10}{6} = \frac{1}{3} \vee a = \frac{-8 - 10}{6} = -3$
$a = \frac{1}{3}$ geeft $b = -9 \cdot \frac{1}{3} = -3$, dus m_1: $y = \frac{1}{3}x - 3$.
$a = -3$ geeft $b = -9 \cdot -3 = 27$, dus m_2: $y = -3x + 27$.

74 **a** $x^2 + y^2 - 2x - 4y - 12 = 0$
$x^2 - 2x + y^2 - 4y - 12 = 0$
$(x-1)^2 - 1 + (y-2)^2 - 4 - 12 = 0$
$(x-1)^2 + (y-2)^2 = 17$
$M(1, 2)$ is het middelpunt van c.
Stel k: $y = ax + b$.
Lijn p door M en A heeft $\text{rc}_p = \frac{2-1}{1--3} = \frac{1}{4}$, dus $a = \text{rc}_k = -4$.
$\left.\begin{array}{l} k: y = -4x + b \\ \text{door } A(-3, 1) \end{array}\right\} -4 \cdot -3 + b = 1$
$12 + b = 1$
$b = -11$
Dus k: $y = -4x - 11$.

b Loodrecht op m: $4x - y = 1$, dus l: $x + 4y = c$.
$d(M, l) = r$ geeft $\frac{|1 + 4 \cdot 2 - c|}{\sqrt{17}} = \sqrt{17}$
$|9 - c| = 17$
$9 - c = 17 \vee 9 - c = -17$
$-c = 8 \vee -c = -26$
$c = -8 \vee c = 26$
Dus l_1: $x + 4y = -8$ en l_2: $x + 4y = 26$.

c Stel n: $y = ax + b$.
$B(6, -1)$ op n geeft $6a + b = -1$, dus $b = -6a - 1$.
n: $y = ax - 6a - 1$ oftewel n: $ax - y - 6a - 1 = 0$

$$d(M, n) = r \text{ geeft } \frac{|a - 2 - 6a - 1|}{\sqrt{a^2 + 1}} = \sqrt{17}$$
$$|-5a - 3| = \sqrt{17a^2 + 17}$$
$$25a^2 + 30a + 9 = 17a^2 + 17$$
$$8a^2 + 30a - 8 = 0$$
$$4a^2 + 15a - 4 = 0$$
$$D = 15^2 - 4 \cdot 4 \cdot -4 = 289$$
$$a = \frac{-15 + 17}{8} = \tfrac{1}{4} \vee a = \frac{-15 - 17}{8} = -4$$

$a = \frac{1}{4}$ geeft $b = -6 \cdot \frac{1}{4} - 1 = -2\frac{1}{2}$, dus n_1: $y = \frac{1}{4}x - 2\frac{1}{2}$.
$a = -4$ geeft $b = -6 \cdot -4 - 1 = 23$, dus n_2: $y = -4x + 23$.

75 $x^2 + y^2 - 10x + 4y + 9 = 0$
$x^2 - 10x + y^2 + 4y + 9 = 0$
$(x - 5)^2 - 25 + (y + 2)^2 - 4 + 9 = 0$
$(x - 5)^2 + (y + 2)^2 = 20$
Dus $M(5, -2)$ en $r = \sqrt{20}$.
Stel k: $y = ax + b$.
$P(-1, -4)$ op k geeft $-a + b = -4$, dus $b = a - 4$.
k: $y = ax + a - 4$ oftewel k: $ax - y + a - 4 = 0$

$$d(M, k) = r \text{ geeft } \frac{|5a + 2 + a - 4|}{\sqrt{a^2 + 1}} = \sqrt{20}$$
$$|6a - 2| = \sqrt{20a^2 + 20}$$
$$36a^2 - 24a + 4 = 20a^2 + 20$$
$$16a^2 - 24a - 16 = 0$$
$$2a^2 - 3a - 2 = 0$$
$$D = (-3)^2 - 4 \cdot 2 \cdot -2 = 25$$
$$a = \frac{3 + 5}{4} = 2 \vee a = \frac{3 - 5}{4} = -\tfrac{1}{2}$$

Dus $\text{rc}_{k_1} = -\frac{1}{2}$ en $\text{rc}_{k_2} = 2$.
$\text{rc}_{k_1} \cdot \text{rc}_{k_2} = -\frac{1}{2} \cdot 2 = -1$, dus $k_1 \perp k_2$, dus $\angle APB = 90°$.
k_1 raakt c in A, dus $\angle PAM = 90°$ en $d(A, M) = d(M, k_1) = r = \sqrt{20}$.
k_2 raakt c in B, dus $\angle PBM = 90°$ en $d(B, M) = d(M, k_2) = r = \sqrt{20}$.

$$\left.\begin{array}{l} \angle APB = 90° \\ \angle PAM = 90° \\ \angle PBM = 90° \\ d(A, M) = d(B, M) \end{array}\right\} AMBP \text{ is een vierkant.}$$

Bladzijde 123

76 $x^2 + y^2 - 14x - 8y + 55 = 0$
$x^2 - 14x + y^2 - 8y + 55 = 0$
$(x - 7)^2 - 49 + (y - 4)^2 - 16 + 55 = 0$
$(x - 7)^2 + (y - 4)^2 = 10$
Dus $M(7, 4)$ en $r = \sqrt{10}$.
Stel k: $y = ax + b$.
$P(2, -1)$ op k geeft $2a + b = -1$, dus $b = -2a - 1$.
k: $y = ax - 2a - 1$ oftewel k: $ax - y - 2a - 1 = 0$

$d(M, k) = r$ geeft $\frac{|7a - 4 - 2a - 1|}{\sqrt{a^2 + 1}} = \sqrt{10}$

$|5a - 5| = \sqrt{10a^2 + 10}$

$25a^2 - 50a + 25 = 10a^2 + 10$

$15a^2 - 50a + 15 = 0$

$3a^2 - 10a + 3 = 0$

$D = (-10)^2 - 4 \cdot 3 \cdot 3 = 64$

$a = \frac{10 + 8}{6} = 3 \vee a = \frac{10 - 8}{6} = \frac{1}{3}$

$a = 3$ geeft $b = -2 \cdot 3 - 1 = -7$, dus k_1: $y = 3x - 7$.
$a = \frac{1}{3}$ geeft $b = -2 \cdot \frac{1}{3} - 1 = -1\frac{2}{3}$, dus k_2: $y = \frac{1}{3}x - 1\frac{2}{3}$.

$AM \perp k_1$, dus $\text{rc}_{AM} = -\frac{1}{3}$.

AM: $y = -\frac{1}{3}x + b$
door $M(7, 4)$ } $-\frac{1}{3} \cdot 7 + b = 4$

$-2\frac{1}{3} + b = 4$

$b = 6\frac{1}{3}$

Dus AM: $y = -\frac{1}{3}x + 6\frac{1}{3}$.
k_1 snijden met AM geeft
$3x - 7 = -\frac{1}{3}x + 6\frac{1}{3}$
$3\frac{1}{3}x = 13\frac{1}{3}$
$x = 4$
$x = 4$ geeft $y = 3 \cdot 4 - 7 = 5$
Dus $A(4, 5)$.

$BM \perp k_2$, dus $\text{rc}_{BM} = -3$.

BM: $y = -3x + b$
door $M(7, 4)$ } $-3 \cdot 7 + b = 4$

$-21 + b = 4$

$b = 25$

Dus BM: $y = -3x + 25$.
k_2 snijden met BM geeft
$\frac{1}{3}x - 1\frac{2}{3} = -3x + 25$
$3\frac{1}{3}x = 26\frac{2}{3}$
$x = 8$
$x = 8$ geeft $y = -3 \cdot 8 + 25 = 1$
Dus $B(8, 1)$.

N is het midden van AB, dus $N(6, 3)$.
De straal van c_2 is $r_2 = d(A, N) = \sqrt{(6 - 4)^2 + (3 - 5)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$.
Dus $d(S, N) = 2\sqrt{2}$.
$d(P, N) = \sqrt{(6 - 2)^2 + (3 - -1)^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$.
$d(P, N) = 2 \cdot d(S, N)$, dus S is het midden van NP.

Diagnostische toets

Bladzijde 126

1 **a** Snijden, dus $\frac{p}{-4} \neq \frac{-3}{p}$

$p^2 \neq 12$

$p \neq \sqrt{12} \wedge p \neq -\sqrt{12}$

$p \neq 2\sqrt{3} \wedge p \neq -2\sqrt{3}$

Dus voor $p \neq 2\sqrt{3} \wedge p \neq -2\sqrt{3}$ en q elk getal van $\mathbb{R}$ hebben de lijnen een snijpunt.

b Evenwijdig, dus $\frac{p}{-4} = \frac{-3}{p} \neq \frac{q}{8}$.

Uit $\frac{p}{-4} = \frac{-3}{p}$ volgt $p = 2\sqrt{3} \vee p = -2\sqrt{3}$.

$p = 2\sqrt{3}$ geeft $\frac{2\sqrt{3}}{-4} = \frac{-3}{2\sqrt{3}} \neq \frac{q}{8}$ oftewel $\frac{\sqrt{3}}{-2} \neq \frac{q}{8}$, dus $q \neq \frac{8\sqrt{3}}{-2} = -4\sqrt{3}$.

$p = -2\sqrt{3}$ geeft $\frac{-2\sqrt{3}}{-4} = \frac{-3}{-2\sqrt{3}} \neq \frac{q}{8}$ oftewel $\frac{\sqrt{3}}{2} \neq \frac{q}{8}$, dus $q \neq \frac{8\sqrt{3}}{2} = 4\sqrt{3}$.

Dus voor $p = 2\sqrt{3} \wedge q \neq -4\sqrt{3}$ en voor $p = -2\sqrt{3} \wedge q \neq 4\sqrt{3}$ zijn de lijnen evenwijdig.

c (2, 8) op $k_{p,q}$ geeft $2p - 3 \cdot 8 = q$ oftewel $q = 2p - 24$.
(2, 8) op l_p geeft $-4 \cdot 2 + 8p = 8$

$$-8 + 8p = 8$$
$$8p = 16$$
$$p = 2$$

$p = 2$ geeft $q = 2 \cdot 2 - 24 = -20$
Dus voor $p = 2$ en $q = -20$ snijden de lijnen elkaar in het punt (2, 8).

2 **a** $\left.\begin{array}{l} k\colon \dfrac{x}{2p} + \dfrac{y}{p+1} = 1 \\ A(8, -3) \text{ op } k \end{array}\right\} \dfrac{8}{2p} + \dfrac{-3}{p+1} = 1$

$$8(p+1) - 3 \cdot 2p = 2p(p+1)$$
$$8p + 8 - 6p = 2p^2 + 2p$$
$$2p^2 = 8$$
$$p^2 = 4$$
$$p = 2 \vee p = -2$$

vold. vold.

b $\text{rc}_l = \dfrac{2p + 3 - 0}{0 - 5} = \dfrac{2p+3}{-5}$

Van de lijn door (2, 0) en (0, 3) is $\text{rc} = \dfrac{3-0}{0-2} = \dfrac{3}{-2} = -1\frac{1}{2}$.

De lijnen zijn evenwijdig, dus $\dfrac{2p+3}{-5} = -1\frac{1}{2}$

$$2p + 3 = 7\tfrac{1}{2}$$
$$2p = 4\tfrac{1}{2}$$
$$p = 2\tfrac{1}{4}$$

c $\text{rc}_k = \dfrac{p + 1 - 0}{0 - 2p} = \dfrac{p+1}{-2p}$

k en l zijn evenwijdig, dus $\dfrac{p+1}{-2p} = \dfrac{2p+3}{-5}$

$$-2p(2p+3) = -5(p+1)$$
$$-4p^2 - 6p = -5p - 5$$
$$4p^2 + p - 5 = 0$$
$$D = 1^2 - 4 \cdot 4 \cdot -5 = 81$$
$$p = \frac{-1+9}{8} = 1 \vee p = \frac{-1-9}{8} = -1\tfrac{1}{4}$$

vold. vold.

3 **a** $k\colon y = 4x + 2$
$\tan(\alpha) = \text{rc}_k = 4$ geeft $\alpha = 75{,}96...°$
$l\colon y = -\frac{1}{2}x + 6$
$\tan(\beta) = \text{rc}_l = -\frac{1}{2}$ geeft $\beta = -26{,}56...°$
$\alpha - \beta = 75{,}96...° - -26{,}56...° \approx 102{,}5°$
Dus $\angle(k, l) \approx 180° - 102{,}5° = 77{,}5°$.

b $\text{rc}_m = \dfrac{0 - -2}{3 - 0} = \dfrac{2}{3}$
$\tan(\alpha) = \frac{2}{3}$ geeft $\alpha = 33{,}69...°$
$\text{rc}_n = \dfrac{0-5}{2-0} = -2\frac{1}{2}$
$\tan(\beta) = -2\frac{1}{2}$ geeft $\beta = -68{,}19...°$
$\alpha - \beta = 33{,}69...° - -68{,}19...° \approx 101{,}9°$
Dus $\angle(m, n) \approx 180° - 101{,}9° = 78{,}1°$.

c $p\colon 2x + 3y = 6$ geeft $3y = -2x + 6$
$y = -\frac{2}{3}x + 2$
$\tan(\alpha) = \text{rc}_p = -\frac{2}{3}$ geeft $\alpha = -33{,}69...°$
$q\colon y = 8x - 6$
$\tan(\beta) = \text{rc}_q = 8$ geeft $\beta = 82{,}87...°$
$\beta - \alpha = 82{,}87...° - -33{,}69...° \approx 116{,}6°$
Dus $\angle(p, q) \approx 180° - 116{,}6° = 63{,}4°$.

4 **a** $d(A, B) = \sqrt{(3-2p)^2 + (p+1-0)^2} = \sqrt{9 - 12p + 4p^2 + p^2 + 2p + 1} = \sqrt{5p^2 - 10p + 10}$

b $d(A, B) = 5$ geeft $\sqrt{5p^2 - 10p + 10} = 5$

$5p^2 - 10p + 10 = 25$
$5p^2 - 10p - 15 = 0$
$p^2 - 2p - 3 = 0$
$(p+1)(p-3) = 0$
$p = -1 \vee p = 3$

c Het midden van het lijnstuk AB is $M(\frac{1}{2}(2p+3), \frac{1}{2}(0+p+1)) = M(p + 1\frac{1}{2}, \frac{1}{2}p + \frac{1}{2})$.

M op $x + y = 6$ geeft $p + 1\frac{1}{2} + \frac{1}{2}p + \frac{1}{2} = 6$

$1\frac{1}{2}p = 4$
$p = 2\frac{2}{3}$

5 **a** $\left.\begin{array}{l} k \perp l, \text{ dus } 2x - 3y = c \\ \text{door } A(2, -4) \end{array}\right\} c = 2 \cdot 2 - 3 \cdot -4 = 16$

Dus k: $2x - 3y = 16$.

b $m \perp n$, dus $\text{rc}_m = 3$.

$\left.\begin{array}{l} m: y = 3x + b \\ \text{door } B(2, 3) \end{array}\right\} 3 \cdot 2 + b = 3$

$6 + b = 3$
$b = -3$

Dus m: $y = 3x - 3$ oftewel m: $3x - y = 3$.

6 **a** k: $y = \frac{1}{3}x + 10$ oftewel k: $\frac{1}{3}x - y + 10 = 0$ oftewel k: $x - 3y + 30 = 0$

$d(A, k) = \frac{|2 - 3 \cdot -6 + 30|}{\sqrt{1^2 + (-3)^2}} = \frac{50}{\sqrt{10}} = 5\sqrt{10}$

b Stel l: $y = ax + b$.

l door $B(4, 0)$ geeft $4a + b = 0$, dus $b = -4a$.

l: $y = ax - 4a$ oftewel l: $ax - y - 4a = 0$

$d(C, l) = \sqrt{5}$ geeft $\frac{|a - 1 - 4a|}{\sqrt{a^2 + 1}} = \sqrt{5}$

$|-3a - 1| = \sqrt{5a^2 + 5}$
$9a^2 + 6a + 1 = 5a^2 + 5$
$4a^2 + 6a - 4 = 0$
$2a^2 + 3a - 2 = 0$
$D = 3^2 - 4 \cdot 2 \cdot -2 = 25$
$a = \frac{-3 + 5}{4} = \frac{1}{2} \vee a = \frac{-3 - 5}{4} = -2$

$a = \frac{1}{2}$ geeft $b = -4 \cdot \frac{1}{2} = -2$, dus l_1: $y = \frac{1}{2}x - 2$.
$a = -2$ geeft $b = -4 \cdot -2 = 8$, dus l_2: $y = -2x + 8$.

7 **a** Stel l: $x + 2y = c$.

$A(6, 0)$ is een punt op k.

$d(A, l) = 2\sqrt{5}$ geeft $\frac{|6 + 2 \cdot 0 - c|}{\sqrt{1^2 + 2^2}} = 2\sqrt{5}$

$\frac{|6 - c|}{\sqrt{5}} = 2\sqrt{5}$
$|6 - c| = 10$
$6 - c = 10 \vee 6 - c = -10$
$c = -4 \vee c = 16$

Dus l_1: $x + 2y = -4$ en l_2: $x + 2y = 16$.

b Stel $P(p, p)$.

$d(P, k) = \sqrt{5}$ geeft $\dfrac{|p + 2 \cdot p - 6|}{\sqrt{1^2 + 2^2}} = \sqrt{5}$

$\dfrac{|3p - 6|}{\sqrt{5}} = \sqrt{5}$

$|3p - 6| = 5$

$3p - 6 = 5 \vee 3p - 6 = -5$

$3p = 11 \vee 3p = 1$

$p = 3\frac{2}{3} \vee p = \frac{1}{3}$

Dus $P_1(3\frac{2}{3}, 3\frac{2}{3})$ en $P_2(\frac{1}{3}, \frac{1}{3})$.

Bladzijde 127

8 **a** De straal is gelijk aan $d(A, B)$, dus $r = \sqrt{(8 - 2)^2 + (2 - -3)^2} = \sqrt{36 + 25} = \sqrt{61}$

Dus c_1: $(x - 2)^2 + (y + 3)^2 = 61$.

b De cirkel heeft middelpunt B en raakt de y-as, dus $r = d(B, y\text{-as}) = 8$.

Dus c_2: $(x - 8)^2 + (y - 2)^2 = 64$.

c Het middelpunt M van c_3 is het midden van het lijnstuk AB, dus

$M(\frac{1}{2}(2 + 8), \frac{1}{2}(-3 + 2)) = M(5, -\frac{1}{2})$.

De straal van c_3 is de helft van $d(A, B)$, dus $r = \frac{1}{2}\sqrt{61}$. Dit geeft $r^2 = \frac{1}{4} \cdot 61 = 15\frac{1}{4}$.

Dus c_3: $(x - 5)^2 + (y + \frac{1}{2})^2 = 15\frac{1}{4}$.

d De cirkel raakt de x-as in $C(5, 0)$, dus $x_M = 5$ en $M(5, r)$.

$\left.\begin{array}{l} y = 3x - 6 \\ \text{door } M(5, r) \end{array}\right\} r = 3 \cdot 5 - 6 = 9$

Dus c_4: $(x - 5)^2 + (y - 9)^2 = 81$.

9 $M(7, 8)$ geeft c_1: $(x - 7)^2 + (y - 8)^2 = r_1{}^2$

$r_1 = d(M, k) = \dfrac{|2 \cdot 7 + 3 \cdot 8 - 12|}{\sqrt{2^2 + 3^2}} = \dfrac{|26|}{\sqrt{13}} = \dfrac{26}{\sqrt{13}} = 2\sqrt{13}$

Dus c_1: $(x - 7)^2 + (y - 8)^2 = (2\sqrt{13})^2$ oftewel c_1: $(x - 7)^2 + (y - 8)^2 = 52$.

De lijn l gaat door M en staat loodrecht op k.

$\left.\begin{array}{l} l\text{: } 3x - 2y = c \\ \text{door } M(7, 8) \end{array}\right\} c = 3 \cdot 7 - 2 \cdot 8 = 5$

Dus l: $3x - 2y = 5$.

k en l snijden geeft het punt A.

$\begin{cases} 2x + 3y = 12 \\ 3x - 2y = 5 \end{cases} \begin{vmatrix} 2 \\ 3 \end{vmatrix}$ geeft $\begin{cases} 4x + 6y = 24 \\ 9x - 6y = 15 \end{cases}$ +

$13x = 39$

$\left.\begin{array}{l} x = 3 \\ 2x + 3y = 12 \end{array}\right\} 2 \cdot 3 + 3y = 12$

$6 + 3y = 12$

$3y = 6$

$y = 2$

Dus $A(3, 2)$.

Het midden van het lijnstuk AM is $N(5, 5)$ geeft c_2: $(x - 5)^2 + (y - 5)^2 = r_2{}^2$.

$r_2 = d(A, N) = \sqrt{(5 - 3)^2 + (5 - 2)^2} = \sqrt{4 + 9} = \sqrt{13}$

Dus c_2: $(x - 5)^2 + (y - 5)^2 = 13$.

10 $x^2 + y^2 - 16x + 8y - 1 = 0$
$x^2 - 16x + y^2 + 8y - 1 = 0$
$(x - 8)^2 - 64 + (y + 4)^2 - 16 - 1 = 0$
$(x - 8)^2 + (y + 4)^2 = 81$
Dus $M(8, -4)$ en $r = 9$.
$d(A, M) = \sqrt{(8 - 3)^2 + (-4 - -4)^2} = \sqrt{25 + 0} = 5 < r$
Dus A ligt binnen c.
$d(B, M) = \sqrt{(8 - 2)^2 + (-4 - 5)^2} = \sqrt{36 + 81} = \sqrt{117} > r$
Dus B ligt buiten c.
$d(C, M) = \sqrt{(8 - 8)^2 + (-4 - 5)^2} = \sqrt{0 + 81} = 9 = r$
Dus C ligt op c.

11 $d(M, k) = \dfrac{|3 \cdot 5 + 4 \cdot 7 - 18|}{\sqrt{3^2 + 4^2}} = \dfrac{|25|}{\sqrt{25}} = \dfrac{25}{5} = 5$
Uit $d(k, c_2) = d(k, M) - r_2$, $d(k, c_2) = 3$ en $d(k, M) = 5$ volgt $r_2 = 2$.
Dus c_2: $(x - 5)^2 + (y - 7)^2 = 4$.
$x^2 + y^2 + 10y = 0$
$x^2 + (y + 5)^2 - 25 = 0$
$x^2 + (y + 5)^2 = 25$
Van c_1 is het middelpunt $N(0, -5)$ en de straal $r_1 = 5$.
$d(M, N) = \sqrt{(0 - 5)^2 + (-5 - 7)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$
$d(c_1, c_2) = d(M, N) - r_1 - r_2 = 13 - 5 - 2 = 6$

12 **a** $\left.\begin{array}{l} x^2 + y^2 - 4x + 6y = 0 \\ x = 4 \end{array}\right\}$ $4^2 + y^2 - 4 \cdot 4 + 6y = 0$
$16 + y^2 - 16 + 6y = 0$
$y^2 + 6y = 0$
$y(y + 6) = 0$
$y = 0 \vee y = -6$
$y_A > y_B$, dus $A(4, 0)$ en $B(4, -6)$.
$x^2 + y^2 - 4x + 6y = 0$
$x^2 - 4x + y^2 + 6y = 0$
$(x - 2)^2 - 4 + (y + 3)^2 - 9 = 0$
$(x - 2)^2 + (y + 3)^2 = 13$
Dus $M(2, -3)$ en $r = \sqrt{13}$.
$\text{rc}_{MA} = \dfrac{0 - -3}{4 - 2} = \frac{3}{2}$, dus $\text{rc}_k = -\frac{2}{3}$.
$\left.\begin{array}{l} k\text{: } y = -\frac{2}{3}x + b \\ \text{door } A(4, 0) \end{array}\right\}$ $-\frac{2}{3} \cdot 4 + b = 0$
$-2\frac{2}{3} + b = 0$
$b = 2\frac{2}{3}$
Dus k: $y = -\frac{2}{3}x + 2\frac{2}{3}$.
$\text{rc}_{MB} = \dfrac{-6 - -3}{4 - 2} = -\frac{3}{2}$, dus $\text{rc}_l = \frac{2}{3}$.
$\left.\begin{array}{l} l\text{: } y = \frac{2}{3}x + b \\ \text{door } B(4, -6) \end{array}\right\}$ $\frac{2}{3} \cdot 4 + b = -6$
$2\frac{2}{3} + b = -6$
$b = -8\frac{2}{3}$
Dus l: $y = \frac{2}{3}x - 8\frac{2}{3}$.

b $\tan(\alpha) = \text{rc}_k = -\frac{2}{3}$ geeft $\alpha = -33{,}69...°$
$\tan(\beta) = \text{rc}_l = \frac{2}{3}$ geeft $\beta = 33{,}69...°$
$\beta - \alpha = 33{,}69...° - -33{,}69...° \approx 67{,}4°$
Dus $\angle(k, l) \approx 67{,}4°$.

13 **a** $x^2 + y^2 + 4x - 2y = 0$
$x^2 + 4x + y^2 - 2y = 0$
$(x + 2)^2 - 4 + (y - 1)^2 - 1 = 0$
$(x + 2)^2 + (y - 1)^2 = 5$
$M(-2, 1)$ is het middelpunt van c.
Stel k: $y = ax + b$.
Lijn n door M en A heeft $\text{rc}_n = \dfrac{3 - 1}{-1 - -2} = 2$, dus $a = \text{rc}_k = -\frac{1}{2}$.

$$\left.\begin{array}{l} k\text{: } y = -\frac{1}{2}x + b \\ \text{door } A(-1, 3) \end{array}\right\} -\frac{1}{2} \cdot -1 + b = 3$$
$$\tfrac{1}{2} + b = 3$$
$$b = 2\tfrac{1}{2}$$

Dus k: $y = -\frac{1}{2}x + 2\frac{1}{2}$.

b Loodrecht op p: $2x + y = 10$, dus l: $x - 2y = c$.

$d(M, l) = r$ geeft $\dfrac{|-2 - 2 \cdot 1 - c|}{\sqrt{2^2 + 1^2}} = \sqrt{5}$
$$\frac{|-4 - c|}{\sqrt{5}} = \sqrt{5}$$
$$|-4 - c| = 5$$
$$-4 - c = 5 \vee -4 - c = -5$$
$$c = -9 \vee c = 1$$

Dus l_1: $x - 2y = -9$ en l_2: $x - 2y = 1$.

c Stel m: $y = ax + b$.
$B(-1, -2)$ op m geeft $-a + b = -2$, dus $b = a - 2$.
m: $y = ax + a - 2$ oftewel m: $ax - y + a - 2 = 0$

$d(M, m) = r$ geeft $\dfrac{|-2a - 1 + a - 2|}{\sqrt{a^2 + 1}} = \sqrt{5}$
$$|-a - 3| = \sqrt{5a^2 + 5}$$
$$a^2 + 6a + 9 = 5a^2 + 5$$
$$4a^2 - 6a - 4 = 0$$
$$2a^2 - 3a - 2 = 0$$
$$D = (-3)^2 - 4 \cdot 2 \cdot -2 = 25$$
$$a = \frac{3 + 5}{4} = 2 \vee a = \frac{3 - 5}{4} = -\frac{1}{2}$$

$a = 2$ geeft $b = 2 - 2 = 0$, dus m_1: $y = 2x$.
$a = -\frac{1}{2}$ geeft $b = -\frac{1}{2} - 2 = -2\frac{1}{2}$, dus m_2: $y = -\frac{1}{2}x - 2\frac{1}{2}$.

8 Goniometrische functies

Voorkennis Exacte waarden van goniometrische verhoudingen

Bladzijde 132

1 **a** $2\sin(45°) + \sqrt{6} \cdot \sin(60°) = 2 \cdot \frac{1}{2}\sqrt{2} + \sqrt{6} \cdot \frac{1}{2}\sqrt{3} = \sqrt{2} + \frac{1}{2}\sqrt{18} = \sqrt{2} + \frac{1}{2} \cdot 3\sqrt{2} = \sqrt{2} + 1\frac{1}{2}\sqrt{2} = 2\frac{1}{2}\sqrt{2}$

b $4\sin(30°)\tan(30°) + 2\cos(30°)\cos(60°) = 4 \cdot \frac{1}{2} \cdot \frac{1}{3}\sqrt{3} + 2 \cdot \frac{1}{2}\sqrt{3} \cdot \frac{1}{2} = \frac{2}{3}\sqrt{3} + \frac{1}{2}\sqrt{3} = \frac{4}{6}\sqrt{3} + \frac{3}{6}\sqrt{3} = 1\frac{1}{6}\sqrt{3}$

2 **a** $4\cos(30°) + 9\tan(30°) = 4 \cdot \frac{1}{2}\sqrt{3} + 9 \cdot \frac{1}{3}\sqrt{3} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

b $3\sin(30°) - \sqrt{2} \cdot \sin(45°) = 3 \cdot \frac{1}{2} - \sqrt{2} \cdot \frac{1}{2}\sqrt{2} = 1\frac{1}{2} - 1 = \frac{1}{2}$

c $6\tan(30°) - 3\tan(60°) = 6 \cdot \frac{1}{3}\sqrt{3} - 3 \cdot \sqrt{3} = 2\sqrt{3} - 3\sqrt{3} = -\sqrt{3}$

d $\sqrt{2} \cdot \sin(60°) + 3\sqrt{3} \cdot \sin(45°) = \sqrt{2} \cdot \frac{1}{2}\sqrt{3} + 3\sqrt{3} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{6} + 1\frac{1}{2}\sqrt{6} = 2\sqrt{6}$

3 **a** $8\sin(45°)\cos(30°) = 8 \cdot \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3} = 2\sqrt{6}$

b $2\sqrt{3} \cdot \sin(60°)\cos(30°) = 2\sqrt{3} \cdot \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{3} = 1\frac{1}{2}\sqrt{3}$

c $\sqrt{2} \cdot \sin(45°)\sin(60°) - 2\tan(60°) = \sqrt{2} \cdot \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3} - 2 \cdot \sqrt{3} = \frac{1}{2}\sqrt{3} - 2\sqrt{3} = -1\frac{1}{2}\sqrt{3}$

d $(\sin(30°) + \cos(30°))^2 = (\frac{1}{2} + \frac{1}{2}\sqrt{3})^2 = \frac{1}{4} + \frac{1}{2}\sqrt{3} + \frac{3}{4} = 1 + \frac{1}{2}\sqrt{3}$

4 **a** $\dfrac{\cos(30°)}{1 + \sin(60°)} = \dfrac{\frac{1}{2}\sqrt{3}}{1 + \frac{1}{2}\sqrt{3}} = \dfrac{\sqrt{3}}{2 + \sqrt{3}} \cdot \dfrac{2 - \sqrt{3}}{2 - \sqrt{3}} = \dfrac{2\sqrt{3} - 3}{1} = 2\sqrt{3} - 3$

b $\dfrac{\sin(30°) + \sin(60°)}{\sin(30°) - \sin(60°)} = \dfrac{\frac{1}{2} + \frac{1}{2}\sqrt{3}}{\frac{1}{2} - \frac{1}{2}\sqrt{3}} = \dfrac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \dfrac{1 + \sqrt{3}}{1 + \sqrt{3}} = \dfrac{1 + 2\sqrt{3} + 3}{-2} = -2 - \sqrt{3}$

5 **a** $\sin(15°) = \sin(60° - 45°) = \sin(60°)\cos(45°) - \cos(60°)\sin(45°) = \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2}$

b $\sin(75°) = \sin(45° + 30°) = \sin(45°)\cos(30°) + \cos(45°)\sin(30°) = \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{2} \cdot \frac{1}{2} = \frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2}$

8.1 Eenheidscirkel en radiaal

Bladzijde 133

1 **a** $\sin(\alpha) = \dfrac{PQ}{OP} = \dfrac{PQ}{1} = PQ$, dus $PQ = \sin(65°) \approx 0{,}91$.

$\cos(\alpha) = \dfrac{OQ}{OP} = \dfrac{OQ}{1} = OQ$, dus $OQ = \cos(65°) \approx 0{,}42$.

b $P(0{,}42;\ 0{,}91)$

c $\angle POQ = 180° - 115° = 65°$

$PQ \approx 0{,}91$, $OQ \approx 0{,}42$ en $P(-0{,}42;\ 0{,}91)$.

d $\cos(115°) \approx -0{,}42$ en $\sin(115°) \approx 0{,}91$

Dezelfde y-coördinaat en tegengestelde x-coördinaat, oftewel $x_P = \cos(115°)$ en $y_P = \sin(115°)$.

e $\angle POQ = 245° - 180° = 65°$

$P(-0{,}42;\ -0{,}91)$

De GR geeft $\cos(245°) \approx -0{,}42$ en $\sin(245°) \approx -0{,}91$, oftewel $x_P = \cos(245°)$ en $y_P = \sin(245°)$.

Bladzijde 134

2
a $\sin(0°) = 0$
b $\cos(0°) = 1$
c $\sin(90°) = 1$
d $\cos(90°) = 0$
e $\sin(270°) = -1$
f $\cos(270°) = 0$
g $\sin(360°) = 0$
h $\tan(360°) = 0$
i $\sin(450°) = 1$
j $\cos(-90°) = 0$
k $\tan(-360°) = 0$
l $\cos(-180°) = -1$

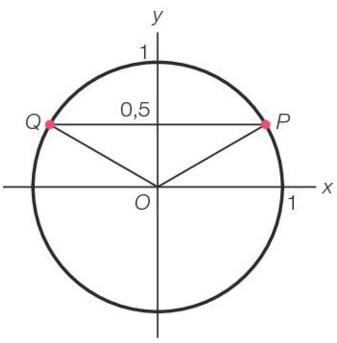
3
a $\alpha = 0°$, $\alpha = 180°$ en $\alpha = 360°$
b $\alpha = 90°$ en $\alpha = 270°$
c $\alpha = 90°$
d $\alpha = 0°$ en $\alpha = 360°$
e $\alpha = 45°$ en $\alpha = 225°$
f $\alpha = 135°$ en $\alpha = 315°$

Bladzijde 135

4
a *
b $P(\cos(110°), \sin(110°)) \approx P(-0{,}34; 0{,}94)$
$Q(\cos(200°), \sin(200°)) \approx Q(-0{,}94; -0{,}34)$
$R(\cos(-102°), \sin(-102°)) \approx R(-0{,}21; -0{,}98)$
$S(\cos(-50°), \sin(-50°)) \approx S(0{,}64; -0{,}77)$

5 $\frac{360°}{5} = 72°$ $\quad 2 \cdot 72° = 144°$ $\quad 3 \cdot 72° = 216°$ $\quad 4 \cdot 72° = 288°$
$B(2\cos(72°), 2\sin(72°)) \approx B(0{,}62; 1{,}90)$
$C(2\cos(144°), 2\sin(144°)) \approx C(-1{,}62; 1{,}18)$
$D(2\cos(216°), 2\sin(216°)) \approx D(-1{,}62; -1{,}18)$
$E(2\cos(288°), 2\sin(288°)) \approx E(0{,}62; -1{,}90)$

6
a



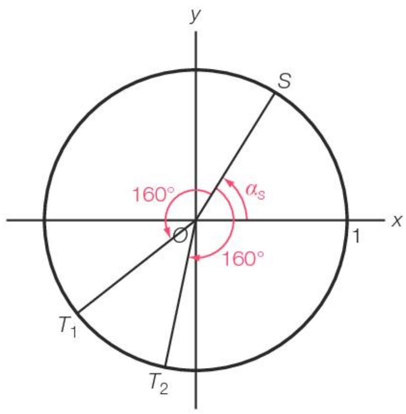
b $\beta = 180° - 30° = 150°$

Bladzijde 136

7
a $x_P = 0{,}81$, dus $\cos(\alpha) = 0{,}81$.
De GR geeft 35,90...°.
Dus $\alpha \approx 35{,}9°$.
b $y_P = 0{,}94$, dus $\sin(\alpha) = 0{,}94$.
De GR geeft 70,05...°.
Dus $\alpha = 180° - 70{,}05...° \approx 109{,}9°$.
c $x_P = 0{,}26$, dus $\cos(\alpha) = 0{,}26$.
De GR geeft 74,92...°.
Dus $\alpha \approx -74{,}9°$.
d $y_P = -0{,}22$, dus $\sin(\alpha) = -0{,}22$.
De GR geeft −12,70...°.
Dus $\alpha = 180° + 12{,}70...° \approx 192{,}7°$.

8 $y_P = 0{,}92$, dus $\sin(\alpha_P) = 0{,}92$.
De GR geeft 66,92...°, dus $\alpha_P = 66{,}92...°$.
$x_Q = -0{,}87$, dus $\cos(\alpha_Q) = -0{,}87$.
De GR geeft 150,45...°, dus $\alpha_Q = 360° - 150{,}45...° = 209{,}54...°$.
$\angle POQ = \alpha_Q - \alpha_P = 209{,}54...° - 66{,}92...° \approx 142{,}6°$

9 Zie de figuur hiernaast.
$x_S = 0{,}527$, dus $\cos(\alpha_S) = 0{,}527$.
De GR geeft 58,197...°, dus $\alpha_S = 58{,}197...°$.
$\cos(58{,}197...° + 160°) = -0{,}785...$
$\sin(58{,}197...° + 160°) = -0{,}618...$
Dus $T_1(-0{,}79; -0{,}62)$.
$\cos(58{,}197...° - 160°) = -0{,}204...$
$\sin(58{,}197...° - 160°) = -0{,}978...$
Dus $T_2(-0{,}20; -0{,}98)$.



Bladzijde 137

10 **a** omtrek cirkel $= 2\pi r$
In de eenheidscirkel is $r = 1$, dus omtrek $= 2\pi \cdot 1 = 2\pi$.
b Bij $\alpha = 90°$ hoort een kwart cirkelboog, dus lengte cirkelboog $= \frac{1}{4} \cdot 2\pi = \frac{1}{2}\pi$.
c

draaiingshoek α	0°	90°	180°	270°	360°
lengte cirkelboog b	0	$\frac{1}{2}\pi$	π	$1\frac{1}{2}\pi$	2π

Bladzijde 138

11
a $\frac{1}{6}\pi$ rad $= \frac{1}{6} \cdot 180° = 30°$
b $\frac{1}{4}\pi$ rad $= \frac{1}{4} \cdot 180° = 45°$
c 2π rad $= 2 \cdot 180° = 360°$
d 2 rad $= 2 \cdot \frac{180°}{\pi} \approx 114{,}6°$
e $1\frac{1}{4}\pi$ rad $= 1\frac{1}{4} \cdot 180° = 225°$
f $1\frac{1}{4}$ rad $= 1\frac{1}{4} \cdot \frac{180°}{\pi} \approx 71{,}6°$
g $-2\frac{1}{3}\pi$ rad $= -2\frac{1}{3} \cdot 180° = -420°$
h $-2\frac{1}{3}$ rad $= -2\frac{1}{3} \cdot \frac{180°}{\pi} \approx -133{,}7°$

Bladzijde 139

12
a $360° = 360 \cdot \frac{\pi}{180}$ rad $= 2\pi$ rad
b $30° = 30 \cdot \frac{\pi}{180}$ rad $= \frac{1}{6}\pi$ rad
c $45° = 45 \cdot \frac{\pi}{180}$ rad $= \frac{1}{4}\pi$ rad
d $60° = 60 \cdot \frac{\pi}{180}$ rad $= \frac{1}{3}\pi$ rad
e $90° = 90 \cdot \frac{\pi}{180}$ rad $= \frac{1}{2}\pi$ rad
f $135° = 135 \cdot \frac{\pi}{180}$ rad $= \frac{3}{4}\pi$ rad
g $300° = 300 \cdot \frac{\pi}{180}$ rad $= 1\frac{2}{3}\pi$ rad
h $210° = 210 \cdot \frac{\pi}{180}$ rad $= 1\frac{1}{6}\pi$ rad

13
a $10° = 10 \cdot \frac{\pi}{180}$ rad $\approx 0{,}17$ rad
b $57{,}3° = 57{,}3 \cdot \frac{\pi}{180}$ rad $\approx 1{,}00$ rad
c $1030° = 1030 \cdot \frac{\pi}{180}$ rad $\approx 17{,}98$ rad
d $90° = 90 \cdot \frac{\pi}{180}$ rad $\approx 1{,}57$ rad

14

hoek in graden	40°	72°	100°	135°	145°	150°
hoek in radialen	$\frac{2}{9}\pi$	$\frac{2}{5}\pi$	$\frac{5}{9}\pi$	$\frac{3}{4}\pi$	$\frac{29}{36}\pi$	$\frac{5}{6}\pi$

15
a $\cos(\frac{5}{8}\pi) \approx -0{,}38$
b $\cos(\frac{5}{8}) \approx 0{,}81$
c $\sin(\frac{4}{5}\pi) \approx 0{,}59$
d $\sin(\frac{4}{5}) \approx 0{,}72$
e $\cos(7{,}6\pi) \approx 0{,}31$
f $\cos(7{,}6) \approx 0{,}25$

16
a $\alpha \approx 1{,}17$
b $\alpha \approx 0{,}55$
c $\alpha \approx 0{,}43$
d $\alpha \approx 1{,}39$
e $\alpha \approx 0{,}84$
f $\alpha \approx 1{,}21$

17 **a** $\sin(\frac{1}{5}\pi) + \cos(\frac{2}{5}\pi) - \sin(\frac{3}{5}) \approx 0{,}33$

b $\cos(2\alpha) = 0{,}6$ geeft $2\alpha = 0{,}927...$, dus $\alpha \approx 0{,}46$.

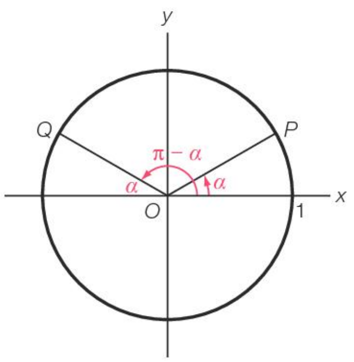
18 **a** $y_P = 0{,}35$, dus $\sin(\alpha) = 0{,}35$.
De GR geeft 0,357...
Dus $\alpha = \pi - 0{,}357... \approx 2{,}78$.

b $x_P = -0{,}35$, dus $\cos(\alpha) = -0{,}35$.
De GR geeft 1,928...
Dus $\alpha = 2\pi - 1{,}928... \approx 4{,}35$.

Bladzijde 140

19 $x_P = -0{,}32$, dus $\cos(\alpha_P) = -0{,}32$.
De GR geeft 1,896...
Dus $\alpha_P = 1{,}896...$
$y_Q = -0{,}88$, dus $\sin(\alpha_Q) = -0{,}88$.
De GR geeft $-1{,}075...$
Dus $\alpha_Q = \pi + 1{,}075... = 4{,}217...$
$\angle POQ = \alpha_Q - \alpha_P = 4{,}217... - 1{,}896... \approx 2{,}32$

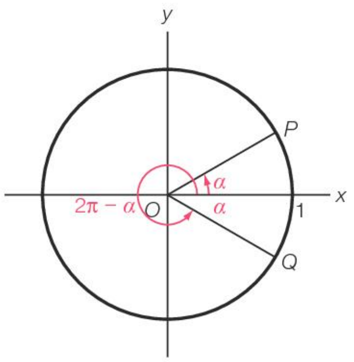
20 **a**



In de figuur zie je dat $y_Q = y_P$.
Dus $\sin(\pi - \alpha) = \sin(\alpha)$.

b In de figuur van vraag a zie je dat $x_Q = -x_P$.
Dus $\cos(\pi - \alpha) = -\cos(\alpha)$.

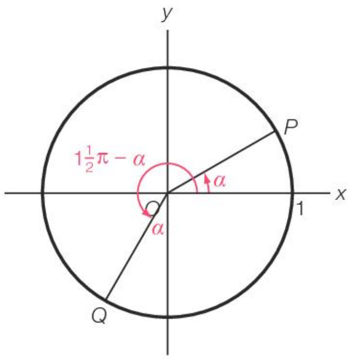
c



In de figuur zie je dat $y_Q = -y_P$.
Dus $\sin(2\pi - \alpha) = -\sin(\alpha)$.

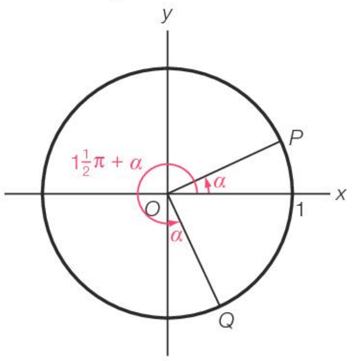
d In de figuur van vraag c zie je dat $x_Q = x_P$.
Dus $\cos(2\pi - \alpha) = \cos(\alpha)$.

e



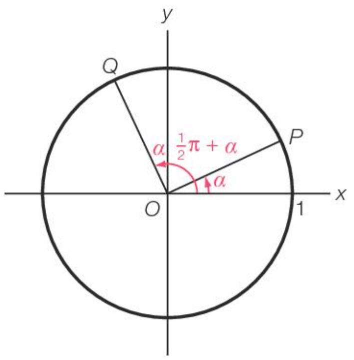
In de figuur zie je dat $y_Q = -x_P$.
Dus $\sin(1\frac{1}{2}\pi - \alpha) = -\cos(\alpha)$.

f



In de figuur zie je dat $x_Q = y_P$.
Dus $\cos(1\frac{1}{2}\pi + \alpha) = \sin(\alpha)$.

g



In de figuur zie je dat $y_Q = x_P$.
Dus $\sin(\frac{1}{2}\pi + \alpha) = \cos(\alpha)$.

h In de figuur van vraag g zie je dat $x_Q = -y_P$.
Dus $\cos(\frac{1}{2}\pi + \alpha) = -\sin(\alpha)$.

21 **a** $\frac{1}{6}\pi = \frac{1}{6} \cdot 180° = 30°$, dus $\cos(\frac{1}{6}\pi) = \cos(30°) = \frac{1}{2}\sqrt{3}$.

b $\frac{1}{4}\pi = \frac{1}{4} \cdot 180° = 45°$, dus $\sin(\frac{1}{4}\pi) = \sin(45°) = \frac{1}{2}\sqrt{2}$.

Bladzijde 141

22 **a** $\sin(\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}$

b $\cos(1\frac{1}{6}\pi) = -\frac{1}{2}\sqrt{3}$

c $\sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$

d $\cos(1\frac{1}{3}\pi) = -\frac{1}{2}$

e $\cos(1\frac{2}{3}\pi) = \frac{1}{2}$

f $\sin(-\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$

23 $\tan(1\frac{5}{6}\pi) - \tan(\frac{2}{3}\pi) = \frac{\sin(1\frac{5}{6}\pi)}{\cos(1\frac{5}{6}\pi)} - \frac{\sin(\frac{2}{3}\pi)}{\cos(\frac{2}{3}\pi)} = \frac{-\frac{1}{2}}{\frac{1}{2}\sqrt{3}} - \frac{\frac{1}{2}\sqrt{3}}{-\frac{1}{2}} = \frac{-1}{\sqrt{3}} + \sqrt{3} = -\frac{1}{3}\sqrt{3} + \sqrt{3} = \frac{2}{3}\sqrt{3}$

24 **a** $\sin(\alpha) = \frac{1}{2}\sqrt{3}$ geeft $\alpha = \frac{1}{3}\pi \vee \alpha = \frac{2}{3}\pi$

b $\cos(\alpha) = -\frac{1}{2}$ geeft $\alpha = \frac{2}{3}\pi \vee \alpha = 1\frac{1}{3}\pi$

c $\sin(\alpha) = -\frac{1}{2}\sqrt{2}$ geeft $\alpha = 1\frac{1}{4}\pi \vee \alpha = 1\frac{3}{4}\pi$

d $\cos(\alpha) = 0$ geeft $\alpha = \frac{1}{2}\pi \vee \alpha = 1\frac{1}{2}\pi$

e $\cos(\alpha) = \frac{1}{2}\sqrt{3}$ geeft $\alpha = \frac{1}{6}\pi \vee \alpha = 1\frac{5}{6}\pi$

f $\cos(\alpha) = \frac{1}{2}\sqrt{2}$ geeft $\alpha = \frac{1}{4}\pi \vee \alpha = 1\frac{3}{4}\pi$

25 **a** $x = \sin(\frac{5}{6}\pi) = \frac{1}{2}$

b $\sin(x) = -\frac{1}{2}\sqrt{3}$ met $0 \leq x \leq 1\frac{1}{2}\pi$ geeft $x = 1\frac{1}{3}\pi$

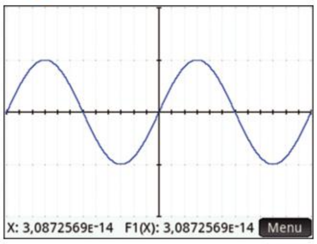
c $x = \cos(1\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}$

d $\cos(x) = \frac{1}{2}$ met $1\frac{1}{2}\pi \leq x \leq 2\pi$ geeft $x = 1\frac{2}{3}\pi$

8.2 Sinusoïden

Bladzijde 143

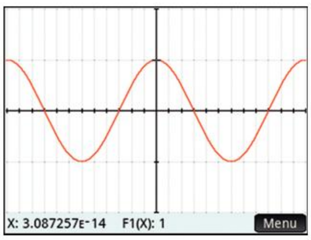
26 **a**



b De toppen zijn $(-1\frac{1}{2}\pi, 1)$, $(-\frac{1}{2}\pi, -1)$, $(\frac{1}{2}\pi, 1)$ en $(1\frac{1}{2}\pi, -1)$.

c De snijpunten met de x-as zijn $(-2\pi, 0)$, $(-\pi, 0)$, $(0, 0)$, $(\pi, 0)$ en $(2\pi, 0)$.

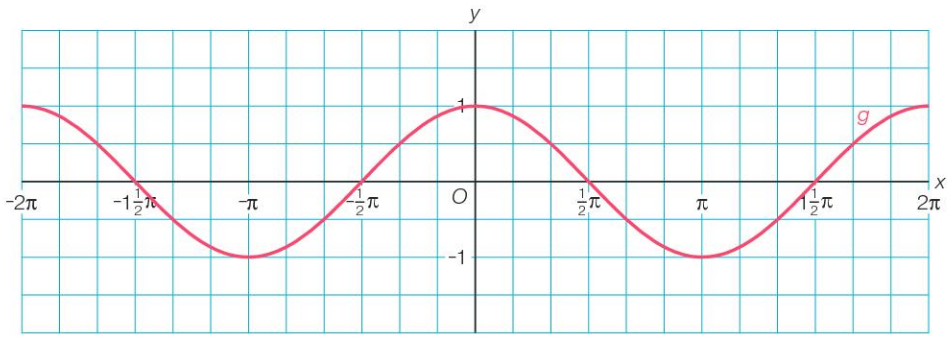
27 **a**



b De toppen zijn $(-2\pi, 1)$, $(-\pi, -1)$, $(0, 1)$, $(\pi, -1)$ en $(2\pi, 1)$.

c De nulpunten zijn $-1\frac{1}{2}\pi$, $-\frac{1}{2}\pi$, $\frac{1}{2}\pi$ en $1\frac{1}{2}\pi$.

d



Bladzijde 144

28 **a** $f(x) = \sin(x)$
$\downarrow$ translatie (0, 2)
$g(x) = 2 + \sin(x)$
evenwichtsstand = 2

b $f(x) = \sin(x)$
$\downarrow$ translatie $(\frac{1}{3}\pi, 0)$
$h(x) = \sin(x - \frac{1}{3}\pi)$
De nulpunten van h zijn ..., $-2\frac{2}{3}\pi$, $-1\frac{2}{3}\pi$, $-\frac{2}{3}\pi$, $\frac{1}{3}\pi$, $1\frac{1}{3}\pi$, $2\frac{1}{3}\pi$, ...

c $f(x) = \sin(x)$
$\downarrow$ verm. x-as, 4
$k(x) = 4\sin(x)$
amplitude = 4

d $f(x) = \sin(x)$
$\downarrow$ verm. y-as, $\frac{1}{5}$
$l(x) = \sin(5x)$
periode $= \dfrac{2\pi}{5} = \frac{2}{5}\pi$

Bladzijde 145

29 **a** Ja, ik ben het met José eens.
b $(\frac{1}{3}\pi, 1\frac{1}{2})$

Bladzijde 146

30 **Effect van transformaties op de standaardgrafiek $y = \cos(x)$**

transformatie	*beeldgrafiek*	*kenmerk*
translatie $(0, a)$	$y = a + \cos(x)$	evenwichtsstand a
verm. x-as, b	$y = b\cos(x)$	amplitude b $(b > 0)$
verm. y-as, $\dfrac{1}{c}$	$y = \cos(cx)$	periode $\dfrac{2\pi}{c}$ $(c > 0)$
translatie $(d, 0)$	$y = \cos(x - d)$	beginpunt $(d, 1)$

31 **a** De evenwichtsstand is 0, de amplitude is 2, de periode is 2π en een beginpunt is $(-3, 0)$.
b De evenwichtsstand is $\frac{1}{5}$, de amplitude is $\frac{1}{3}$, de periode is 2π en een beginpunt is $(0, \frac{1}{5})$.
c De evenwichtsstand is 0, de amplitude is 1, de periode is $\frac{2}{3}\pi$ en een beginpunt is (4, 1).
d De evenwichtsstand is 0, de amplitude is $1\frac{1}{2}$, de periode is 8π en een beginpunt is $(0, 1\frac{1}{2})$.

32 **a** De evenwichtsstand is 5, de amplitude is 1,2, de periode is 2π en een beginpunt is $(\frac{1}{6}\pi; 6{,}2)$.
b De evenwichtsstand is 0,4, de amplitude is 1, de periode is 10π en een beginpunt is $(-\frac{1}{3}\pi; 0{,}4)$.
c De evenwichtsstand is 0, de amplitude is 0,29, de periode is $\frac{2}{3}\pi$ en een beginpunt is $(-1{,}4; 0{,}29)$.
d De evenwichtsstand is $-0{,}8$, de amplitude is 2, de periode is $\frac{2}{3}\pi$ en een beginpunt is $(\frac{1}{2}\pi; -0{,}8)$.

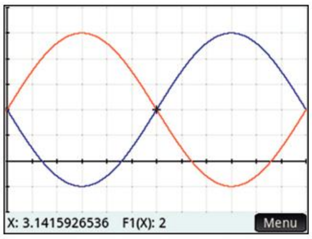
33 **a** $y = \cos(x)$
$\downarrow$ translatie $(\frac{1}{4}\pi, 4)$
$y = 4 + \cos(x - \frac{1}{4}\pi)$
$\downarrow$ verm. x-as, 3
$y = 3(4 + \cos(x - \frac{1}{4}\pi))$
oftewel $y = 12 + 3\cos(x - \frac{1}{4}\pi)$
Dus $f(x) = 12 + 3\cos(x - \frac{1}{4}\pi)$.

b $y = \cos(x)$
$\downarrow$ verm. x-as, 3
$y = 3\cos(x)$
$\downarrow$ translatie $(\frac{1}{4}\pi, 4)$
$y = 4 + 3\cos(x - \frac{1}{4}\pi)$
Dus $g(x) = 4 + 3\cos(x - \frac{1}{4}\pi)$.

34 **a** De evenwichtsstand is $-\frac{1}{2}$, de amplitude is 1, de periode is 2π en een beginpunt is $(\frac{1}{4}\pi, -\frac{1}{2})$.
b De grafiek van f snijdt de lijn van de evenwichtsstand in de punten $(\frac{1}{4}\pi, -\frac{1}{2})$, $(1\frac{1}{4}\pi, -\frac{1}{2})$ en $(2\frac{1}{4}\pi, -\frac{1}{2})$.
c De toppen van de grafiek van f zijn $(\frac{3}{4}\pi, \frac{1}{2})$, $(1\frac{3}{4}\pi, -1\frac{1}{2})$ en $(2\frac{3}{4}\pi, \frac{1}{2})$.
d De afstand tussen A en C is gelijk aan de periode, dus $AC = 2\pi$.

35 $f(x) = 3 + 4\sin(x - \frac{1}{2}\pi)$
$\downarrow$ translatie $(-\frac{3}{4}\pi, -5)$
$y = -5 + 3 + 4\sin(x + \frac{3}{4}\pi - \frac{1}{2}\pi)$
oftewel $y = -2 + 4\sin(x + \frac{1}{4}\pi)$
Dus $m = -\frac{3}{4}\pi$ en $n = -5$ zijn mogelijke waarden van m en n.

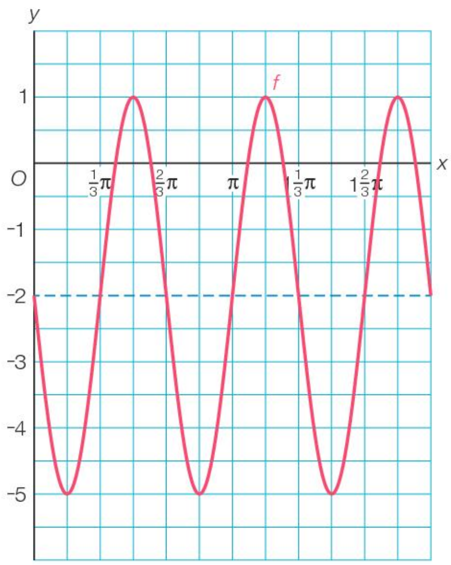
36 **a**



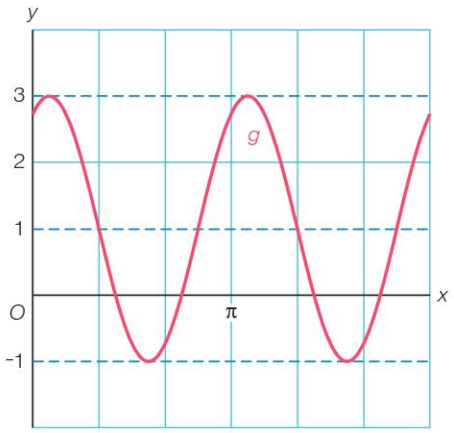
b Van de grafiek van $f(x) = 2 + 3\sin(x)$ is de amplitude 3 en van de grafiek van $g(x) = 2 - 3\sin(x)$ is de amplitude 3.

Bladzijde 149

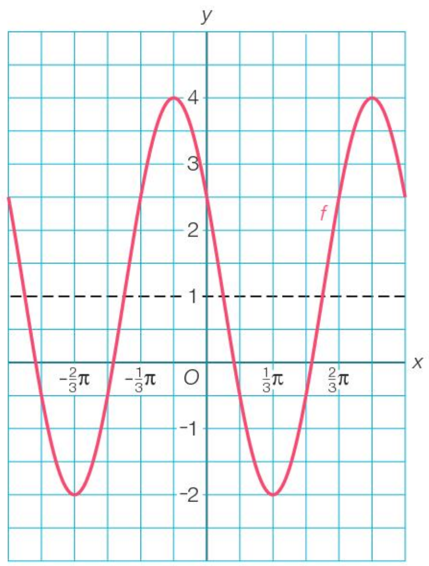
37 **a** $f(x) = -2 + 3\sin(3x + \pi) = -2 + 3\sin(3(x + \frac{1}{3}\pi))$
evenwichtsstand -2
amplitude 3
periode $\frac{2\pi}{3} = \frac{2}{3}\pi$
$3 > 0$, dus grafiek stijgend door het punt $(-\frac{1}{3}\pi, -2)$ en dus ook stijgend door het punt $(\frac{1}{3}\pi, -2)$.



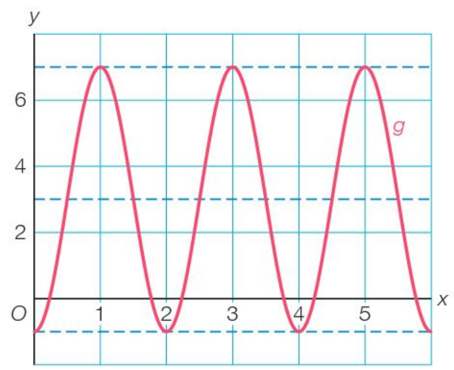
b $g(x) = 1 - 2\sin(2x - \frac{2}{3}\pi) = 1 - 2\sin(2(x - \frac{1}{3}\pi))$
evenwichtsstand 1
amplitude 2
periode $\frac{2\pi}{2} = \pi$
$-2 < 0$, dus grafiek dalend door het punt $(\frac{1}{3}\pi, 1)$.



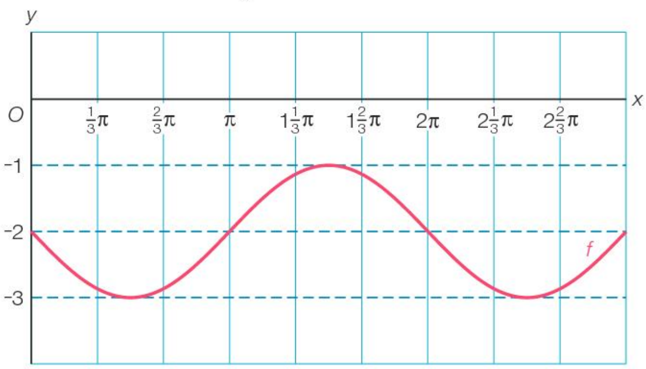
38 **a** $f(x) = 1 + 3\cos(2x + \frac{1}{3}\pi) = 1 + 3\cos(2(x + \frac{1}{6}\pi))$
evenwichtsstand 1
amplitude 3
periode $\frac{2\pi}{2} = \pi$
$3 > 0$, dus $(-\frac{1}{6}\pi, 4)$ is een hoogste punt van de grafiek.



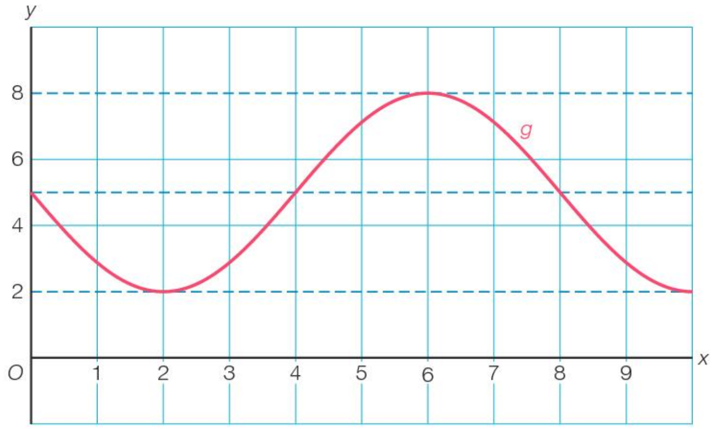
b $g(x) = 3 - 4\cos(\pi x)$
evenwichtsstand 3
amplitude 4
periode $\frac{2\pi}{\pi} = 2$
$-4 < 0$, dus het punt $(0, -1)$ is een laagste punt van de grafiek.



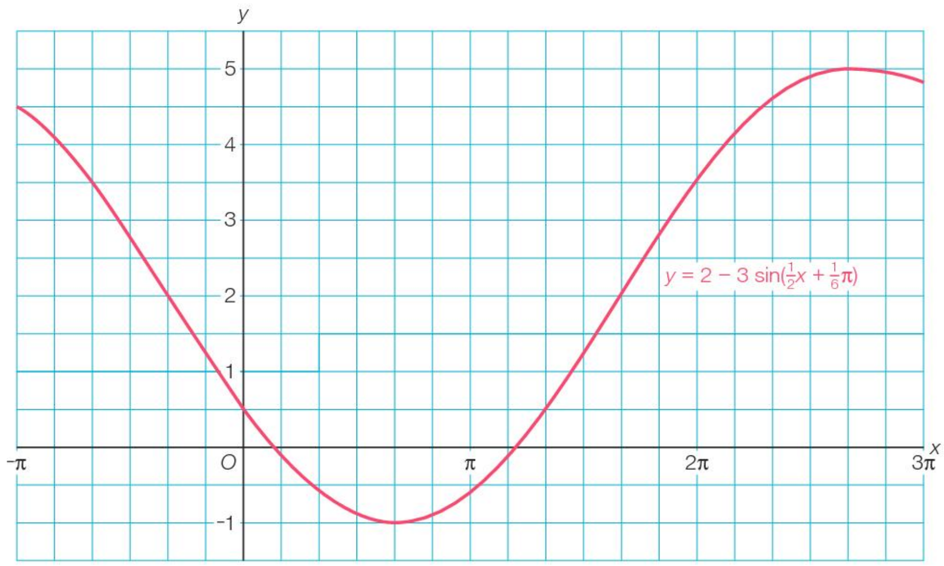
39 **a** $f(x) = -2 - \cos(x - \frac{1}{2}\pi)$
evenwichtsstand -2
amplitude 1
periode 2π
$-1 < 0$, dus het punt $(\frac{1}{2}\pi, -3)$ is een laagste punt van de grafiek.



b $g(x) = 5 - 3\sin(\frac{1}{4}\pi x)$
evenwichtsstand 5
amplitude 3
periode $\frac{2\pi}{\frac{1}{4}\pi} = 8$
$-3 < 0$, dus de grafiek gaat dalend door het punt (0, 5).



40 $y = 2 - 3\sin(\frac{1}{2}x + \frac{1}{6}\pi)$ oftewel $y = 2 - 3\sin(\frac{1}{2}(x + \frac{1}{3}\pi))$
evenwichtsstand 2
amplitude 3
Dalend door de evenwichtsstand bij $x = -\frac{1}{3}\pi$.
periode $\frac{2\pi}{\frac{1}{2}} = 4\pi$
Tussen het laagste en het hoogste punt zitten horizontaal 12 roosterhokjes,
dus $\frac{1}{2}$ periode $= 2\pi$ komt overeen met 12 roosterhokjes, dus horizontaal komt 1 roosterhokje overeen met $\frac{1}{6}\pi$.
Tussen het laagste en het hoogste punt zitten verticaal 12 roosterhokjes,
dus $2 \cdot$ amplitude $= 6$ komt overeen met 12 roosterhokjes, dus verticaal komt 1 roosterhokje overeen met $\frac{1}{2}$.
Het laagste punt is $(-\frac{1}{3}\pi + \frac{1}{4} \cdot 4\pi, 2 - 3) = (\frac{2}{3}\pi, -1)$.
Teken de verticale as 4 roosterhokjes links van het laagste punt en teken de horizontale as 2 roosterhokjes boven het laagste punt. Je krijgt de figuur hieronder.



41 **a** evenwichtsstand 3
amplitude 2
periode π

b $a = 3$
$b = 2$
$c = \frac{2\pi}{\pi} = 2$
$d = \frac{1}{3}\pi$

Bladzijde 150

42 **a** De grafiek gaat dalend door de evenwichtsstand bij $x = \pi$, dus
$y = 1 - 2\frac{1}{2}\sin(1\frac{1}{2}(x - \pi))$.

b Het punt $(0, -1\frac{1}{2})$ is een laagste punt, dus $y = 1 - 2\frac{1}{2}\cos(1\frac{1}{2}x)$.

43 **a** De grafiek gaat dalend door de evenwichtsstand bij $x = 3$, dus
$y = 1 - 2\frac{1}{2}\sin(\frac{1}{2}\pi(x - 3))$.

b Het punt $(0, -1\frac{1}{2})$ is een laagste punt, dus $y = 1 - 2\frac{1}{2}\cos(\frac{1}{2}\pi x)$.

44 **a** $a = \frac{3\frac{1}{2} + \frac{1}{2}}{2} = 2$
$b = 3\frac{1}{2} - 2 = 1\frac{1}{2}$
$\frac{1}{2}$ periode $= \pi - \frac{1}{2}\pi = \frac{1}{2}\pi$, dus periode $= \pi$, dus $c = \frac{2\pi}{\pi} = 2$.
Stijgend door de evenwichtsstand bij $x = \frac{1}{2}\pi$, dus $d = \frac{1}{2}\pi$.
Dus $y = 2 + 1\frac{1}{2}\sin(2(x - \frac{1}{2}\pi))$.

b Een hoogste punt is $(\frac{3}{4}\pi, 3\frac{1}{2})$, dus $y = 2 + 1\frac{1}{2}\cos(2(x - \frac{3}{4}\pi))$.

Bladzijde 151

45 **a** $a = \frac{4 + -2}{2} = 1$
$b = 4 - 1 = 3$
$\frac{1}{2}$ periode $= 2\frac{3}{4} - 1\frac{1}{4} = 1\frac{1}{2}$, dus periode $= 3$, dus $c = \frac{2\pi}{3} = \frac{2}{3}\pi$.
Stijgend door de evenwichtsstand bij $x = \frac{1\frac{1}{4} + 2\frac{3}{4}}{2} = 2$, dus $d = 2$.
Dus $y = 1 + 3\sin(\frac{2}{3}\pi(x - 2))$.

b De grafiek gaat dalend door de evenwichtsstand bij $x = 2 - \frac{1}{2} \cdot$ periode $= 2 - \frac{1}{2} \cdot 3 = \frac{1}{2}$,
dus $y = 1 - 3\sin(\frac{2}{3}\pi(x - \frac{1}{2}))$.

46 **a** $a = \frac{175 + 25}{2} = 100$
$b = 175 - 100 = 75$
$\frac{1}{2}$ periode $= 9 - 4 = 5$, dus periode $= 10$, dus $c = \frac{2\pi}{10} = \frac{1}{5}\pi$.
Stijgend door de evenwichtsstand bij $t = 4$, dus $d = 4$.
Dus $N = 100 + 75\sin(\frac{1}{5}\pi(t - 4))$.

b Er is een hoogste punt bij $t = 4 + \frac{1}{4} \cdot$ periode $= 4 + \frac{1}{4} \cdot 10 = 6\frac{1}{2}$.
Dus $N = 100 + 75\cos(\frac{1}{5}\pi(t - 6\frac{1}{2}))$.

47 **a** Stel $y = a + b\cos(c(x - d))$.
$a = 12 - 4 = 8$
$b = 4$
De periode is $18 - 3 = 15$, dus $c = \frac{2\pi}{15} = \frac{2}{15}\pi$.
Het punt (3, 12) is een hoogste punt, dus $d = 3$.
Dus $y = 8 + 4\cos(\frac{2}{15}\pi(x - 3))$.

b Stel $y = a + b\cos(c(x - d))$.
$a = 650$
$b = 812 - 650 = 162$
De periode is 48, dus $c = \frac{2\pi}{48} = \frac{1}{24}\pi$.
Het punt (16, 812) is een hoogste punt, dus $d = 16$.
Dus $y = 650 + 162\cos(\frac{1}{24}\pi(x - 16))$.

c Stel $y = a + b\cos(c(x - d))$.
$a = \frac{250 + 110}{2} = 180$
$b = 250 - 180 = 70$
$\frac{1}{2}$ periode $= 26 - 2 = 24$, dus periode $= 48$, dus $c = \frac{2\pi}{48} = \frac{1}{24}\pi$.
Het punt (2, 250) is een hoogste punt, dus $d = 2$.
Dus $y = 180 + 70\cos(\frac{1}{24}\pi(x - 2))$.

8.3 Goniometrische vergelijkingen

Bladzijde 153

48 $\ldots, -3\frac{1}{2}\pi, -2\frac{1}{2}\pi, -1\frac{1}{2}\pi, -\frac{1}{2}\pi, \frac{1}{2}\pi, 1\frac{1}{2}\pi, 2\frac{1}{2}\pi, 3\frac{1}{2}\pi, \ldots$

Bladzijde 154

49 **a** $\sin(3x - \frac{1}{2}\pi) = 0$
$3x - \frac{1}{2}\pi = k \cdot \pi$
$3x = \frac{1}{2}\pi + k \cdot \pi$
$x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$

b $\cos(\frac{1}{2}x - \frac{1}{6}\pi) = 0$
$\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$
$\frac{1}{2}x = \frac{2}{3}\pi + k \cdot \pi$
$x = 1\frac{1}{3}\pi + k \cdot 2\pi$

c $\sin^2(x) - \sin(x) = 0$
$\sin(x)(\sin(x) - 1) = 0$
$\sin(x) = 0 \vee \sin(x) = 1$
$x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi$

d $\cos^2(2x) + \cos(2x) = 0$
$\cos(2x)(\cos(2x) + 1) = 0$
$\cos(2x) = 0 \vee \cos(2x) = -1$
$2x = \frac{1}{2}\pi + k \cdot \pi \vee 2x = \pi + k \cdot 2\pi$
$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi \vee x = \frac{1}{2}\pi + k \cdot \pi$

Bladzijde 155

50 **a** $\cos^2(x - \frac{1}{5}\pi) = 1$
$\cos(x - \frac{1}{5}\pi) = 1 \vee \cos(x - \frac{1}{5}\pi) = -1$
$x - \frac{1}{5}\pi = k \cdot 2\pi \vee x - \frac{1}{5}\pi = \pi + k \cdot 2\pi$
$x = \frac{1}{5}\pi + k \cdot 2\pi \vee x = 1\frac{1}{5}\pi + k \cdot 2\pi$
Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = \frac{1}{5}\pi + k \cdot \pi$.

b $\sin^2(2x - \frac{1}{4}\pi) = 1$
$\sin(2x - \frac{1}{4}\pi) = 1 \vee \sin(2x - \frac{1}{4}\pi) = -1$
$2x - \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x - \frac{1}{4}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$
$2x = \frac{3}{4}\pi + k \cdot 2\pi \vee 2x = 1\frac{3}{4}\pi + k \cdot 2\pi$
$x = \frac{3}{8}\pi + k \cdot \pi \vee x = \frac{7}{8}\pi + k \cdot \pi$
Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = \frac{3}{8}\pi + k \cdot \frac{1}{2}\pi$.

c $\sin^3(x) - \sin(x) = 0$
$\sin(x)(\sin^2(x) - 1) = 0$
$\sin(x) = 0 \vee \sin^2(x) = 1$
$\sin(x) = 0 \vee \sin(x) = 1 \vee \sin(x) = -1$
$x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi \vee x = 1\frac{1}{2}\pi + k \cdot 2\pi$
Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = k \cdot \frac{1}{2}\pi$.

d $\cos^3(2x) - \cos(2x) = 0$
$\cos(2x)(\cos^2(2x) - 1) = 0$
$\cos(2x) = 0 \vee \cos^2(2x) = 1$
$\cos(2x) = 0 \vee \cos(2x) = 1 \vee \cos(2x) = -1$
$2x = \frac{1}{2}\pi + k \cdot \pi \vee 2x = k \cdot 2\pi \vee 2x = \pi + k \cdot 2\pi$
$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi \vee x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot \pi$
Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = k \cdot \frac{1}{4}\pi$.

51 **a** $\sin(4x - \frac{1}{3}\pi) = 1$
$4x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot 2\pi$
$4x = \frac{5}{6}\pi + k \cdot 2\pi$
$x = \frac{5}{24}\pi + k \cdot \frac{1}{2}\pi$

b $\cos(4\pi x) = -1$
$4\pi x = \pi + k \cdot 2\pi$
$x = \frac{1}{4} + k \cdot \frac{1}{2}$

c $\sin^2(\frac{1}{4}\pi x) = 1$
$\sin(\frac{1}{4}\pi x) = 1 \vee \sin(\frac{1}{4}\pi x) = -1$
$\frac{1}{4}\pi x = \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{4}\pi x = 1\frac{1}{2}\pi + k \cdot 2\pi$
$x = 2 + k \cdot 8 \vee x = 6 + k \cdot 8$
Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = 2 + k \cdot 4$.

d $\sin(2x)\cos(2x) + \sin(2x) = 0$
$\sin(2x)(\cos(2x) + 1) = 0$
$\sin(2x) = 0 \vee \cos(2x) = -1$
$2x = k \cdot \pi \vee 2x = \pi + k \cdot 2\pi$
$x = k \cdot \frac{1}{2}\pi \vee x = \frac{1}{2}\pi + k \cdot \pi$
Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = k \cdot \frac{1}{2}\pi$.

52 **a** $2\sin(x)\cos(2x) = 0$
$\sin(x) = 0 \vee \cos(2x) = 0$
$x = k \cdot \pi \vee 2x = \frac{1}{2}\pi + k \cdot \pi$
$x = k \cdot \pi \vee x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$
Het domein van f is $[0, 2\pi]$ geeft $x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{1}{4}\pi \vee x = \frac{3}{4}\pi \vee x = 1\frac{1}{4}\pi \vee x = 1\frac{3}{4}\pi$.
De nulpunten zijn $0, \frac{1}{4}\pi, \frac{3}{4}\pi, \pi, 1\frac{1}{4}\pi, 1\frac{3}{4}\pi$ en 2π.

b $\text{rc}_{AB} = \dfrac{2 - -2}{1\frac{1}{2}\pi - \frac{1}{2}\pi} = \dfrac{4}{\pi}$

$\left.\begin{array}{l} y = \dfrac{4}{\pi}x + b \\ \text{door } A(\frac{1}{2}\pi, -2) \end{array}\right\} \begin{array}{l} \dfrac{4}{\pi} \cdot \frac{1}{2}\pi + b = -2 \\ 2 + b = -2 \\ b = -4 \end{array}$

Dus $AB: y = \dfrac{4}{\pi}x - 4$.

Snijden met de x-as geeft $\dfrac{4}{\pi}x - 4 = 0$
$\dfrac{4}{\pi}x = 4$
$x = \pi$

Dus de lijn door A en B snijdt de x-as in $(\pi, 0)$, een van de snijpunten van de grafiek van f met de x-as.

c $f(\frac{1}{6}\pi) = 2\sin(\frac{1}{6}\pi) \cdot \cos(\frac{1}{3}\pi) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$, dus C ligt op de grafiek van f.
$f(1\frac{1}{6}\pi) = 2\sin(1\frac{1}{6}\pi) \cdot \cos(2\frac{1}{3}\pi) = 2 \cdot -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{2}$, dus D ligt op de grafiek van f.

d $rc_{CD} = \dfrac{-\frac{1}{2} - \frac{1}{2}}{1\frac{1}{6}\pi - \frac{1}{6}\pi} = \dfrac{-1}{\pi} = -\dfrac{1}{\pi}$

$\left.\begin{array}{l} y = -\dfrac{1}{\pi}x + b \\ \text{door } C(\frac{1}{6}\pi, \frac{1}{2}) \end{array}\right\} -\dfrac{1}{\pi} \cdot \frac{1}{6}\pi + b = \frac{1}{2}$

$-\frac{1}{6} + b = \frac{1}{2}$

$b = \frac{2}{3}$

Dus CD: $y = -\dfrac{1}{\pi}x + \frac{2}{3}$.

Snijden met de x-as geeft $-\dfrac{1}{\pi}x + \frac{2}{3} = 0$

$-\dfrac{1}{\pi}x = -\frac{2}{3}$

$x = \frac{2}{3}\pi$

Dus de lijn door C en D snijdt de x-as in $(\frac{2}{3}\pi, 0)$, dat is geen snijpunt van de grafiek van f met de x-as.

53 **a** $x = \frac{1}{6}\pi$ geeft $\sin(\frac{1}{6}\pi) = \frac{1}{2}$, dus $x = \frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.

b $x = 2\frac{1}{6}\pi$ geeft $\sin(2\frac{1}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}$, dus $x = 2\frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.
$x = 4\frac{1}{6}\pi$ geeft $\sin(4\frac{1}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}$, dus $x = 4\frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.

c $x = \frac{5}{6}\pi$ geeft $\sin(\frac{5}{6}\pi) = \frac{1}{2}$, dus $x = \frac{5}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.

d $x = 2\frac{5}{6}\pi$ geeft $\sin(2\frac{5}{6}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$, dus $x = 2\frac{5}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.
$x = -1\frac{1}{6}\pi$ geeft $\sin(-1\frac{1}{6}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$, dus $x = -1\frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.

Bladzijde 157

54 **a** $2\sin(\frac{1}{2}x) = 1$

$\sin(\frac{1}{2}x) = \frac{1}{2}$

$\frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 4\pi \vee x = 1\frac{2}{3}\pi + k \cdot 4\pi$

b $2\cos(x - \frac{1}{3}\pi) = 1$

$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$

$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot 2\pi$

c $2\sin(2x - \frac{1}{4}\pi) = -\sqrt{3}$

$\sin(2x - \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$

$2x - \frac{1}{4}\pi = 1\frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{1}{4}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$

$2x = 1\frac{7}{12}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{12}\pi + k \cdot 2\pi$

$x = \frac{19}{24}\pi + k \cdot \pi \vee x = -\frac{1}{24}\pi + k \cdot \pi$

d $2\cos(3x - \pi) = -1$

$\cos(3x - \pi) = -\frac{1}{2}$

$3x - \pi = \frac{2}{3}\pi + k \cdot 2\pi \vee 3x - \pi = -\frac{2}{3}\pi + k \cdot 2\pi$

$3x = 1\frac{2}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{5}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

55 **a** $2\sin(2x - \frac{1}{6}\pi) = \sqrt{2}$

$\sin(2x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{2}$

$2x - \frac{1}{6}\pi = \frac{1}{4}\pi + k \cdot 2\pi \vee 2x - \frac{1}{6}\pi = \frac{3}{4}\pi + k \cdot 2\pi$

$2x = \frac{5}{12}\pi + k \cdot 2\pi \vee 2x = \frac{11}{12}\pi + k \cdot 2\pi$

$x = \frac{5}{24}\pi + k \cdot \pi \vee x = \frac{11}{24}\pi + k \cdot \pi$

x in $[0, 2\pi]$ geeft $x = \frac{5}{24}\pi \vee x = 1\frac{5}{24}\pi \vee x = \frac{11}{24}\pi \vee x = 1\frac{11}{24}\pi$

b $2\cos(3x - \frac{1}{2}\pi) = \sqrt{3}$

$\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{3}$

$3x - \frac{1}{2}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$

$3x = \frac{2}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

x in $[0, 2\pi]$ geeft $x = \frac{2}{9}\pi \vee x = \frac{8}{9}\pi \vee x = 1\frac{5}{9}\pi \vee x = \frac{1}{9}\pi \vee x = \frac{7}{9}\pi \vee x = 1\frac{4}{9}\pi$

c $\sin(\frac{2}{3}x) = \frac{1}{2}\sqrt{2}$

$\frac{2}{3}x = \frac{1}{4}\pi + k \cdot 2\pi \vee \frac{2}{3}x = \frac{3}{4}\pi + k \cdot 2\pi$

$x = \frac{3}{8}\pi + k \cdot 3\pi \vee x = 1\frac{1}{8}\pi + k \cdot 3\pi$

x in $[0, 2\pi]$ geeft $x = \frac{3}{8}\pi \vee x = 1\frac{1}{8}\pi$

d $\cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{3}$

$\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{5}{6}\pi + k \cdot 2\pi$

$x = 1\frac{2}{3}\pi + k \cdot 4\pi \vee x = -1\frac{2}{3}\pi + k \cdot 4\pi$

x in $[0, 2\pi]$ geeft $x = 1\frac{2}{3}\pi$

56 **a** $2\sin^2(x) = 1$

$\sin^2(x) = \frac{1}{2}$

$\sin(x) = \sqrt{\frac{1}{2}} \vee \sin(x) = -\sqrt{\frac{1}{2}}$

$\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$

Dus uit $2\sin^2(x) = 1$ volgt $\sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2}$.

b Uit $\sin(x) = \frac{1}{2}\sqrt{2}$ volgt $x = \frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{4}\pi + k \cdot 2\pi$.

Uit $\sin(x) = -\frac{1}{2}\sqrt{2}$ volgt $x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = 1\frac{1}{4}\pi + k \cdot 2\pi$.

Dus uit $\sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2}$ volgt

$x = \frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{4}\pi + k \cdot 2\pi \vee x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = 1\frac{1}{4}\pi + k \cdot 2\pi$.

c $x = \frac{1}{4}\pi + k \cdot 2\pi$: ..., $-1\frac{3}{4}\pi$, $\frac{1}{4}\pi$, $2\frac{1}{4}\pi$, ...

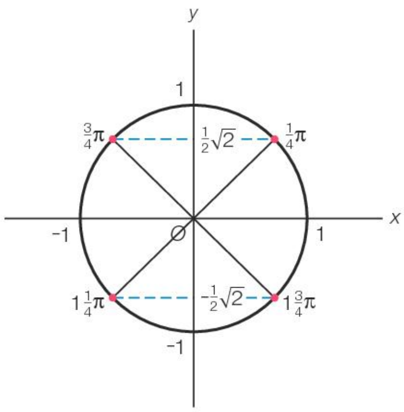
$x = \frac{3}{4}\pi + k \cdot 2\pi$: ..., $-1\frac{1}{4}\pi$, $\frac{3}{4}\pi$, $2\frac{3}{4}\pi$, ...

$x = 1\frac{1}{4}\pi + k \cdot 2\pi$: ..., $-\frac{3}{4}\pi$, $1\frac{1}{4}\pi$, $3\frac{1}{4}\pi$, ...

$x = -\frac{1}{4}\pi + k \cdot 2\pi$: ..., $-\frac{1}{4}\pi$, $1\frac{3}{4}\pi$, $3\frac{3}{4}\pi$, ...

Dus $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$.

d



Uit de cirkel is af te lezen dat $\sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2}$ geeft $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$.

Bladzijde 158

57 **a** $2\cos^2(\frac{1}{2}x) = 1$

$\cos^2(\frac{1}{2}x) = \frac{1}{2}$

$\cos(\frac{1}{2}x) = \sqrt{\frac{1}{2}} \vee \cos(\frac{1}{2}x) = -\sqrt{\frac{1}{2}}$

$\cos(\frac{1}{2}x) = \frac{1}{2}\sqrt{2} \vee \cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{2}$

$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

$x = \frac{1}{2}\pi + k \cdot \pi$

b $4\sin^2(x - \frac{1}{6}\pi) = 1$

$\sin^2(x - \frac{1}{6}\pi) = \frac{1}{4}$

$\sin(x - \frac{1}{6}\pi) = \frac{1}{2} \vee \sin(x - \frac{1}{6}\pi) = -\frac{1}{2}$

$x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = \pi + k \cdot 2\pi \vee x = k \cdot 2\pi \vee x = 1\frac{1}{3}\pi + k \cdot 2\pi$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot

$x = \frac{1}{3}\pi + k \cdot \pi \vee x = k \cdot \pi$.

c $4\cos^2(x + \frac{1}{4}\pi) = 3$

$\cos^2(x + \frac{1}{4}\pi) = \frac{3}{4}$

$\cos(x + \frac{1}{4}\pi) = \sqrt{\frac{3}{4}} \vee \cos(x + \frac{1}{4}\pi) = -\sqrt{\frac{3}{4}}$

$\cos(x + \frac{1}{4}\pi) = \frac{1}{2}\sqrt{3} \vee \cos(x + \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$

$x + \frac{1}{4}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$

$x = -\frac{1}{12}\pi + k \cdot 2\pi \vee x = -\frac{5}{12}\pi + k \cdot 2\pi \vee x = \frac{7}{12}\pi + k \cdot 2\pi \vee x = -\frac{13}{12}\pi + k \cdot 2\pi$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot

$x = -\frac{1}{12}\pi + k \cdot \pi \vee x = -\frac{5}{12}\pi + k \cdot \pi$.

d $4\sin^3(x) - \sin(x) = 0$

$\sin(x)(4\sin^2(x) - 1) = 0$

$\sin(x) = 0 \vee 4\sin^2(x) = 1$

$\sin(x) = 0 \vee \sin^2(x) = \frac{1}{4}$

$\sin(x) = 0 \vee \sin(x) = \frac{1}{2} \vee \sin(x) = -\frac{1}{2}$

$x = k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot

$x = k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot \pi \vee x = \frac{5}{6}\pi + k \cdot \pi$.

58 **a** $\sin(\frac{1}{2}\pi x) = \frac{1}{2}\sqrt{3}$

$\frac{1}{2}\pi x = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = \frac{2}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3} + k \cdot 4 \vee x = 1\frac{1}{3} + k \cdot 4$

x in [0, 10] geeft $x = \frac{2}{3} \vee x = 4\frac{2}{3} \vee x = 8\frac{2}{3} \vee x = 1\frac{1}{3} \vee x = 5\frac{1}{3} \vee x = 9\frac{1}{3}$

b $\cos(\frac{1}{3}\pi x) = -\frac{1}{2}\sqrt{3}$

$\frac{1}{3}\pi x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{3}\pi x = -\frac{5}{6}\pi + k \cdot 2\pi$

$x = 2\frac{1}{2} + k \cdot 6 \vee x = -2\frac{1}{2} + k \cdot 6$

x in $[0, 10]$ geeft $x = 2\frac{1}{2} \vee x = 8\frac{1}{2} \vee x = 3\frac{1}{2} \vee x = 9\frac{1}{2}$

c $4\sin^2(\frac{1}{5}\pi x) = 1$

$\sin^2(\frac{1}{5}\pi x) = \frac{1}{4}$

$\sin(\frac{1}{5}\pi x) = \frac{1}{2} \vee \sin(\frac{1}{5}\pi x) = -\frac{1}{2}$

$\frac{1}{5}\pi x = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{5}\pi x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{5}\pi x = -\frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{5}\pi x = 1\frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{6} + k \cdot 10 \vee x = 4\frac{1}{6} + k \cdot 10 \vee x = -\frac{5}{6} + k \cdot 10 \vee x = 5\frac{5}{6} + k \cdot 10$

x in $[0, 10]$ geeft $x = \frac{5}{6} \vee x = 4\frac{1}{6} \vee x = 9\frac{1}{6} \vee x = 5\frac{5}{6}$

d $2\cos^2(0{,}1\pi x) + \cos(0{,}1\pi x) = 0$

$\cos(0{,}1\pi x)(2\cos(0{,}1\pi x) + 1) = 0$

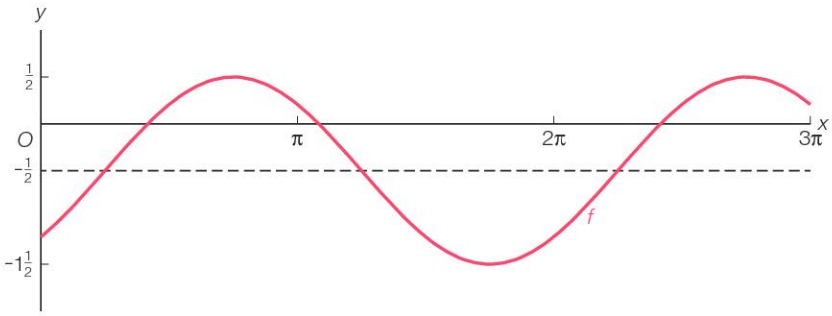
$\cos(0{,}1\pi x) = 0 \vee \cos(0{,}1\pi x) = -\frac{1}{2}$

$0{,}1\pi x = \frac{1}{2}\pi + k \cdot \pi \vee 0{,}1\pi x = \frac{2}{3}\pi + k \cdot 2\pi \vee 0{,}1\pi x = -\frac{2}{3}\pi + k \cdot 2\pi$

$x = 5 + k \cdot 10 \vee x = 6\frac{2}{3} + k \cdot 20 \vee x = -6\frac{2}{3} + k \cdot 20$

x in $[0, 10]$ geeft $x = 5 \vee x = 6\frac{2}{3}$

59 **a** Voer in $y_1 = -\frac{1}{2} + \sin(x - \frac{1}{4}\pi)$.



b $f(x) = -\frac{1}{2}$ geeft $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -\frac{1}{2}$

$\sin(x - \frac{1}{4}\pi) = 0$

$x - \frac{1}{4}\pi = k \cdot \pi$

$x = \frac{1}{4}\pi + k \cdot \pi$

x in $[0, 3\pi]$ geeft $x = \frac{1}{4}\pi \vee x = 1\frac{1}{4}\pi \vee x = 2\frac{1}{4}\pi$

De grafiek van f snijdt de lijn van de evenwichtsstand in de punten $(\frac{1}{4}\pi, -\frac{1}{2})$, $(1\frac{1}{4}\pi, -\frac{1}{2})$ en $(2\frac{1}{4}\pi, -\frac{1}{2})$.

c $f(x) = \frac{1}{2}$ geeft $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = \frac{1}{2}$

$\sin(x - \frac{1}{4}\pi) = 1$

$x - \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot 2\pi$

$x = \frac{3}{4}\pi + k \cdot 2\pi$

x in $[0, 3\pi]$ geeft $x = \frac{3}{4}\pi \vee x = 2\frac{3}{4}\pi$

$f(x) = -1\frac{1}{2}$ geeft $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -1\frac{1}{2}$

$\sin(x - \frac{1}{4}\pi) = -1$

$x - \frac{1}{4}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$

$x = 1\frac{3}{4}\pi + k \cdot 2\pi$

x in $[0, 3\pi]$ geeft $x = 1\frac{3}{4}\pi$

De toppen van de grafiek van f zijn $(\frac{3}{4}\pi, \frac{1}{2})$, $(1\frac{3}{4}\pi, -1\frac{1}{2})$ en $(2\frac{3}{4}\pi, \frac{1}{2})$.

d $f(x) = 0$ geeft $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = 0$

$\sin(x - \frac{1}{4}\pi) = \frac{1}{2}$

$x - \frac{1}{4}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{12}\pi + k \cdot 2\pi \vee x = 1\frac{1}{12}\pi + k \cdot 2\pi$

x in $[0, 3\pi]$ geeft $x = \frac{5}{12}\pi \vee x = 2\frac{5}{12}\pi \vee x = 1\frac{1}{12}\pi$

Dus $x_A = \frac{5}{12}\pi$, $x_B = 1\frac{1}{12}\pi$ en $x_C = 2\frac{5}{12}\pi$.

$AB = x_B - x_A = 1\frac{1}{12}\pi - \frac{5}{12}\pi = \frac{8}{12}\pi = \frac{2}{3}\pi$

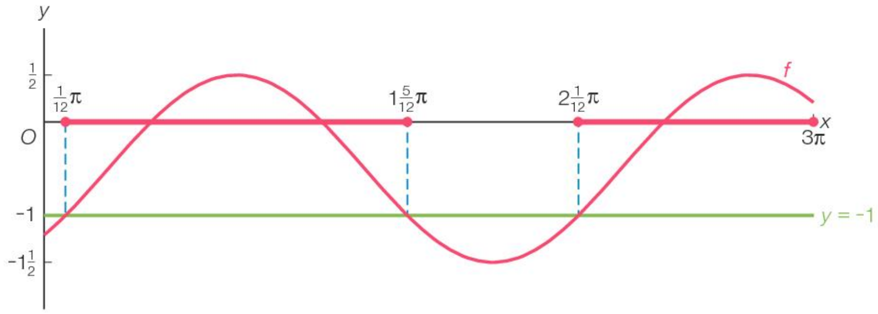
e $f(x) = -1$ geeft $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -1$

$\sin(x - \frac{1}{4}\pi) = -\frac{1}{2}$

$x - \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{12}\pi + k \cdot 2\pi \vee x = 1\frac{5}{12}\pi + k \cdot 2\pi$

x in $[0, 3\pi]$ geeft $x = \frac{1}{12}\pi \vee x = 2\frac{1}{12}\pi \vee x = 1\frac{5}{12}\pi$



$f(x) \geq -1$ geeft $\frac{1}{12}\pi \leq x \leq 1\frac{5}{12}\pi \vee 2\frac{1}{12}\pi \leq x \leq 3\pi$

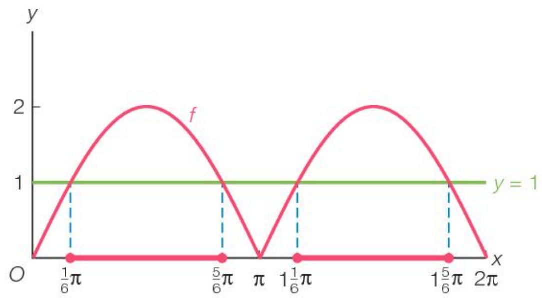
60 $f(x) = 1$ geeft $|2\sin(x)| = 1$

$2\sin(x) = 1 \vee 2\sin(x) = -1$

$\sin(x) = \frac{1}{2} \vee \sin(x) = -\frac{1}{2}$

$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$



$f(x) \geq 1$ geeft $\frac{1}{6}\pi \leq x \leq \frac{5}{6}\pi \vee 1\frac{1}{6}\pi \leq x \leq 1\frac{5}{6}\pi$

61 **a** $\sin(3x) = \sin(\frac{1}{6}\pi)$

$\sin(3x) = \frac{1}{2}$

$3x = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x = \frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$

b $\cos(3x) = \cos(\frac{1}{6}\pi)$

$\cos(3x) = \frac{1}{2}\sqrt{3}$

$3x = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \vee x = -\frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$

Bladzijde 159

62 **a** $\sin(x+1) = \sin(2x+3)$

$x+1 = 2x+3+k \cdot 2\pi \vee x+1 = \pi - (2x+3) + k \cdot 2\pi$

$-x = 2 + k \cdot 2\pi \vee x+1 = \pi - 2x - 3 + k \cdot 2\pi$

$x = -2 + k \cdot 2\pi \vee 3x = -4 + \pi + k \cdot 2\pi$

$x = -2 + k \cdot 2\pi \vee x = -1\frac{1}{3} + \frac{1}{3}\pi + k \cdot \frac{2}{3}\pi$

b $\cos(2x-1) = \cos(x+1)$

$2x - 1 = x + 1 + k \cdot 2\pi \vee 2x - 1 = -(x+1) + k \cdot 2\pi$

$x = 2 + k \cdot 2\pi \vee 2x - 1 = -x - 1 + k \cdot 2\pi$

$x = 2 + k \cdot 2\pi \vee 3x = k \cdot 2\pi$

$x = 2 + k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$

c $\sin(2x - \frac{1}{2}\pi) = \sin(x + \frac{1}{3}\pi)$

$2x - \frac{1}{2}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$

$x = \frac{5}{6}\pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = \pi - x - \frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{5}{6}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$

d $\cos(x - \frac{1}{3}\pi) = \cos(2x)$

$x - \frac{1}{3}\pi = 2x + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -2x + k \cdot 2\pi$

$-x = \frac{1}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$

$x = -\frac{1}{3}\pi + k \cdot 2\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

e $\sin(2\pi x) = \sin(\pi(x-1))$

$\sin(2\pi x) = \sin(\pi x - \pi)$

$2\pi x = \pi x - \pi + k \cdot 2\pi \vee 2\pi x = \pi - (\pi x - \pi) + k \cdot 2\pi$

$\pi x = -\pi + k \cdot 2\pi \vee 2\pi x = \pi - \pi x + \pi + k \cdot 2\pi$

$x = -1 + k \cdot 2 \vee 3\pi x = 2\pi + k \cdot 2\pi$

$x = -1 + k \cdot 2 \vee x = \frac{2}{3} + k \cdot \frac{2}{3}$

$x = -1 + k \cdot 2 \vee x = k \cdot \frac{2}{3}$

f $\cos(\frac{1}{2}\pi x) = \cos(\pi(x-2))$

$\cos(\frac{1}{2}\pi x) = \cos(\pi x - 2\pi)$

$\frac{1}{2}\pi x = \pi x - 2\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = -(\pi x - 2\pi) + k \cdot 2\pi$

$-\frac{1}{2}\pi x = -2\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = -\pi x + 2\pi + k \cdot 2\pi$

$x = 4 + k \cdot 4 \vee 1\frac{1}{2}\pi x = 2\pi + k \cdot 2\pi$

$x = k \cdot 4 \vee x = 1\frac{1}{3} + k \cdot 1\frac{1}{3}$

$x = k \cdot 4 \vee x = k \cdot 1\frac{1}{3}$

Deze rijtjes oplossingen kunnen (dit is niet verplicht) samengenomen worden tot $x = k \cdot 1\frac{1}{3}$.

63 **a** $\sin(2x - \frac{1}{3}\pi) = \sin(x + \frac{1}{4}\pi)$

$2x - \frac{1}{3}\pi = x + \frac{1}{4}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \pi - (x + \frac{1}{4}\pi) + k \cdot 2\pi$

$x = \frac{7}{12}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \pi - x - \frac{1}{4}\pi + k \cdot 2\pi$

$x = \frac{7}{12}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{12}\pi + k \cdot 2\pi$

$x = \frac{7}{12}\pi + k \cdot 2\pi \vee x = \frac{13}{36}\pi + k \cdot \frac{2}{3}\pi$

x in $[0, 2\pi]$ geeft $x = \frac{7}{12}\pi \vee x = \frac{13}{36}\pi \vee x = 1\frac{1}{36}\pi \vee x = 1\frac{25}{36}\pi$

b $\cos(3x + \frac{1}{2}\pi) = \cos(2x - \frac{1}{4}\pi)$

$3x + \frac{1}{2}\pi = 2x - \frac{1}{4}\pi + k \cdot 2\pi \vee 3x + \frac{1}{2}\pi = -(2x - \frac{1}{4}\pi) + k \cdot 2\pi$

$x = -\frac{3}{4}\pi + k \cdot 2\pi \vee 3x + \frac{1}{2}\pi = -2x + \frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{3}{4}\pi + k \cdot 2\pi \vee 5x = -\frac{1}{4}\pi + k \cdot 2\pi$

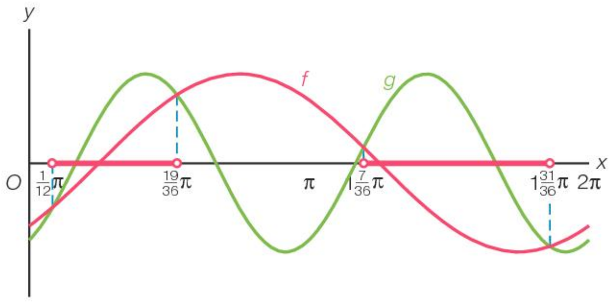
$x = -\frac{3}{4}\pi + k \cdot 2\pi \vee x = -\frac{1}{20}\pi + k \cdot \frac{2}{5}\pi$

x in $[0, 2\pi]$ geeft $x = 1\frac{1}{4}\pi \vee x = \frac{7}{20}\pi \vee x = \frac{3}{4}\pi \vee x = 1\frac{3}{20}\pi \vee x = 1\frac{11}{20}\pi \vee x = 1\frac{19}{20}\pi$

64 $f(x) = g(x)$ geeft $\sin(x - \frac{1}{4}\pi) = \sin(2x - \frac{1}{3}\pi)$

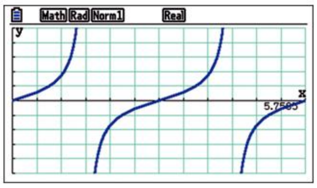
$$x - \frac{1}{4}\pi = 2x - \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \pi - (2x - \frac{1}{3}\pi) + k \cdot 2\pi$$
$$-x = -\frac{1}{12}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \pi - 2x + \frac{1}{3}\pi + k \cdot 2\pi$$
$$x = \frac{1}{12}\pi + k \cdot 2\pi \vee 3x = 1\frac{7}{12}\pi + k \cdot 2\pi$$
$$x = \frac{1}{12}\pi + k \cdot 2\pi \vee x = \frac{19}{36}\pi + k \cdot \frac{2}{3}\pi$$

x in $[0, 2\pi]$ geeft $x = \frac{1}{12}\pi \vee x = \frac{19}{36}\pi \vee x = 1\frac{7}{36}\pi \vee x = 1\frac{31}{36}\pi$



$f(x) < g(x)$ geeft $\frac{1}{12}\pi < x < \frac{19}{36}\pi \vee 1\frac{7}{36}\pi < x < 1\frac{31}{36}\pi$

65 **a**



b Dat zijn de lijnen $x = \frac{1}{2}\pi$ en $x = 1\frac{1}{2}\pi$.

66 **a** $\left.\begin{array}{l} \tan(A) = \dfrac{\sin(A)}{\cos(A)} \\ \tan(A) = 0 \end{array}\right\} \begin{array}{l} \sin(A) = 0 \wedge \cos(A) \neq 0 \\ A = k \cdot \pi \end{array}$

b $\tan(2x - \frac{1}{6}\pi) = 0$

$2x - \frac{1}{6}\pi = k \cdot \pi$

$2x = \frac{1}{6}\pi + k \cdot \pi$

$x = \frac{1}{12}\pi + k \cdot \frac{1}{2}\pi$

Bladzijde 161

67 **a** $\tan(3x - \frac{1}{6}\pi) = \frac{1}{3}\sqrt{3}$

$3x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot \pi$

$3x = \frac{1}{3}\pi + k \cdot \pi$

$x = \frac{1}{9}\pi + k \cdot \frac{1}{3}\pi$

b $1 - \tan(\frac{3}{4}x) = 2$

$\tan(\frac{3}{4}x) = -1$

$\frac{3}{4}x = \frac{3}{4}\pi + k \cdot \pi$

$x = \pi + k \cdot 1\frac{1}{3}\pi$

c $\tan(\frac{1}{2}x) = \tan(2x - \frac{1}{6}\pi)$

$\frac{1}{2}x = 2x - \frac{1}{6}\pi + k \cdot \pi$

$-1\frac{1}{2}x = -\frac{1}{6}\pi + k \cdot \pi$

$x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

d $3\tan(\frac{1}{2}\pi x) = \sqrt{3}$

$\tan(\frac{1}{2}\pi x) = \frac{1}{3}\sqrt{3}$

$\frac{1}{2}\pi x = \frac{1}{6}\pi + k \cdot \pi$

$x = \frac{1}{3} + k \cdot 2$

e $2 + \sqrt{3} \cdot \tan(\frac{1}{8}\pi x) = 5$

$\sqrt{3} \cdot \tan(\frac{1}{8}\pi x) = 3$

$\tan(\frac{1}{8}\pi x) = \sqrt{3}$

$\frac{1}{8}\pi x = \frac{1}{3}\pi + k \cdot \pi$

$x = 2\frac{2}{3} + k \cdot 8$

f $\tan(\frac{1}{3}\pi x) = \tan(\frac{1}{4}\pi(x - 1))$

$\frac{1}{3}\pi x = \frac{1}{4}\pi(x - 1) + k \cdot \pi$

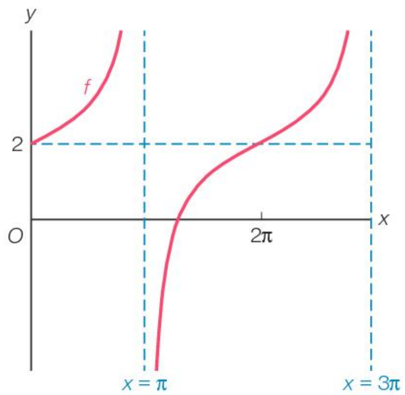
$\frac{1}{3}\pi x = \frac{1}{4}\pi x - \frac{1}{4}\pi + k \cdot \pi$

$\frac{1}{12}\pi x = -\frac{1}{4}\pi + k \cdot \pi$

$x = -3 + k \cdot 12$

68 **a** Beginpunt (0, 2), periode $\frac{\pi}{\frac{1}{2}} = 2\pi$ en asymptoten zijn de lijnen $x = \pi$ en $x = 3\pi$.

b



c Voer in $y_1 = 2 + \tan(\frac{1}{2}x)$.
De optie nulpunt geeft $x \approx 4{,}07$.
Dus het nulpunt is 4,07.

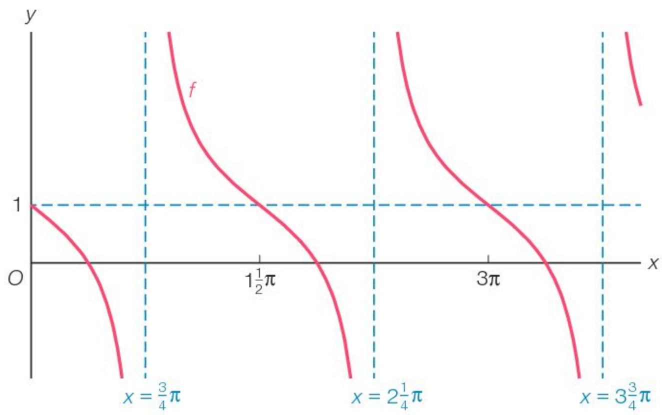
d $f(x) = 3$ geeft $2 + \tan(\frac{1}{2}x) = 3$

$$\tan(\tfrac{1}{2}x) = 1$$
$$\tfrac{1}{2}x = \tfrac{1}{4}\pi + k \cdot \pi$$
$$x = \tfrac{1}{2}\pi + k \cdot 2\pi$$

x in $[0, 3\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 2\frac{1}{2}\pi$, dus de snijpunten zijn $(\frac{1}{2}\pi, 3)$ en $(2\frac{1}{2}\pi, 3)$.

69 **a** Beginpunt (0, 1), periode $\frac{\pi}{\frac{2}{3}} = 1\frac{1}{2}\pi$ en asymptoten zijn de lijnen $x = \frac{3}{4}\pi$, $x = 2\frac{1}{4}\pi$ en $x = 3\frac{3}{4}\pi$.

b



c $f(x) = 0$ geeft $1 - \tan(\frac{2}{3}x) = 0$

$$\tan(\tfrac{2}{3}x) = 1$$
$$\tfrac{2}{3}x = \tfrac{1}{4}\pi + k \cdot \pi$$
$$x = \tfrac{3}{8}\pi + k \cdot 1\tfrac{1}{2}\pi$$

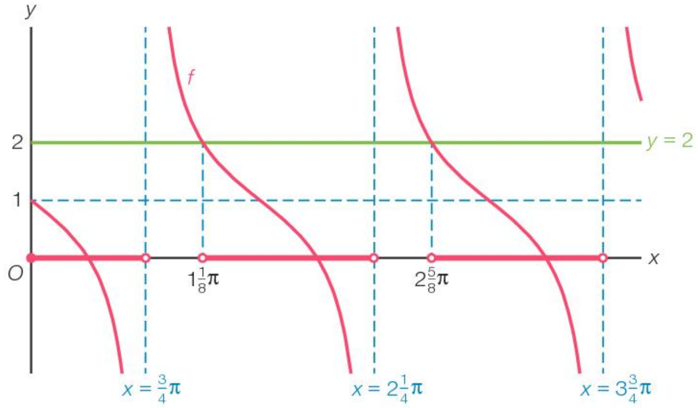
x in $[0, 4\pi]$ geeft $x = \frac{3}{8}\pi \vee x = 1\frac{7}{8}\pi \vee x = 3\frac{3}{8}\pi$

Dus de nulpunten zijn $\frac{3}{8}\pi$, $1\frac{7}{8}\pi$ en $3\frac{3}{8}\pi$.

d $f(x) = 2$ geeft $1 - \tan(\frac{2}{3}x) = 2$

$$\tan(\tfrac{2}{3}x) = -1$$
$$\tfrac{2}{3}x = \tfrac{3}{4}\pi + k \cdot \pi$$
$$x = 1\tfrac{1}{8}\pi + k \cdot 1\tfrac{1}{2}\pi$$

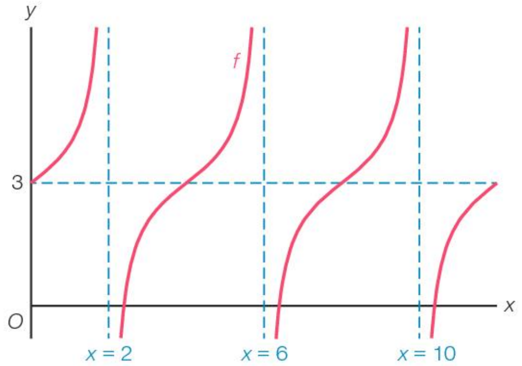
x in $[0, 4\pi]$ geeft $x = 1\frac{1}{8}\pi \vee x = 2\frac{5}{8}\pi$



$f(x) < 2$ geeft $0 \leq x < \frac{3}{4}\pi \vee 1\frac{1}{8}\pi < x < 2\frac{1}{4}\pi \vee 2\frac{5}{8}\pi < x < 3\frac{3}{4}\pi$

70 **a** Beginpunt (0, 3), periode $\dfrac{\pi}{\frac{1}{4}\pi} = 4$ en asymptoten zijn de lijnen $x = 2$, $x = 6$ en $x = 10$.

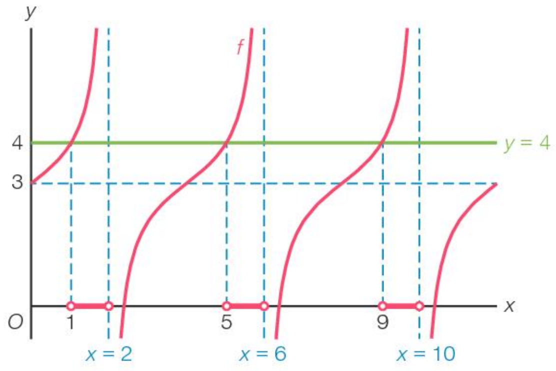
b



c $f(x) = 4$ geeft $3 + \tan(\frac{1}{4}\pi x) = 4$

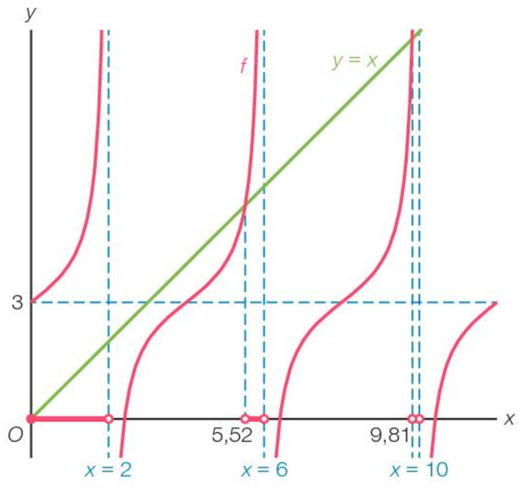
$$\tan(\tfrac{1}{4}\pi x) = 1$$
$$\tfrac{1}{4}\pi x = \tfrac{1}{4}\pi + k \cdot \pi$$
$$x = 1 + k \cdot 4$$

x in $[0, 12]$ geeft $x = 1 \vee x = 5 \vee x = 9$



$f(x) > 4$ geeft $1 < x < 2 \vee 5 < x < 6 \vee 9 < x < 10$

d Voer in $y_1 = 3 + \tan(\frac{1}{4}\pi x)$ en $y_2 = x$.
De optie snijpunt geeft $x \approx 5{,}52$ en $x \approx 9{,}81$.



$f(x) > x$ geeft $0 \leq x < 2 \vee 5{,}52 < x < 6 \vee 9{,}81 < x < 10$

71 $\sin(2x - \frac{1}{3}\pi) = -\cos(2x - \frac{1}{3}\pi)$

$\frac{\sin(2x - \frac{1}{3}\pi)}{\cos(2x - \frac{1}{3}\pi)} = -1$

$\tan(2x - \frac{1}{3}\pi) = -1$

$2x - \frac{1}{3}\pi = \frac{3}{4}\pi + k \cdot \pi$

$2x = 1\frac{1}{12}\pi + k \cdot \pi$

$x = \frac{13}{24}\pi + k \cdot \frac{1}{2}\pi$

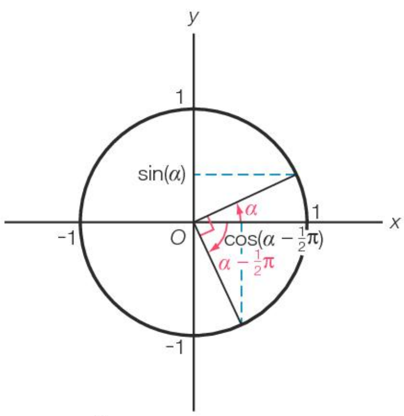
8.4 Herleiden en differentiëren

Bladzijde 163

72 $\sin(-\alpha) = y_Q = -y_P = -\sin(\alpha)$
$\cos(-\alpha) = x_Q = x_P = \cos(\alpha)$

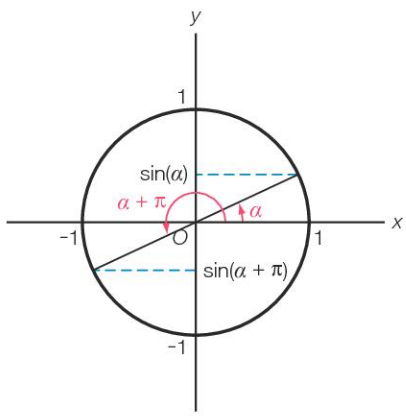
Bladzijde 164

73 **a**



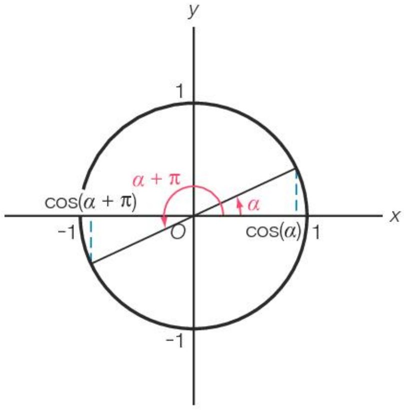
$\cos(\alpha - \frac{1}{2}\pi) = \sin(\alpha)$

b



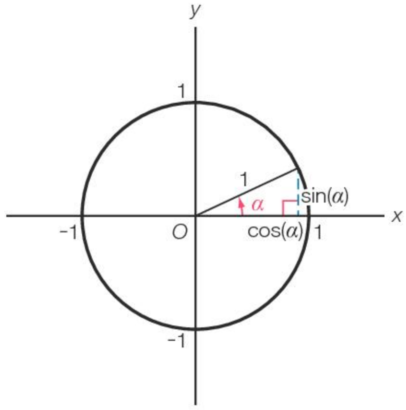
$\sin(\alpha + \pi) = -\sin(\alpha)$

c



$\cos(\alpha + \pi) = -\cos(\alpha)$

d



De stelling van Pythagoras geeft $\sin^2(\alpha) + \cos^2(\alpha) = 1$.

74 **a** $\sin(x + \frac{1}{6}\pi) = \cos(x + \frac{1}{6}\pi - \frac{1}{2}\pi) = \cos(x - \frac{1}{3}\pi)$

b $\cos(2x + \frac{1}{3}\pi) = \sin(2x + \frac{1}{3}\pi + \frac{1}{2}\pi) = \sin(2x + \frac{5}{6}\pi)$

c $-\sin(3x - \frac{2}{3}\pi) = \sin(3x - \frac{2}{3}\pi + \pi) = \sin(3x + \frac{1}{3}\pi) = \cos(3x + \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(3x - \frac{1}{6}\pi)$

d $-\cos(4x + 1\frac{1}{6}\pi) = \cos(4x + 1\frac{1}{6}\pi + \pi) = \cos(4x + 2\frac{1}{6}\pi) = \sin(4x + 2\frac{1}{6} + \frac{1}{2}\pi) = \sin(4x + 2\frac{2}{3}\pi) = \sin(4x + \frac{2}{3}\pi)$

75 **a** $(\sin(x) - \cos(x))^2 = \sin^2(x) - 2\sin(x)\cos(x) + \cos^2(x) = \sin^2(x) + \cos^2(x) - 2\sin(x)\cos(x) = 1 - 2\sin(x)\cos(x)$

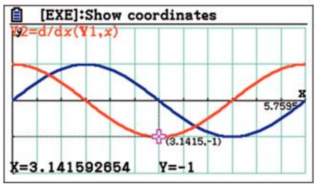
b $\dfrac{2\sin^2(x) + \cos^2(x)}{\cos^2(x)} = \dfrac{2\sin^2(x)}{\cos^2(x)} + \dfrac{\cos^2(x)}{\cos^2(x)} = 2\left(\dfrac{\sin(x)}{\cos(x)}\right)^2 + 1 = 2\tan^2(x) + 1$

76 **a** $\sin^2(x) + 4\cos(x) = 1 - \cos^2(x) + 4\cos(x)$

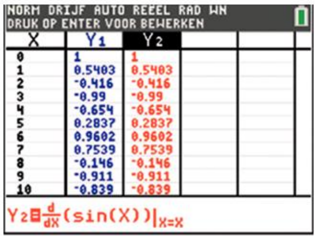
b $2\cos^2(x) + \sin(x) - 2 = 2(1 - \sin^2(x)) + \sin(x) - 2 = 2 - 2\sin^2(x) + \sin(x) - 2 = -2\sin^2(x) + \sin(x)$

c $2\sin^2(x) + \cos^2(x) + \cos(x) = 2(1 - \cos^2(x)) + \cos^2(x) + \cos(x) =$
$2 - 2\cos^2(x) + \cos^2(x) + \cos(x) = -\cos^2(x) + \cos(x) + 2$

77 **a** Voer in $y_1 = \sin(x)$ en $y_2 = \frac{d}{dx}(y_1)\Big|_{x=x}$.



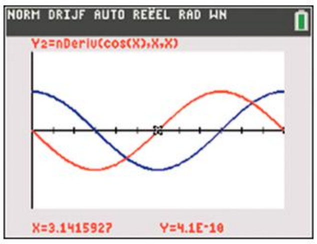
b Waarschijnlijk is $y = \cos(x)$ de afgeleide van $y = \sin(x)$.


NORM DRIJF AUTO REËEL RAD WN
DRUK OP ENTER VOOR BEWERKEN

X	Y₁	Y₂
0	1	1
1	0.5403	0.5403
2	-0.416	-0.416
3	-0.99	-0.99
4	-0.654	-0.654
5	0.2837	0.2837
6	0.9602	0.9602
7	0.7539	0.7539
8	-0.146	-0.146
9	-0.911	-0.911
10	-0.839	-0.839

Y₂=d/dX(sin(X))|X=X

c Voer in $y_1 = \cos(x)$ en $y_2 = \frac{d}{dx}(y_1)\Big|_{x=x}$.



Waarschijnlijk is $y = -\sin(x)$ de afgeleide van $y = \cos(x)$.

NORM DRIJF AUTO REËEL RAD WN
DRUK OP ENTER VOOR BEWERKEN

X	Y1	Y2
0	0	0
1	-0.841	-0.841
2	-0.909	-0.909
3	-0.141	-0.141
4	0.7568	0.7568
5	0.9589	0.9589
6	0.2794	0.2794
7	-0.657	-0.657
8	-0.989	-0.989
9	-0.412	-0.412
10	0.544	0.544

Y1=-sin(X)

d $g(x) = \cos(x) = \sin(x + \frac{1}{2}\pi)$ geeft $g'(x) = \cos(x + \frac{1}{2}\pi) = \sin(x + \frac{1}{2}\pi + \frac{1}{2}\pi) = \sin(x + \pi) = -\sin(x)$
Dus $g(x) = \cos(x)$ geeft $g'(x) = -\sin(x)$.

Bladzijde 166

78 **a** $f(x) = 3 + 4\sin(2x - \frac{1}{3}\pi)$ geeft $f'(x) = 4\cos(2x - \frac{1}{3}\pi) \cdot 2 = 8\cos(2x - \frac{1}{3}\pi)$

b $g(x) = x\cos(x)$ geeft $g'(x) = 1 \cdot \cos(x) + x \cdot -\sin(x) = \cos(x) - x\sin(x)$

c $h(x) = \dfrac{x^2 + \sin(x)}{\cos(x)}$ geeft $h'(x) = \dfrac{\cos(x) \cdot (2x + \cos(x)) - (x^2 + \sin(x)) \cdot -\sin(x)}{\cos^2(x)}$

$$= \frac{2x\cos(x) + \cos^2(x) + x^2\sin(x) + \sin^2(x)}{\cos^2(x)} = \frac{2x\cos(x) + x^2\sin(x) + 1}{\cos^2(x)}$$

d $k(x) = \cos^2(x)$ geeft $k'(x) = 2\cos(x) \cdot -\sin(x) = -2\sin(x)\cos(x)$

e $l(x) = x + 3\sin^2(x)$ geeft $l'(x) = 1 + 6\sin(x)\cos(x)$

f $j(x) = \dfrac{\cos(x)}{\sin(x)}$ geeft $j'(x) = \dfrac{\sin(x) \cdot -\sin(x) - \cos(x) \cdot \cos(x)}{\sin^2(x)} = \dfrac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \dfrac{-1}{\sin^2(x)}$

79 **a** $f(x) = 10 + 16\cos(\frac{1}{2}(x - 1))$ geeft $f'(x) = -16\sin(\frac{1}{2}(x - 1)) \cdot \frac{1}{2} = -8\sin(\frac{1}{2}(x - 1))$

b $g(x) = x^2\sin(3x)$ geeft $g'(x) = 2x \cdot \sin(3x) + x^2 \cdot \cos(3x) \cdot 3 = 2x\sin(3x) + 3x^2\cos(3x)$

c $h(x) = \dfrac{\sin(x)}{x^2 + \cos(x)}$ geeft $h'(x) = \dfrac{(x^2 + \cos(x)) \cdot \cos(x) - \sin(x) \cdot (2x - \sin(x))}{(x^2 + \cos(x))^2}$

$$= \frac{x^2\cos(x) + \cos^2(x) - 2x\sin(x) + \sin^2(x)}{(x^2 + \cos(x))^2} = \frac{x^2\cos(x) - 2x\sin(x) + 1}{(x^2 + \cos(x))^2}$$

d $j(x) = 2\sin^2(x)$ geeft $j'(x) = 4\sin(x)\cos(x)$

80 **a** $f(x) = x\cos(2x)$ geeft $f'(x) = 1 \cdot \cos(2x) + x \cdot -\sin(2x) \cdot 2 = \cos(2x) - 2x\sin(2x)$

b $g(x) = 2x\sin(3x - 1)$ geeft $g'(x) = 2 \cdot \sin(3x - 1) + 2x \cdot \cos(3x - 1) \cdot 3 = 2\sin(3x - 1) + 6x\cos(3x - 1)$

c $h(x) = \dfrac{x\sin(x)}{x + \sin(x)}$ geeft $h'(x) = \dfrac{(x + \sin(x)) \cdot (1 \cdot \sin(x) + x \cdot \cos(x)) - x\sin(x) \cdot (1 + \cos(x))}{(x + \sin(x))^2}$

$$= \frac{x\sin(x) + x^2\cos(x) + \sin^2(x) + x\sin(x)\cos(x) - x\sin(x) - x\sin(x)\cos(x)}{(x + \sin(x))^2}$$

$$= \frac{x^2\cos(x) + \sin^2(x)}{(x + \sin(x))^2}$$

d $j(x) = 1 + 2\cos^2(x)$ geeft $j'(x) = 4\cos(x) \cdot -\sin(x) = -4\sin(x)\cos(x)$

Bladzijde 167

81 **a** $f(x) = 3\tan(2x)$ geeft $f'(x) = 3 \cdot \dfrac{1}{\cos^2(2x)} \cdot 2 = \dfrac{6}{\cos^2(2x)}$

b $g(x) = \tan^2(x)$ geeft $g'(x) = 2\tan(x) \cdot \dfrac{1}{\cos^2(x)} = 2 \cdot \dfrac{\sin(x)}{\cos(x)} \cdot \dfrac{1}{\cos^2(x)} = \dfrac{2\sin(x)}{\cos^3(x)}$

82 **a** $f(x) = \cos^3(x)$ geeft $f'(x) = 3\cos^2(x) \cdot -\sin(x) = -3\sin(x)\cos^2(x)$

b $g(x) = \cos(x^3)$ geeft $g'(x) = -\sin(x^3) \cdot 3x^2 = -3x^2\sin(x^3)$

c $h(x) = \sqrt{\sin(x)}$ geeft $h'(x) = \dfrac{1}{2\sqrt{\sin(x)}} \cdot \cos(x) = \dfrac{\cos(x)}{2\sqrt{\sin(x)}}$

d $j(x) = \cos^2(x) - \sin^2(x)$ geeft $j'(x) = 2\cos(x) \cdot -\sin(x) - 2\sin(x) \cdot \cos(x) = -4\sin(x)\cos(x)$

83 **a** $f(x) = \sin^3(x) + \sin(x)$ geeft $f'(x) = 3\sin^2(x)\cos(x) + \cos(x)$

$= 3(1 - \cos^2(x))\cos(x) + \cos(x)$

$= 3\cos(x) - 3\cos^3(x) + \cos(x)$

$= 4\cos(x) - 3\cos^3(x)$

b $g(x) = \sin^2(x)\cos(x)$ geeft $g'(x) = 2\sin(x)\cos(x) \cdot \cos(x) + \sin^2(x) \cdot -\sin(x)$

$= 2\sin(x)\cos^2(x) - \sin^3(x)$

$= 2\sin(x)(1 - \sin^2(x)) - \sin^3(x)$

$= 2\sin(x) - 2\sin^3(x) - \sin^3(x)$

$= 2\sin(x) - 3\sin^3(x)$

c $h(x) = \dfrac{\tan(x)}{\sin(x)} = \dfrac{\left(\dfrac{\sin(x)}{\cos(x)}\right)}{\sin(x)} = \dfrac{1}{\cos(x)}$ geeft $h'(x) = \dfrac{\cos(x) \cdot 0 - 1 \cdot -\sin(x)}{\cos^2(x)} = \dfrac{\sin(x)}{\cos^2(x)}$

84 **a** $f(x) = \dfrac{3\cos(x)}{2 - \sin(x)}$ geeft $f'(x) = \dfrac{(2 - \sin(x)) \cdot -3\sin(x) - 3\cos(x) \cdot -\cos(x)}{(2 - \sin(x))^2}$

$= \dfrac{-6\sin(x) + 3\sin^2(x) + 3\cos^2(x)}{(2 - \sin(x))^2} = \dfrac{-6\sin(x) + 3}{(2 - \sin(x))^2}$

b $f(x) = 0$ geeft $\dfrac{3\cos(x)}{2 - \sin(x)} = 0$

$\cos(x) = 0$

$x = \frac{1}{2}\pi + k \cdot \pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$

Dus $A(\frac{1}{2}\pi, 0)$.

Stel k: $y = ax + b$ met $a = f'(\frac{1}{2}\pi) = \dfrac{-6 \cdot 1 + 3}{(2 - 1)^2} = \dfrac{-3}{1} = -3$.

$\left.\begin{array}{l} y = -3x + b \\ \text{door } A(\frac{1}{2}\pi, 0) \end{array}\right\}$ $-3 \cdot \frac{1}{2}\pi + b = 0$

$b = 1\frac{1}{2}\pi$

Dus k: $y = -3x + 1\frac{1}{2}\pi$.

c $f'(x) = 0$ geeft $\dfrac{-6\sin(x) + 3}{(2 - \sin(x))^2} = 0$

$-6\sin(x) + 3 = 0$

$\sin(x) = \frac{1}{2}$

$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$

max. is $f(\frac{1}{6}\pi) = \dfrac{3\cos(\frac{1}{6}\pi)}{2 - \sin(\frac{1}{6}\pi)} = \dfrac{3 \cdot \frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = \sqrt{3}$

min. is $f(\frac{5}{6}\pi) = \dfrac{3\cos(\frac{5}{6}\pi)}{2 - \sin(\frac{5}{6}\pi)} = \dfrac{3 \cdot -\frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = -\sqrt{3}$

Dus $B_f = [-\sqrt{3}, \sqrt{3}]$.

85 $f_p(x) = \cos^2(x) + \cos(x) + p$ geeft $f_p'(x) = 2\cos(x) \cdot -\sin(x) - \sin(x) = -2\sin(x)\cos(x) - \sin(x)$

$f_p'(x) = 0$ geeft $-2\sin(x)\cos(x) - \sin(x) = 0$

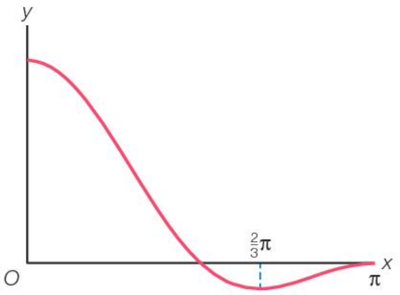
$-\sin(x)(2\cos(x) + 1) = 0$

$\sin(x) = 0 \vee \cos(x) = -\frac{1}{2}$

$x = k \cdot \pi \vee x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$

x in $[0, \pi]$ geeft $x = 0 \vee x = \pi \vee x = \frac{2}{3}\pi$

Hieronder zie je een schets van de grafiek van $y = \cos^2(x) + \cos(x)$.



De grafiek van $y = \cos^2(x) + \cos(x)$ heeft een minimum voor $x = \frac{2}{3}\pi$, dus ook de grafiek van $f_p(x) = \cos^2(x) + \cos(x) + p$ heeft een minimum voor $x = \frac{2}{3}\pi$.

$f_p(\frac{2}{3}\pi) = 1$ geeft $\cos^2(\frac{2}{3}\pi) + \cos(\frac{2}{3}\pi) + p = 1$

$(-\frac{1}{2})^2 - \frac{1}{2} + p = 1$

$\frac{1}{4} - \frac{1}{2} + p = 1$

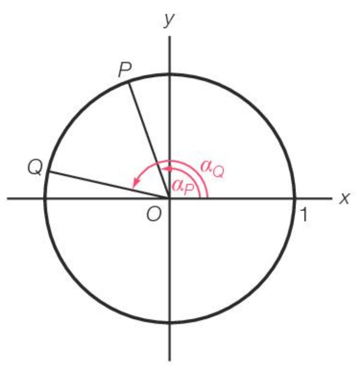
$p = 1\frac{1}{4}$

Diagnostische toets

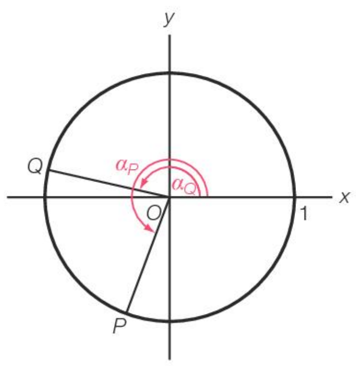
Bladzijde 170

1 $A(\cos(40°), \sin(40°)) \approx A(0{,}77; 0{,}64)$
De draaiingshoek die bij het punt B hoort is $40° + 120° = 160°$.
$B(\cos(160°), \sin(160°)) \approx B(-0{,}94; 0{,}34)$
De draaiingshoek die bij het punt C hoort is $40° + 240° = 280°$.
$C(\cos(280°), \sin(280°)) \approx C(0{,}17; -0{,}98)$

2 Mogelijkheid 1, zie de figuur hiernaast.
$x_P = -0{,}33$, dus $\cos(\alpha_P) = -0{,}33$.
De GR geeft 1,907...
Dus $\alpha_P = 1{,}907...$
$y_Q = 0{,}22$, dus $\sin(\alpha_Q) = 0{,}22$.
De GR geeft 0,221...
Dus $\alpha_Q = \pi - 0{,}221... = 2{,}919...$
Dus $\angle POQ = 2{,}919... - 1{,}907... \approx 1{,}01$.



Mogelijkheid 2, zie de figuur hiernaast.
$\alpha_P = 2\pi - 1{,}907... = 4{,}376...$
$\alpha_Q = 2{,}919...$
Dus $\angle POQ = 4{,}376... - 2{,}919... \approx 1{,}46$.



3 **a** $\sin(x) = -\frac{1}{2}$ met $-\frac{1}{2}\pi \le x \le \frac{1}{2}\pi$ geeft $x = -\frac{1}{6}\pi$

b $x = \cos(\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}$

c $\cos(x) = \frac{1}{2}$ en $\frac{1}{2}\pi \le x \le 2\pi$ geeft $x = 1\frac{2}{3}\pi$

d $x = \sin(1\frac{2}{3}\pi) = -\frac{1}{2}\sqrt{3}$

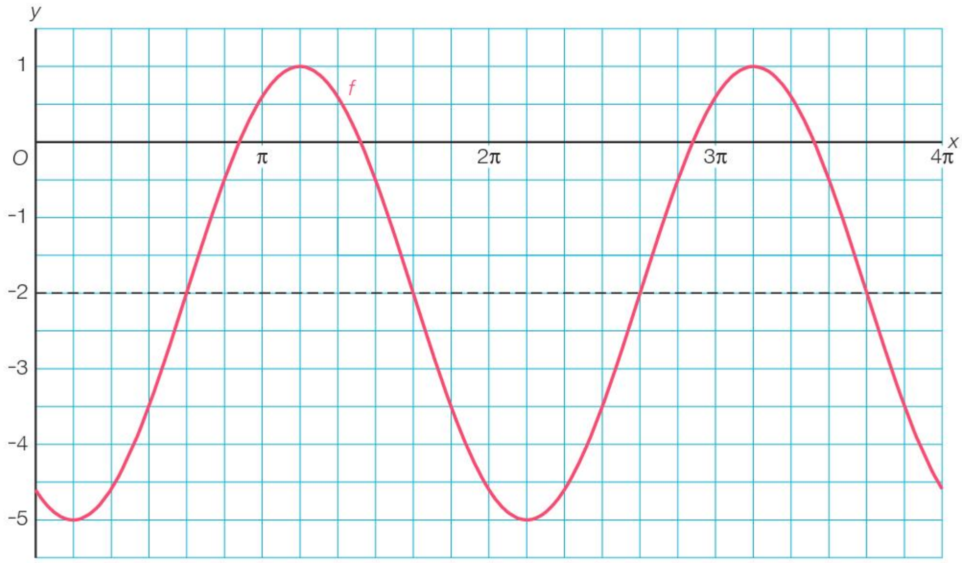
e $\sin(x) = \frac{1}{2}\sqrt{3}$ en $\frac{1}{2}\pi \le x \le 1\frac{1}{2}\pi$ geeft $x = \frac{2}{3}\pi$

f $x = \cos(1\frac{2}{3}\pi) = \frac{1}{2}$

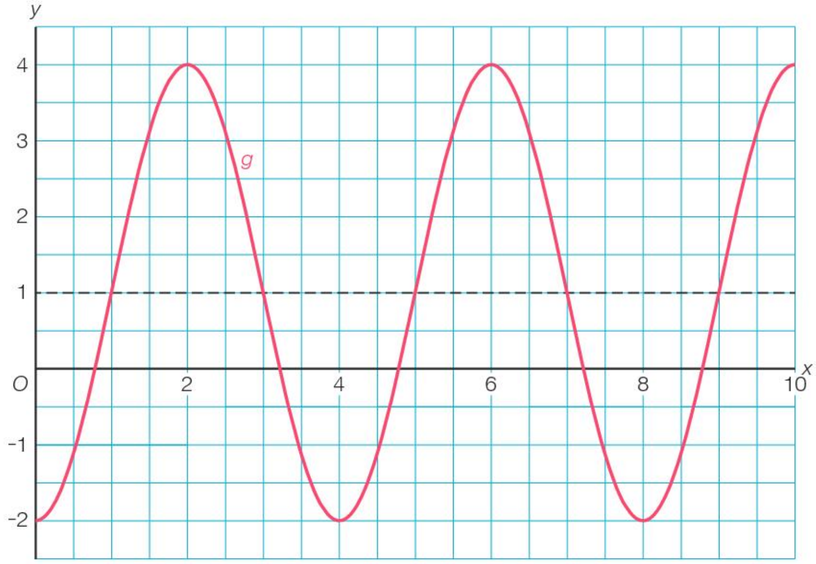
4 **a** De evenwichtsstand is −4, de amplitude is 3,1, de periode is 2π en een beginpunt is $(-\frac{2}{3}\pi, -4)$.

b De evenwichtsstand is 3,4, de amplitude is 4, de periode is $\frac{2}{5}\pi$ en een beginpunt is (0,4; −0,6).

5 **a** $f(x) = -2 - 3\sin(x + \frac{1}{3}\pi)$
evenwichtsstand −2
amplitude 3
periode 2π
$-3 < 0$, dus de grafiek gaat dalend door het punt $(1\frac{2}{3}\pi, -2)$.



b $g(x) = 1 + 3\cos(\frac{1}{2}\pi x - \pi) = 1 + 3\cos(\frac{1}{2}\pi(x - 2))$
evenwichtsstand 1
amplitude 3
periode $\frac{2\pi}{\frac{1}{2}\pi} = 4$
$3 > 0$, dus (2, 4) is een hoogste punt van de grafiek.



6 **a** $a = \dfrac{10 + -30}{2} = -10$

$b = 10 - -10 = 20$

$c = \dfrac{2\pi}{30} = \frac{1}{15}\pi$

$y = a + b\sin(c(x - d))$ met $b > 0$

Stijgend door de evenwichtsstand bij $x = 10$, dus $d = 10$.

$y = -10 + 20\sin(\frac{1}{15}\pi(x - 10))$

b $y = a + b\cos(c(x - d))$ met $b > 0$

Hoogste punt is $(17\frac{1}{2}, 10)$, dus $d = 17\frac{1}{2}$.

$y = -10 + 20\cos(\frac{1}{15}\pi(x - 17\frac{1}{2}))$

c $y = a + b\sin(c(x - d))$ met $b < 0$

Dalend door de evenwichtsstand bij $x = 25$, dus $d = 25$.

$y = -10 - 20\sin(\frac{1}{15}\pi(x - 25))$

d $y = a + b\cos(c(x - d))$ met $b < 0$

Laagste punt is $(2\frac{1}{2}, -30)$, dus $d = 2\frac{1}{2}$.

$y = -10 - 20\cos(\frac{1}{15}\pi(x - 2\frac{1}{2}))$

Bladzijde 171

7 **a** $\sin(2x - \frac{1}{2}\pi) = -1$

$2x - \frac{1}{2}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$

$2x = 2\pi + k \cdot 2\pi$

$x = \pi + k \cdot \pi$

$x = k \cdot \pi$

b $\cos(2x + \frac{1}{3}\pi) = 0$

$2x + \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$

$2x = \frac{1}{6}\pi + k \cdot \pi$

$x = \frac{1}{12}\pi + k \cdot \frac{1}{2}\pi$

c $\cos(1\frac{1}{2}\pi x) \cdot (\cos(2\pi x) + 1) = 0$

$\cos(1\frac{1}{2}\pi x) = 0 \vee \cos(2\pi x) = -1$

$1\frac{1}{2}\pi x = \frac{1}{2}\pi + k \cdot \pi \vee 2\pi x = \pi + k \cdot 2\pi$

$x = \frac{1}{3} + k \cdot \frac{2}{3} \vee x = \frac{1}{2} + k \cdot 1$

d $\sin^3(\frac{1}{3}\pi x) + \sin^2(\frac{1}{3}\pi x) = 0$

$\sin^2(\frac{1}{3}\pi x)(\sin(\frac{1}{3}\pi x) + 1) = 0$

$\sin(\frac{1}{3}\pi x) = 0 \vee \sin(\frac{1}{3}\pi x) = -1$

$\frac{1}{3}\pi x = k \cdot \pi \vee \frac{1}{3}\pi x = 1\frac{1}{2}\pi + k \cdot 2\pi$

$x = k \cdot 3 \vee x = 4\frac{1}{2} + k \cdot 6$

8 **a** $4\sin(2x + \frac{1}{2}\pi) = 2\sqrt{2}$

$\sin(2x + \frac{1}{2}\pi) = \frac{1}{2}\sqrt{2}$

$2x + \frac{1}{2}\pi = \frac{1}{4}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = \frac{3}{4}\pi + k \cdot 2\pi$

$2x = -\frac{1}{4}\pi + k \cdot 2\pi \vee 2x = \frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{1}{8}\pi + k \cdot \pi \vee x = \frac{1}{8}\pi + k \cdot \pi$

x in $[0, 2\pi]$ geeft $x = \frac{7}{8}\pi \vee x = 1\frac{7}{8}\pi \vee x = \frac{1}{8}\pi \vee x = 1\frac{1}{8}\pi$

b $\cos(x - \frac{1}{6}\pi) = -\frac{1}{2}\sqrt{3}$

$x - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$

$x = \pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \pi \vee x = 1\frac{1}{3}\pi$

c $4\cos^2(\frac{1}{2}x) = 2$

$\cos^2(\frac{1}{2}x) = \frac{1}{2}$

$\cos(\frac{1}{2}x) = \frac{1}{2}\sqrt{2} \vee \cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{2}$

$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{3}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{3}{4}\pi + k \cdot 2\pi$

$x = \frac{1}{2}\pi + k \cdot 4\pi \vee x = -\frac{1}{2}\pi + k \cdot 4\pi \vee x = 1\frac{1}{2}\pi + k \cdot 4\pi \vee x = -1\frac{1}{2}\pi + k \cdot 4\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$

d $\sin^2(\frac{3}{4}x + 1\frac{1}{2}\pi) = \frac{3}{4}$

$\sin(\frac{3}{4}x + 1\frac{1}{2}\pi) = \frac{1}{2}\sqrt{3} \vee \sin(\frac{3}{4}x + 1\frac{1}{2}\pi) = -\frac{1}{2}\sqrt{3}$

$\frac{3}{4}x + 1\frac{1}{2}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{3}{4}x + 1\frac{1}{2}\pi = \frac{2}{3}\pi + k \cdot 2\pi \vee \frac{3}{4}x + 1\frac{1}{2}\pi = -\frac{1}{3}\pi + k \cdot 2\pi \vee \frac{3}{4}x + 1\frac{1}{2}\pi = 1\frac{1}{3}\pi + k \cdot 2\pi$

$\frac{3}{4}x = -1\frac{1}{6}\pi + k \cdot 2\pi \vee \frac{3}{4}x = -\frac{5}{6}\pi + k \cdot 2\pi \vee \frac{3}{4}x = -1\frac{5}{6}\pi + k \cdot 2\pi \vee \frac{3}{4}x = -\frac{1}{6}\pi + k \cdot 2\pi$

$x = -1\frac{5}{9}\pi + k \cdot 2\frac{2}{3}\pi \vee x = -1\frac{1}{9}\pi + k \cdot 2\frac{2}{3}\pi \vee x = -2\frac{4}{9}\pi + k \cdot 2\frac{2}{3}\pi \vee x = -\frac{2}{9}\pi + k \cdot 2\frac{2}{3}\pi$

x in $[0, 2\pi]$ geeft $x = 1\frac{1}{9}\pi \vee x = 1\frac{5}{9}\pi \vee x = \frac{2}{9}\pi$

9 **a** $f(x) = -1$ geeft $-1 + 2\cos(x - \frac{1}{3}\pi) = -1$

$2\cos(x - \frac{1}{3}\pi) = 0$

$\cos(x - \frac{1}{3}\pi) = 0$

$x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{5}{6}\pi + k \cdot \pi$

x in $[-\pi, 3\pi]$ geeft $x = -\frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = 1\frac{5}{6}\pi \vee x = 2\frac{5}{6}\pi$

Dus de grafiek van f snijdt de lijn van de evenwichtsstand in de punten $(-\frac{1}{6}\pi, -1)$, $(\frac{5}{6}\pi, -1)$, $(1\frac{5}{6}\pi, -1)$ en $(2\frac{5}{6}\pi, -1)$.

b $f(x) = 1$ geeft $-1 + 2\cos(x - \frac{1}{3}\pi) = 1$

$2\cos(x - \frac{1}{3}\pi) = 2$

$\cos(x - \frac{1}{3}\pi) = 1$

$x - \frac{1}{3}\pi = k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 2\pi$

x in $[-\pi, 3\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 2\frac{1}{3}\pi$

$f(x) = -3$ geeft $-1 + 2\cos(x - \frac{1}{3}\pi) = -3$

$2\cos(x - \frac{1}{3}\pi) = -2$

$\cos(x - \frac{1}{3}\pi) = -1$

$x - \frac{1}{3}\pi = \pi + k \cdot 2\pi$

$x = 1\frac{1}{3}\pi + k \cdot 2\pi$

x in $[-\pi, 3\pi]$ geeft $x = -\frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi$

De toppen van de grafiek van f zijn $(-\frac{2}{3}\pi, -3)$, $(\frac{1}{3}\pi, 1)$, $(1\frac{1}{3}\pi, -3)$ en $(2\frac{1}{3}\pi, 1)$.

c $f(x) = 0$ geeft $-1 + 2\cos(x - \frac{1}{3}\pi) = 0$

$2\cos(x - \frac{1}{3}\pi) = 1$

$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$

$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot 2\pi$

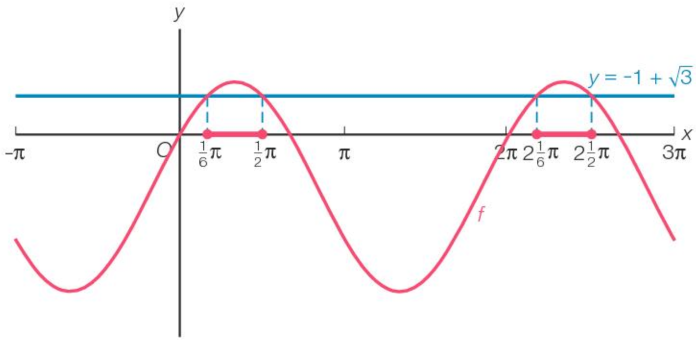
x in $[-\pi, 3\pi]$ geeft $x = \frac{2}{3}\pi \vee x = 2\frac{2}{3}\pi \vee x = 0 \vee x = 2\pi$

Dus de afstand tussen B en C is $2\pi - \frac{2}{3}\pi = 1\frac{1}{3}\pi$.

d $f(x) = -1 + \sqrt{3}$ geeft $-1 + 2\cos(x - \frac{1}{3}\pi) = -1 + \sqrt{3}$

$$2\cos(x - \tfrac{1}{3}\pi) = \sqrt{3}$$
$$\cos(x - \tfrac{1}{3}\pi) = \tfrac{1}{2}\sqrt{3}$$
$$x - \tfrac{1}{3}\pi = \tfrac{1}{6}\pi + k \cdot 2\pi \vee x - \tfrac{1}{3}\pi = -\tfrac{1}{6}\pi + k \cdot 2\pi$$
$$x = \tfrac{1}{2}\pi + k \cdot 2\pi \vee x = \tfrac{1}{6}\pi + k \cdot 2\pi$$

x in $[-\pi, 3\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 2\frac{1}{2}\pi \vee x = \frac{1}{6}\pi \vee x = 2\frac{1}{6}\pi$



$f(x) \geq -1 + \sqrt{3}$ geeft $\frac{1}{6}\pi \leq x \leq \frac{1}{2}\pi \vee 2\frac{1}{6}\pi \leq x \leq 2\frac{1}{2}\pi$

10 **a** $\sin(2x + \frac{1}{2}\pi) = \sin(x - \frac{1}{3}\pi)$

$2x + \frac{1}{2}\pi = x - \frac{1}{3}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = \pi - (x - \frac{1}{3}\pi) + k \cdot 2\pi$

$x = -\frac{5}{6}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = \pi - x + \frac{1}{3}\pi + k \cdot 2\pi$

$x = -\frac{5}{6}\pi + k \cdot 2\pi \vee 3x = \frac{5}{6}\pi + k \cdot 2\pi$

$x = -\frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$

b $\cos(x - \frac{1}{6}\pi) = \cos(4x + \frac{1}{4}\pi)$

$x - \frac{1}{6}\pi = 4x + \frac{1}{4}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -(4x + \frac{1}{4}\pi) + k \cdot 2\pi$

$-3x = \frac{5}{12}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -4x - \frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{5}{36}\pi + k \cdot \frac{2}{3}\pi \vee 5x = -\frac{1}{12}\pi + k \cdot 2\pi$

$x = -\frac{5}{36}\pi + k \cdot \frac{2}{3}\pi \vee x = -\frac{1}{60}\pi + k \cdot \frac{2}{5}\pi$

c $\tan(\frac{2}{3}\pi x) = -\frac{1}{3}\sqrt{3}$

$\frac{2}{3}\pi x = \frac{5}{6}\pi + k \cdot \pi$

$x = 1\frac{1}{4} + k \cdot 1\frac{1}{2}$

d $\tan(2x - \frac{1}{3}\pi) = \tan(x + \frac{1}{2}\pi)$

$2x - \frac{1}{3}\pi = x + \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{5}{6}\pi + k \cdot \pi$

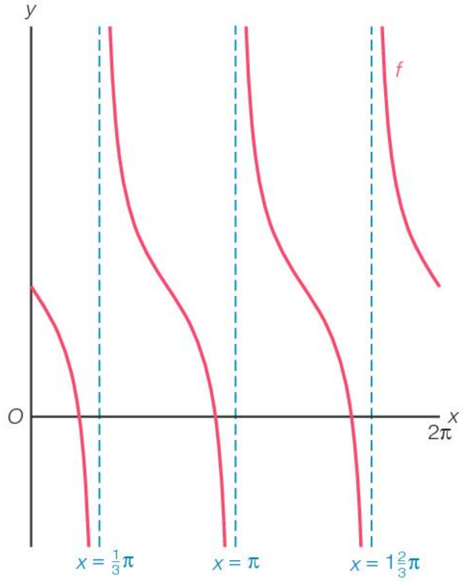
11 **a** $f(x) = 2 - \tan(1\frac{1}{2}x)$

Een beginpunt is $(0, 2)$.

De periode is $\dfrac{\pi}{1\frac{1}{2}} = \frac{2}{3}\pi$.

De asymptoten zijn de lijnen $x = \frac{1}{3}\pi$, $x = \pi$ en $x = 1\frac{2}{3}\pi$.

b Voer in $y_1 = 2 - \tan(1\frac{1}{2}x)$.



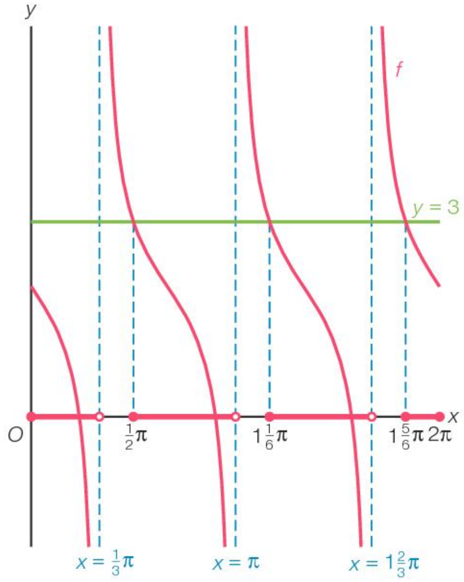
c $f(x) = 3$ geeft $2 - \tan(1\frac{1}{2}x) = 3$

$$\tan(1\tfrac{1}{2}) = -1$$

$$1\tfrac{1}{2}x = \tfrac{3}{4}\pi + k \cdot \pi$$

$$x = \tfrac{1}{2}\pi + k \cdot \tfrac{2}{3}\pi$$

x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$



$f(x) \leq 3$ geeft $0 \leq x < \frac{1}{3}\pi \vee \frac{1}{2}\pi \leq x < \pi \vee 1\frac{1}{6}\pi \leq x < 1\frac{2}{3}\pi \vee 1\frac{5}{6}\pi \leq x \leq 2\pi$

12 **a** $\sin(2x - \frac{2}{3}\pi) = \cos(2x - \frac{2}{3}\pi - \frac{1}{2}\pi) = \cos(2x - 1\frac{1}{6}\pi)$

b $-\cos(5x + 1\frac{1}{4}\pi) = \cos(5x + 1\frac{1}{4}\pi + \pi) = \cos(5x + 2\frac{1}{4}\pi) = \sin(5x + 2\frac{1}{4}\pi + \frac{1}{2}\pi) = \sin(5x + 2\frac{3}{4}\pi) = \sin(5x + \frac{3}{4}\pi)$

c $(\sin(x) - \cos(x)) \cdot (\sin(x) + \cos(x)) \cdot \cos(x) = (\sin^2(x) - \cos^2(x)) \cdot \cos(x) =$

$(1 - \cos^2(x) - \cos^2(x)) \cdot \cos(x) = (1 - 2\cos^2(x)) \cdot \cos(x) = \cos(x) - 2\cos^3(x)$

13 **a** $f(x) = x^3 \sin(4x)$ geeft $f'(x) = 3x^2 \cdot \sin(4x) + x^3 \cdot \cos(4x) \cdot 4 = 3x^2 \sin(4x) + 4x^3 \cos(4x)$

b $g(x) = \dfrac{\sin(2x)}{2 + \cos(x)}$ geeft

$$g'(x) = \frac{(2 + \cos(x)) \cdot \cos(2x) \cdot 2 - \sin(2x) \cdot -\sin(x)}{(2 + \cos(x))^2} = \frac{4\cos(2x) + 2\cos(x)\cos(2x) + \sin(x)\sin(2x)}{(2 + \cos(x))^2}$$

c $h(x) = 5\sin^2(x)$ geeft $h'(x) = 10\sin(x)\cos(x)$

d $j(x) = \tan(x^3)$ geeft $j'(x) = \dfrac{1}{\cos^2(x^3)} \cdot 3x^2 = \dfrac{3x^2}{\cos^2(x^3)}$

14 $f(x) = \dfrac{\sin(2x)}{2 + \sin(x)}$ geeft

$$f'(x) = \frac{(2 + \sin(x)) \cdot \cos(2x) \cdot 2 - \sin(2x) \cdot \cos(x)}{(2 + \sin(x))^2} = \frac{4\cos(2x) + 2\sin(x)\cos(2x) - \sin(2x)\cos(x)}{(2 + \sin(x))^2}$$

Stel k: $y = ax + b$ met $a = f'(\tfrac{1}{2}\pi) = \dfrac{4\cos(\pi) + 2\sin(\tfrac{1}{2}\pi)\cos(\pi) - \sin(\pi)\cos(\tfrac{1}{2}\pi)}{(2 + \sin(\tfrac{1}{2}\pi))^2} = \dfrac{-4 - 2 - 0}{(2 + 1)^2} = -\tfrac{2}{3}$.

$$\left.\begin{array}{l} y = -\tfrac{2}{3}x + b \\ f(\tfrac{1}{2}\pi) = 0, \text{ dus } A(\tfrac{1}{2}\pi, 0) \end{array}\right\} \begin{array}{l} -\tfrac{2}{3} \cdot \tfrac{1}{2}\pi + b = 0 \\ b = \tfrac{1}{3}\pi \end{array}$$

Dus k: $y = -\tfrac{2}{3}x + \tfrac{1}{3}\pi$.

Gemengde opgaven

5 Machten, exponenten en logaritmen

Bladzijde 172

1 **a** $a^{-\frac{1}{3}}b^{\frac{3}{4}} = \dfrac{1}{a^{\frac{1}{3}}} \cdot \sqrt[4]{b^3} = \dfrac{\sqrt[4]{b^3}}{\sqrt[3]{a}}$

$(81p^2)^{-\frac{3}{4}} = 81^{-\frac{3}{4}} \cdot (p^2)^{-\frac{3}{4}} = (3^4)^{-\frac{3}{4}} \cdot p^{-1\frac{1}{2}} = 3^{-3} \cdot \dfrac{1}{p^{1\frac{1}{2}}} = \frac{1}{27} \cdot \dfrac{1}{p\sqrt{p}} = \dfrac{1}{27p\sqrt{p}}$

$\dfrac{10a^{-5}}{(4a)^{-1\frac{1}{2}}} = \dfrac{10a^{-5}}{4^{-1\frac{1}{2}} \cdot a^{-1\frac{1}{2}}} = \dfrac{10a^{-5}}{(2^2)^{-1\frac{1}{2}} \cdot a^{-1\frac{1}{2}}} = \dfrac{10a^{-5}}{2^{-3} \cdot a^{-1\frac{1}{2}}} = \dfrac{10}{\frac{1}{8}} \cdot a^{-3\frac{1}{2}} = 80 \cdot \dfrac{1}{a^{3\frac{1}{2}}} = \dfrac{80}{a^3 \cdot \sqrt{a}}$

b $x^4 \cdot \sqrt[3]{x} \cdot \sqrt[4]{x^3} = x^4 \cdot x^{\frac{1}{3}} \cdot x^{\frac{3}{4}} = x^{4+\frac{1}{3}+\frac{3}{4}} = x^{5\frac{1}{12}}$

$\dfrac{x^2}{x \cdot \sqrt[3]{x^2}} = \dfrac{x^2}{x \cdot x^{\frac{2}{3}}} = \dfrac{x^2}{x^{1\frac{2}{3}}} = x^{2-1\frac{2}{3}} = x^{\frac{1}{3}}$

$\dfrac{x^2 \cdot \sqrt{x}}{\sqrt[4]{x^3}} = \dfrac{x^2 \cdot x^{\frac{1}{2}}}{x^{\frac{3}{4}}} = \dfrac{x^{2\frac{1}{2}}}{x^{\frac{3}{4}}} = x^{2\frac{1}{2}-\frac{3}{4}} = x^{1\frac{3}{4}}$

2 **a** $y = \dfrac{6x^4}{(2\sqrt{x})^3} = \dfrac{6x^4}{2^3 \cdot (\sqrt{x})^3} = \dfrac{6x^4}{8 \cdot (x^{\frac{1}{2}})^3} = \dfrac{6x^4}{8 \cdot x^{1\frac{1}{2}}} = \frac{3}{4}x^{4-1\frac{1}{2}} = \frac{3}{4}x^{2\frac{1}{2}}$

Dus $y = \frac{3}{4}x^{2\frac{1}{2}}$.

b $y = (\frac{1}{4})^{2-\frac{1}{2}x} = (\frac{1}{4})^2 \cdot (\frac{1}{4})^{-\frac{1}{2}x} = \frac{1}{16} \cdot (2^{-2})^{-\frac{1}{2}x} = \frac{1}{16} \cdot 2^x$

Dus $y = \frac{1}{16} \cdot 2^x$.

c $y = 4x^{-1\frac{2}{3}} \cdot (3x^4)^2 = 4x^{-1\frac{2}{3}} \cdot 3^2 \cdot (x^4)^2 = 36x^{-1\frac{2}{3}} \cdot x^8 = 36x^{6\frac{1}{3}}$

$36x^{6\frac{1}{3}} = y$

$x^{\frac{19}{3}} = \frac{1}{36}y$

$x = (\frac{1}{36}y)^{\frac{3}{19}}$

$x = (\frac{1}{36})^{\frac{3}{19}} \cdot y^{\frac{3}{19}}$

$x \approx 0{,}57 \cdot y^{0{,}16}$

Dus $x = 0{,}57y^{0{,}16}$.

d $A = 6 + 2\sqrt{3B+2}$

$6 + 2\sqrt{3B+2} = A$

$2\sqrt{3B+2} = A - 6$

kwadrateren geeft

$4(3B+2) = A^2 - 12A + 36$

$3B + 2 = \frac{1}{4}A^2 - 3A + 9$

$3B = \frac{1}{4}A^2 - 3A + 7$

$B = \frac{1}{12}A^2 - A + 2\frac{1}{3}$

3 **a** $(x^{\frac{1}{2}} - 4)(2x^{\frac{2}{3}} - 16) = 0$

$x^{\frac{1}{2}} - 4 = 0 \vee 2x^{\frac{2}{3}} - 16 = 0$

$\sqrt{x} = 4 \vee 2x^{\frac{2}{3}} = 16$

$x = 16 \vee x^{\frac{2}{3}} = 8$

$x = 16 \vee x = 8^{\frac{3}{2}}$

$x = 16 \vee x = (2^3)^{\frac{3}{2}}$

$x = 16 \vee x = 2^{4\frac{1}{2}}$

$x = 16 \vee x = 16\sqrt{2}$

b $\frac{1}{2}(2x^2 - 5)^{1\frac{1}{2}} = 4$

$(2x^2 - 5)^{1\frac{1}{2}} = 8$

$2x^2 - 5 = 8^{\frac{2}{3}}$

$2x^2 - 5 = (2^3)^{\frac{2}{3}}$

$2x^2 - 5 = 4$

$2x^2 = 9$

$x^2 = 4\frac{1}{2}$

$x = \sqrt{4\frac{1}{2}} \vee x = -\sqrt{4\frac{1}{2}}$

$x = 1\frac{1}{2}\sqrt{2} \vee x = -1\frac{1}{2}\sqrt{2}$

c $2^{x+4} - 2^{x+3} = 16\sqrt{2}$

$2^x \cdot 2^4 - 2^x \cdot 2^3 = 16\sqrt{2}$

$16 \cdot 2^x - 8 \cdot 2^x = 16\sqrt{2}$

$8 \cdot 2^x = 16\sqrt{2}$

$2^x = 2\sqrt{2}$

$2^x = 2^{1\frac{1}{2}}$

$x = 1\frac{1}{2}$

d $5 \cdot 2^{5x-1} + 6 = 56$

$5 \cdot 2^{5x-1} = 50$

$2^{5x-1} = 10$

$5x - 1 = {}^2\log(10)$

$5x = 1 + {}^2\log(10)$

$x = \frac{1}{5} + \frac{1}{5} \cdot {}^2\log(10)$

e ${}^3\log(3^{x-1} + 5) = 2$

$3^{x-1} + 5 = 3^2$

$3^{x-1} + 5 = 9$

$3^{x-1} = 4$

$x - 1 = {}^3\log(4)$

$x = 1 + {}^3\log(4)$

f $2^{2x+3} + 2^{x+5} = 168$

$2^{2x} \cdot 2^3 + 2^x \cdot 2^5 = 168$

$8 \cdot 2^{2x} + 32 \cdot 2^x - 168 = 0$

$(2^x)^2 + 4 \cdot 2^x - 21 = 0$

Stel $2^x = u$.

$u^2 + 4u - 21 = 0$

$(u - 3)(u + 7) = 0$

$u = 3 \vee u = -7$

$2^x = 3 \vee 2^x = -7$

$x = {}^2\log(3)$

4 **a** $y = \sqrt{x}$

$\downarrow$ verm. y-as, $-\frac{1}{3}$

$y = \sqrt{-3x}$

$\downarrow$ translatie $(2, -2)$

$f(x) = -2 + \sqrt{-3(x-2)} = -2 + \sqrt{6 - 3x}$

b $6 - 3x \geq 0$

$-3x \geq -6$

$x \leq 2$

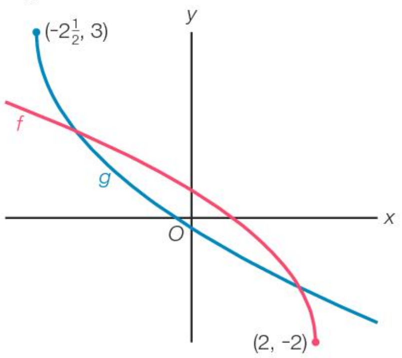
$D_f = \langle \leftarrow, 2]$ en het randpunt van de grafiek van f is $(2, -2)$.

$4x + 10 \geq 0$

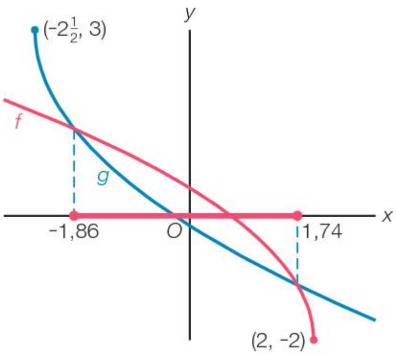
$4x \geq -10$

$x \geq -2\frac{1}{2}$

$D_g = [-2\frac{1}{2}, \rightarrow\rangle$ en het randpunt van de grafiek van g is $(-2\frac{1}{2}, 3)$.

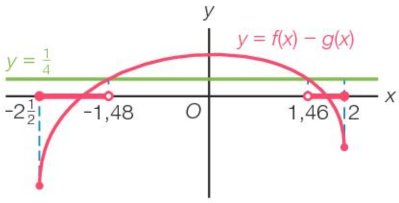


c Voer in $y_1 = -2 + \sqrt{6 - 3x}$ en $y_2 = 3 - \sqrt{4x + 10}$.
De optie snijpunt geeft $x \approx -1{,}86$ en $x \approx 1{,}74$.



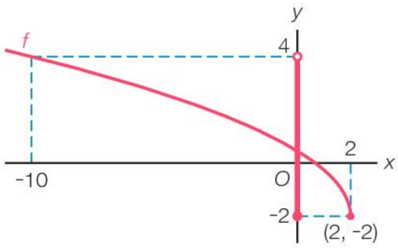
$f(x) \geq g(x)$ geeft $-1{,}86 \leq x \leq 1{,}74$

d Zet y_1 en y_2 uit en voer in $y_3 = y_1 - y_2$ en $y_4 = \frac{1}{4}$.
De optie snijpunt geeft $x \approx -1{,}48$ en $x \approx 1{,}46$.



$f(x) - g(x) < \frac{1}{4}$ geeft $-2\frac{1}{2} \leq x < -1{,}48 \vee 1{,}46 < x \leq 2$

e $f(-10) = -2 + \sqrt{36} = 4$



Voor $x > -10$ is $-2 \leq f(x) < 4$.

5 **a** $(2x-5)^{-\frac{3}{4}} = \frac{1}{27}$

$2x - 5 = (\frac{1}{27})^{-\frac{4}{3}}$

$2x - 5 = (3^{-3})^{-\frac{4}{3}}$

$2x - 5 = 3^4$

$2x - 5 = 81$

$2x = 86$

$x = 43$

b $(\frac{1}{2}x^2 + 9) \cdot \sqrt[3]{\frac{1}{2}x^2 + 9} = 81$

$(\frac{1}{2}x^2 + 9) \cdot (\frac{1}{2}x^2 + 9)^{\frac{1}{3}} = 81$

$(\frac{1}{2}x^2 + 9)^{\frac{4}{3}} = 81$

$\frac{1}{2}x^2 + 9 = 81^{\frac{3}{4}}$

$\frac{1}{2}x^2 + 9 = (3^4)^{\frac{3}{4}}$

$\frac{1}{2}x^2 + 9 = 3^3$

$\frac{1}{2}x^2 = 18$

$x^2 = 36$

$x = 6 \vee x = -6$

c $(\sqrt{3})^{x+2} = 9\sqrt{3}$

$(3^{\frac{1}{2}})^{x+2} = 3^2 \cdot 3^{\frac{1}{2}}$

$3^{\frac{1}{2}x+1} = 3^{2\frac{1}{2}}$

$\frac{1}{2}x + 1 = 2\frac{1}{2}$

$\frac{1}{2}x = 1\frac{1}{2}$

$x = 3$

d $4^x + 48 = 2^{x+4}$

$(2^2)^x + 48 = 2^x \cdot 2^4$

$(2^x)^2 + 48 = 16 \cdot 2^x$

$(2^x)^2 - 16 \cdot 2^x + 48 = 0$

Stel $2^x = u$.

$u^2 - 16u + 48 = 0$

$(u-4)(u-12) = 0$

$u = 4 \vee u = 12$

$2^x = 4 \vee 2^x = 12$

$x = 2 \vee x = {}^2\log(12)$

e $^5\log(x^2 \cdot \sqrt{x} - 7) = 2$

$x^2 \cdot \sqrt{x} - 7 = 5^2$

$x^2 \cdot \sqrt{x} = 32$

$x^{2\frac{1}{2}} = 32$

$x = 32^{\frac{2}{5}}$

$x = (2^5)^{\frac{2}{5}}$

$x = 2^2$

$x = 4$

f $5^x - 6 \cdot (\frac{1}{5})^x = 5$

$5^x - 6 \cdot \dfrac{1}{5^x} = 5$

$(5^x)^2 - 6 = 5 \cdot 5^x$

Stel $5^x = u$.

$u^2 - 6 = 5u$

$u^2 - 5u - 6 = 0$

$(u + 1)(u - 6) = 0$

$u = -1 \vee u = 6$

$5^x = -1 \vee 5^x = 6$

$x = {^5\log(6)}$

Bladzijde 173

6 **a** $y = 2^x$

$\downarrow$ verm. x-as, 0,4

$y = 0{,}4 \cdot 2^x$

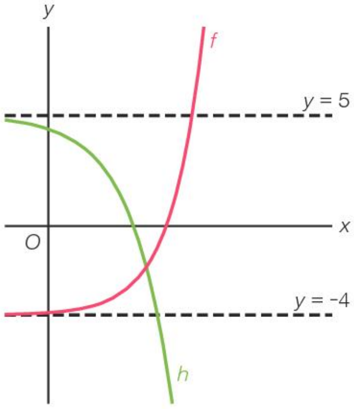
$\downarrow$ translatie (2, −4)

$f(x) = 0{,}4 \cdot 2^{x-2} - 4$

b $k(x) = 5 - 0{,}2 \cdot 3^{\frac{1}{2}(x-6)+1} - 4 = 1 - 0{,}2 \cdot 3^{\frac{1}{2}x - 2} = 1 - 0{,}2 \cdot 3^{\frac{1}{2}x} \cdot 3^{-2} = 1 - 0{,}2 \cdot (3^{\frac{1}{2}})^x \cdot \frac{1}{9} = 1 - \frac{1}{45} \cdot (\sqrt{3})^x$

Dus $a = 1$, $b = -\frac{1}{45}$ en $g = \sqrt{3}$.

c



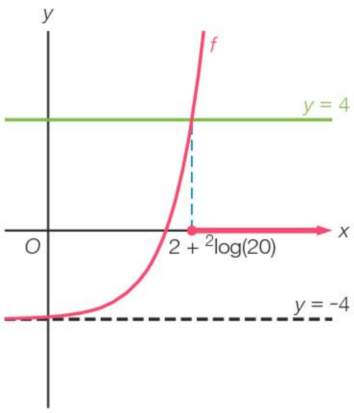
d $f(x) = 4$ geeft $0{,}4 \cdot 2^{x-2} - 4 = 4$

$0{,}4 \cdot 2^{x-2} = 8$

$2^{x-2} = 20$

$x - 2 = {^2\log(20)}$

$x = 2 + {^2\log(20)}$



$f(x) \geq 4$ geeft $x \geq 2 + {^2\log(20)}$

e De lengte van AB is 3 geeft $f(p) - h(p) = 3 \vee f(p) - h(p) = -3$.

$f(p) - h(p) = 0{,}4 \cdot 2^{p-2} - 4 - (5 - 0{,}2 \cdot 3^{\frac{1}{2}p+1}) = 0{,}4 \cdot 2^{p-2} + 0{,}2 \cdot 3^{\frac{1}{2}p+1} - 9$

Voer in $y_1 = 0{,}4 \cdot 2^{x-2} + 0{,}2 \cdot 3^{\frac{1}{2}x+1} - 9$, $y_2 = 3$ en $y_3 = -3$.

De optie snijpunt met y_1 en y_2 geeft $x \approx 4{,}92$.

De optie snijpunt met y_1 en y_3 geeft $x \approx 3{,}74$.

Dus voor $p \approx 4{,}92$ en voor $p \approx 3{,}74$.

f $B_f = \langle -4, \rightarrow\rangle$, dus de vergelijking $f(x) = q$ heeft één oplossing voor $q > -4$.
$B_h = \langle \leftarrow, 5\rangle$, dus de vergelijking $h(x) = q$ heeft geen oplossing voor $q \geq 5$.
Dus $f(x) = q$ heeft één en $h(x) = q$ heeft geen oplossing voor $q \geq 5$.

7 De beeldgrafiek van de grafiek van f is $y = \sqrt{\frac{2}{a}x - 4} + b$.

De beeldgrafiek van de grafiek van g is $y = \sqrt{\frac{1}{a}x + 1} + b$.

De beeldgrafieken gaan door $A(4, 5)$ geeft $\sqrt{\frac{8}{a} - 4} + b = 5 \wedge \sqrt{\frac{4}{a} + 1} + b = 5$.

Hieruit volgt $\sqrt{\frac{8}{a} - 4} = 5 - b \wedge \sqrt{\frac{4}{a} + 1} = 5 - b$.

$\sqrt{\frac{8}{a} - 4} = \sqrt{\frac{4}{a} + 1}$
kwadrateren geeft
$\frac{8}{a} - 4 = \frac{4}{a} + 1$
$\frac{4}{a} = 5$
$a = \frac{4}{5}$

$a = \frac{4}{5}$ en $\sqrt{\frac{4}{a} + 1} + b = 5$ geeft $\sqrt{5 + 1} + b = 5$, dus $b = 5 - \sqrt{6}$.

8 **a** $T = a \cdot R^{1,5}$ geeft $a = \frac{T}{R^{1,5}}$

$R = 5{,}28$ en $T = 4{,}5$ geeft $a = \frac{4{,}5}{5{,}28^{1,5}} \approx 0{,}37$

$R = 3{,}78$ en $T = 2{,}7$ geeft $a = \frac{2{,}7}{3{,}78^{1,5}} \approx 0{,}37$

$R = 2{,}95$ en $T = 1{,}9$ geeft $a = \frac{1{,}9}{2{,}95^{1,5}} \approx 0{,}37$

Dus $a \approx 0{,}37$.

b Straal is $3{,}56 \cdot 10^6$ km $= 35{,}6 \cdot 10^5$ km, dus $R = 35{,}6$.
$R = 35{,}6$ geeft $T = 0{,}37 \cdot 35{,}6^{1,5} \approx 78{,}6$
Dus de omlooptijd is ongeveer 78,6 dagen.

c $R_{\text{Rhea}} = 5{,}28$ geeft $R_{\text{Titan}} = 2{,}3 \cdot 5{,}28 = 12{,}144$
$T_{\text{Titan}} = 0{,}37 \cdot 12{,}144^{1,5} = 15{,}65...$
De omlooptijd is $\frac{15{,}65...}{4{,}5} \approx 3{,}5$ keer zo groot.

d $T = 0{,}37R^{1,5}$
$0{,}37R^{1,5} = T$
$R^{1,5} = \frac{1}{0{,}37} \cdot T$
$R = (\frac{1}{0{,}37} \cdot T)^{\frac{2}{3}}$
$R = (\frac{1}{0{,}37})^{\frac{2}{3}} \cdot T^{\frac{2}{3}}$
$R \approx 1{,}94 \cdot T^{0,67}$
Dus $R = 1{,}94 \cdot T^{0,67}$.

e Omlooptijd 15 uur geeft $T = \frac{15}{24}$.
$R = 1{,}94 \cdot (\frac{15}{24})^{0,67} \approx 1{,}42$
De straal is ongeveer $1{,}42 \cdot 10^5$ km.

Bladzijde 174

9 **a** $2x + 4 > 0$

$2x > -4$

$x > -2$

$D_f = \langle -2, \rightarrow \rangle$ en de verticale asymptoot van de grafiek van f is de lijn $x = -2$.

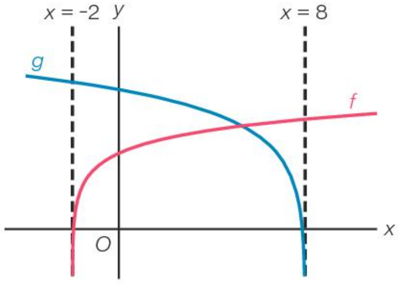
$8 - x > 0$

$-x > -8$

$x < 8$

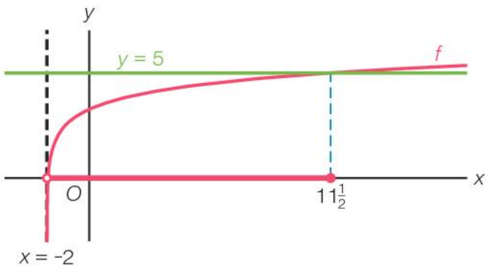
$D_g = \langle \leftarrow, 8 \rangle$ en de verticale asymptoot van de grafiek van g is de lijn $x = 8$.

b



c $f(x) = 5$ geeft $2 + {}^3\log(2x + 4) = 5$

$${}^3\log(2x + 4) = 3$$
$$2x + 4 = 3^3$$
$$2x = 23$$
$$x = 11\tfrac{1}{2}$$



$f(x) \le 5$ geeft $-2 < x \le 11\frac{1}{2}$

d $f(x) = 2$ geeft $2 + {}^3\log(2x + 4) = 2$

$${}^3\log(2x + 4) = 0$$
$$2x + 4 = 1$$
$$2x = -3$$
$$x = -1\tfrac{1}{2}$$

Dus $x_A = -1\frac{1}{2}$.

$g(x) = 2$ geeft $3 - {}^{\frac{1}{2}}\log(8 - x) = 2$

$${}^{\frac{1}{2}}\log(8 - x) = 1$$
$$8 - x = \tfrac{1}{2}$$
$$x = 7\tfrac{1}{2}$$

Dus $x_B = 7\frac{1}{2}$.

$AB = x_B - x_A = 7\frac{1}{2} - -1\frac{1}{2} = 9$

10 **a** $L = 34$ geeft $7{,}5G^{0{,}25} = 34$

$$G^{0{,}25} = \frac{34}{7{,}5}$$
$$G = \left(\frac{34}{7{,}5}\right)^4 \approx 422$$

Het gemiddelde gewicht is 422 kg.

b $H = 900$ geeft $280G^{-0{,}25} = 900$

$$G^{-0{,}25} = \frac{900}{280}$$
$$G = \left(\frac{900}{280}\right)^{-4} = 0{,}009...$$

$G = 0{,}009...$ geeft $L = 7{,}5 \cdot 0{,}009...^{0{,}25} = 2{,}33...$

Het aantal hartslagen is $900 \cdot 60 \cdot 24 \cdot 365 \cdot 2{,}33... \approx 1{,}1 \cdot 10^9$.

Dus ongeveer 1,1 miljard hartslagen.

c L is de levensverwachting in jaren, dus $365 \cdot 24 \cdot 60 \cdot L = 525\,600L$ is de levensverwachting in minuten.
H is het aantal hartslagen per minuut, dus $525\,600L \cdot H$ is het aantal hartslagen in het gehele leven.
Als $L \cdot H$ voor elk zoogdier gelijk is, dan moet $L \cdot H$ voor elk zoogdier hetzelfde getal zijn.
$L \cdot H = 7{,}5G^{0{,}25} \cdot 280G^{-0{,}25} = 2100G^0 = 2100$
Dus $L \cdot H = 2100$.

6 Differentiaalrekening

11 **a** $f(x) = \frac{x^5 - 5x^2}{x^6} = x^{-1} - 5x^{-4}$ geeft $f'(x) = -x^{-2} + 20x^{-5} = -\frac{1}{x^2} + \frac{20}{x^5} = \frac{-x^3 + 20}{x^5}$

b $g(x) = 4x^3 \cdot \sqrt{x} - \frac{4}{x^3 \cdot \sqrt{x}} = 4x^{3\frac{1}{2}} - 4x^{-3\frac{1}{2}}$ geeft $g'(x) = 14x^{2\frac{1}{2}} + 14x^{-4\frac{1}{2}} = 14x^2 \cdot \sqrt{x} + \frac{14}{x^4 \cdot \sqrt{x}}$

c $h(x) = (x\sqrt{x} + 2)^4$ geeft $h'(x) = 4(x\sqrt{x} + 2)^3 \cdot 1\frac{1}{2}\sqrt{x} = 6\sqrt{x} \cdot (x\sqrt{x} + 2)^3$

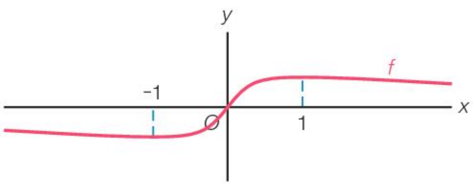
12 **a** $f_p(x) = x^4 + px^3 + \frac{1}{2}x^2 + 4x + 1$
$f_p'(x) = 4x^3 + 3px^2 + x + 4$
$f_p''(x) = 12x^2 + 6px + 1$
$f_p''(x) = 0$ geeft $12x^2 + 6px + 1 = 0$
$D = (6p)^2 - 4 \cdot 12 \cdot 1 = 36p^2 - 48$
$D > 0$ geeft $36p^2 > 48$
$p^2 > \frac{4}{3}$
$p < -\sqrt{\frac{4}{3}} = -\frac{2}{3}\sqrt{3} \vee p > \frac{2}{3}\sqrt{3}$
Dus de grafiek van f_p heeft twee buigpunten voor $p < -\frac{2}{3}\sqrt{3} \vee p > \frac{2}{3}\sqrt{3}$.

b $f_p'(x) = 4$ geeft $4x^3 + 3px^2 + x + 4 = 4$
$4x^3 + 3px^2 + x = 0$
$x(4x^2 + 3px + 1) = 0$
$x = 0 \vee 4x^2 + 3px + 1 = 0$
Van $4x^2 + 3px + 1 = 0$ is $D = (3p)^2 - 4 \cdot 4 \cdot 1 = 9p^2 - 16$
$D > 0$ geeft $9p^2 > 16$
$p^2 > \frac{16}{9}$
$p < -\frac{4}{3} \vee p > \frac{4}{3}$
Dus $4x^2 + 3px + 1 = 0$ heeft twee (van 0 verschillende) oplossingen voor $p < -\frac{4}{3} \vee p > \frac{4}{3}$.
Dus de grafiek van f_p heeft voor $p < -1\frac{1}{3} \vee p > 1\frac{1}{3}$ drie raaklijnen die evenwijdig zijn met k.

Bladzijde 175

13 $f(x) = \frac{x}{x^2 + 4} + \frac{x}{4x^2 + 1}$ geeft $f'(x) = \frac{(x^2 + 4) \cdot 1 - x \cdot 2x}{(x^2 + 4)^2} + \frac{(4x^2 + 1) \cdot 1 - x \cdot 8x}{(4x^2 + 1)^2}$
$= \frac{x^2 + 4 - 2x^2}{(x^2 + 4)^2} + \frac{4x^2 + 1 - 8x^2}{(4x^2 + 1)^2} = \frac{-x^2 + 4}{(x^2 + 4)^2} + \frac{-4x^2 + 1}{(4x^2 + 1)^2}$

$f'(1) = f'(-1) = \frac{-1 + 4}{(1 + 4)^2} + \frac{-4 + 1}{(4 + 1)^2} = \frac{3}{25} - \frac{3}{25} = 0$



$f'(1) = 0$ en $f'(-1) = 0$ en in de schets is te zien dat de grafiek toppen heeft voor $x = 1$ en $x = -1$.
Dus f heeft extremen voor $x = 1$ en $x = -1$.

14 **a** $f(x) = \frac{3x^2 - 9}{x^3} = 3x^{-1} - 9x^{-3}$ geeft $f'(x) = -3x^{-2} + 27x^{-4} = -\frac{3}{x^2} + \frac{27}{x^4} = \frac{-3x^2 + 27}{x^4}$

$f'(x) = 0$ geeft $-3x^2 + 27 = 0$

$3x^2 = 27$

$x^2 = 9$

$x = 3 \vee x = -3$

Zie figuur G.2 in het leerboek.

min. is $f(-3) = -\frac{2}{3}$ en max. is $f(3) = \frac{2}{3}$.

b $f'(x) = 24$ geeft $\frac{-3x^2 + 27}{x^4} = 24$

$-3x^2 + 27 = 24x^4$

$24x^4 + 3x^2 - 27 = 0$

$8x^4 + x^2 - 9 = 0$

Stel $x^2 = u$.

$8u^2 + u - 9 = 0$

$D = 1^2 - 4 \cdot 8 \cdot -9 = 289$

$u = \frac{-1 + 17}{16} = 1 \vee u = \frac{-1 - 17}{16} = -1\frac{1}{8}$

$x^2 = 1 \vee x^2 = -1\frac{1}{8}$

$x = 1 \vee x = -1$

De raakpunten zijn (1, −6) en (−1, 6).

c Voor raken geldt $f(x) = ax^2 \wedge f'(x) = 2ax$

$\frac{3x^2 - 9}{x^3} = ax^2 \wedge \frac{-3x^2 + 27}{x^4} = 2ax$

$a = \frac{3x^2 - 9}{x^5} \wedge a = \frac{-3x^2 + 27}{2x^5}$

Hieruit volgt $\frac{3x^2 - 9}{x^5} = \frac{-3x^2 + 27}{2x^5}$

$\frac{6x^2 - 18}{2x^5} = \frac{-3x^2 + 27}{2x^5}$

$6x^2 - 18 = -3x^2 + 27$

$9x^2 = 45$

$x^2 = 5$

$x = \sqrt{5} \vee x = -\sqrt{5}$

$x = \sqrt{5}$ geeft $a = \frac{3 \cdot 5 - 9}{25\sqrt{5}} = \frac{6}{125}\sqrt{5}$

$x = -\sqrt{5}$ geeft $a = \frac{3 \cdot 5 - 9}{-25\sqrt{5}} = -\frac{6}{125}\sqrt{5}$

Dus voor $a = \frac{6}{125}\sqrt{5} \vee a = -\frac{6}{125}\sqrt{5}$.

15 **a** $f(x) = x^2 \cdot \sqrt{x^2 + 4x}$ geeft $f'(x) = 2x \cdot \sqrt{x^2 + 4x} + x^2 \cdot \frac{2x + 4}{2\sqrt{x^2 + 4x}} = \frac{2x(x^2 + 4x) + x^2(x + 2)}{\sqrt{x^2 + 4x}}$

$= \frac{2x^3 + 8x^2 + x^3 + 2x^2}{\sqrt{x^2 + 4x}} = \frac{3x^3 + 10x^2}{\sqrt{x^2 + 4x}}$

b $g(x) = \frac{x^2}{\sqrt{x^2 + 4x}}$ geeft $g'(x) = \frac{\sqrt{x^2 + 4x} \cdot 2x - x^2 \cdot \frac{2x + 4}{2\sqrt{x^2 + 4x}}}{x^2 + 4x} = \frac{2x(x^2 + 4x) - x^2(x + 2)}{(x^2 + 4x)\sqrt{x^2 + 4x}}$

$= \frac{2x^3 + 8x^2 - x^3 - 2x^2}{(x^2 + 4x)\sqrt{x^2 + 4x}} = \frac{x^3 + 6x^2}{(x^2 + 4x)\sqrt{x^2 + 4x}} = \frac{x^2 + 6x}{(x + 4)\sqrt{x^2 + 4x}}$

c $h(x) = \frac{5x}{(x^2 - 2x)^3}$ geeft $h'(x) = \frac{(x^2 - 2x)^3 \cdot 5 - 5x \cdot 3(x^2 - 2x)^2 \cdot (2x - 2)}{(x^2 - 2x)^6} = \frac{(x^2 - 2x) \cdot 5 - 5x \cdot 3 \cdot (2x - 2)}{(x^2 - 2x)^4}$

$= \frac{5x^2 - 10x - 30x^2 + 30x}{(x^2 - 2x)^4} = \frac{-25x^2 + 20x}{(x^2 - 2x)^4} = \frac{5x(4 - 5x)}{(x^2 - 2x)^4}$

16 **a** $f_p(x) = x^3 + px^2 - 3x + 8$ geeft $f_p'(x) = 3x^2 + 2px - 3$

$f_p'(-\frac{1}{3}) = 0$ geeft $\frac{1}{3} - \frac{2}{3}p - 3 = 0$

$-\frac{2}{3}p = \frac{8}{3}$

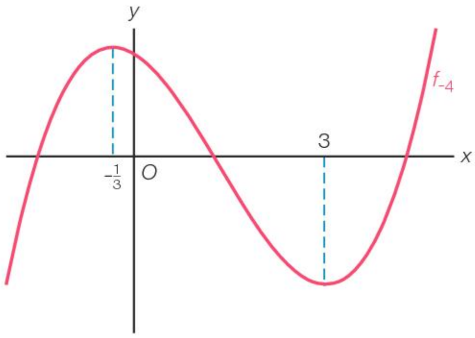
$p = -4$

$f_{-4}(x) = x^3 - 4x^2 - 3x + 8$ geeft $f_{-4}'(x) = 3x^2 - 8x - 3$

$f_{-4}'(x) = 0$ geeft $3x^2 - 8x - 3 = 0$

$D = (-8)^2 - 4 \cdot 3 \cdot -3 = 100$

$x = \dfrac{8+10}{6} = 3 \vee x = \dfrac{8-10}{6} = -\frac{1}{3}$



min. is $f_{-4}(3) = 27 - 36 - 9 + 8 = -10$.

b $f_p'(x) = 3x^2 + 2px - 3$ geeft $f_p''(x) = 6x + 2p$

$f_p''(1) = 0$ geeft $6 + 2p = 0$

$2p = -6$

$p = -3$

$y_A = f_{-3}(1) = 1 - 3 - 3 + 8 = 3$

c Er moet gelden $f_p(x) = -3x + 12 \wedge f_p'(x) = -3$.

$f_p'(x) = -3$ geeft $3x^2 + 2px - 3 = -3$

$3x^2 + 2px = 0$

$x(3x + 2p) = 0$

$x = 0 \vee 3x + 2p = 0$

$x = 0 \vee x = -\frac{2}{3}p$

$x = 0$ geeft het punt (0, 8) op de grafiek van f_p en dit punt ligt niet op de lijn $y = -3x + 12$, dus $x = 0$ voldoet niet.

$x = -\frac{2}{3}p$ substitueren in $f_p(x) = -3x + 12$ geeft $f_p(-\frac{2}{3}p) = -3 \cdot -\frac{2}{3}p + 12$

$(-\frac{2}{3}p)^3 + p \cdot (-\frac{2}{3}p)^2 - 3 \cdot -\frac{2}{3}p + 8 = -3 \cdot -\frac{2}{3}p + 12$

$-\frac{8}{27}p^3 + \frac{4}{9}p^3 = 4$

$\frac{4}{27}p^3 = 4$

$p^3 = 27$

$p = 3$

d $f_p'(x) = 0$ geeft $3x^2 + 2px - 3 = 0$

$2px = -3x^2 + 3$

$p = \dfrac{-3x^2 + 3}{2x}$

Substitutie van $p = \dfrac{-3x^2 + 3}{2x}$ in $y = f_p(x)$ geeft $y = x^3 + \dfrac{-3x^2 + 3}{2x} \cdot x^2 - 3x + 8$

$y = x^3 - 1\frac{1}{2}x^3 + 1\frac{1}{2}x - 3x + 8$

$y = -\frac{1}{2}x^3 - 1\frac{1}{2}x + 8$

Dus alle toppen liggen op de kromme $y = -\frac{1}{2}x^3 - 1\frac{1}{2}x + 8$.

Bladzijde 176

17 **a** $f(x) = \dfrac{x^3 - x^2 - 2x}{\sqrt{x}} = x^{2\frac{1}{2}} - x^{1\frac{1}{2}} - 2\sqrt{x}$ geeft

$f'(x) = 2\frac{1}{2}x^{1\frac{1}{2}} - 1\frac{1}{2}x^{\frac{1}{2}} - \dfrac{2}{2\sqrt{x}} = \dfrac{5x\sqrt{x}}{2} - \dfrac{3\sqrt{x}}{2} - \dfrac{2}{2\sqrt{x}} = \dfrac{5x^2 - 3x - 2}{2\sqrt{x}}$

b $f'(x) = 0$ geeft $5x^2 - 3x - 2 = 0$

$D = (-3)^2 - 4 \cdot 5 \cdot -2 = 49$

$x = \dfrac{3+7}{10} = 1 \vee x = \dfrac{3-7}{10} = -\frac{2}{5}$

vold. vold. niet

Zie figuur G.3 in het leerboek.

min. is $f(1) = -2$.

c $f(x) = 0$ geeft $x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0$

$x(x+1)(x-2) = 0$

$x = 0 \vee x = -1 \vee x = 2$

vold. niet vold. niet vold.

Dus $A(2, 0)$.

De lijn l raakt de grafiek van f in A.

$\text{rc}_l = f'(2) = \dfrac{20 - 6 - 2}{2\sqrt{2}} = \dfrac{6}{\sqrt{2}} = 3\sqrt{2}$

$\text{rc}_k \cdot \text{rc}_l = -1$, dus $\text{rc}_k = -\dfrac{1}{3\sqrt{2}}$.

$\left.\begin{array}{l} k: y = -\dfrac{1}{3\sqrt{2}}x + b \\ \text{door } A(2, 0) \end{array}\right\} -\dfrac{1}{3\sqrt{2}} \cdot 2 + b = 0$

$b = \dfrac{2}{3\sqrt{2}}$

$k: y = -\dfrac{1}{3\sqrt{2}}x + \dfrac{2}{3\sqrt{2}}$ oftewel $k: 3y\sqrt{2} = -x + 2$

Dus $k: x + 3y\sqrt{2} = 2$.

d $f'(x) = 16\frac{1}{2}$ geeft $\dfrac{5x^2 - 3x - 2}{2\sqrt{x}} = \dfrac{33}{2}$

$5x^2 - 3x - 2 = 33\sqrt{x}$

Voer in $y_1 = 5x^2 - 3x - 2$ en $y_2 = 33\sqrt{x}$.

De optie snijpunt geeft $x = 4$.

$f(4) = 20$, dus $B(4, 20)$.

$\left.\begin{array}{l} y = 16\frac{1}{2}x + b \\ \text{door } B(4, 20) \end{array}\right\} 66 + b = 20$

$b = -46$

Dus $m: y = 16\frac{1}{2}x - 46$.

18 $9 - x^2 \geq 0$

$-x^2 \geq -9$

$x^2 \leq 9$

$-3 \leq x \leq 3$

Dus $D_f = [-3, 3]$.

$f(x) = \sqrt{9 - x^2} + \frac{1}{2}x^2 - 2$ geeft $f'(x) = \dfrac{1}{2\sqrt{9 - x^2}} \cdot -2x + x = \dfrac{-x}{\sqrt{9 - x^2}} + x$

$f'(x) = 0$ geeft $\dfrac{-x}{\sqrt{9 - x^2}} + x = 0$

$\dfrac{x}{\sqrt{9 - x^2}} = x$

$x = 0 \vee \sqrt{9 - x^2} = 1$

$x = 0 \vee 9 - x^2 = 1$

$x = 0 \vee x^2 = 8$

$x = 0 \vee x = 2\sqrt{2} \vee x = -2\sqrt{2}$

max. is $f(-2\sqrt{2}) = f(2\sqrt{2}) = \sqrt{9 - 8} + \frac{1}{2} \cdot 8 - 2 = 3$ en min. is $f(0) = \sqrt{9} + 0 - 2 = 1$.

$f(-3) = f(3) = \sqrt{9 - 9} + \frac{1}{2} \cdot 3^2 - 2 = 2\frac{1}{2}$

Dus $B_f = [1, 3]$.

19 **a** $f_p(x) = x\sqrt{x} + p\sqrt{x} = x^{1\frac{1}{2}} + p\sqrt{x}$ geeft $f_p'(x) = 1\frac{1}{2}x^{\frac{1}{2}} + \dfrac{p}{2\sqrt{x}} = \dfrac{3\sqrt{x}}{2} + \dfrac{p}{2\sqrt{x}} = \dfrac{3x + p}{2\sqrt{x}}$

$f_p'(4) = 5$ geeft $\dfrac{12 + p}{2\sqrt{4}} = 5$

$12 + p = 20$

$p = 8$

$f_8(x) = x\sqrt{x} + 8\sqrt{x}$ geeft $f_8(4) = 8 + 16 = 24$

$\left.\begin{array}{l} k: y = 5x + q \\ \text{door } A(4, 24) \end{array}\right\} \; 20 + q = 24$

$q = 4$

b $f_p'(x) = 0$ geeft $3x + p = 0$

$\left.\begin{array}{l} p = -3x \\ y = x\sqrt{x} + p\sqrt{x} \end{array}\right\} \; y = x\sqrt{x} - 3x\sqrt{x} = -2x\sqrt{x}$

Dus de formule van de kromme waarop alle toppen liggen is $y = -2x\sqrt{x}$.

c $-2x\sqrt{x} = -2$

$x\sqrt{x} = 1$

$x^3 = 1$

$x = 1$ vold.

$f_p'(1) = 0$ geeft $3 + p = 0$, dus $p = -3$.

20 **a** $f(x) = (x + 1)\sqrt{4x^2 + 1}$ geeft $f'(x) = 1 \cdot \sqrt{4x^2 + 1} + (x + 1) \cdot \dfrac{1}{2\sqrt{4x^2 + 1}} \cdot 8x = \sqrt{4x^2 + 1} + \dfrac{4x(x + 1)}{\sqrt{4x^2 + 1}}$

$rc_k = f'(0) = 1$

$g_a(x) = 4 + \dfrac{8}{(ax - 2)^2} = 4 + 8(ax - 2)^{-2}$ geeft $g_a'(x) = -16(ax - 2)^{-3} \cdot a = \dfrac{-16a}{(ax - 2)^3}$

$rc_l = g_a'(0) = \dfrac{-16a}{-8} = 2a$

$k \perp l$ geeft $rc_k \cdot rc_l = -1$

$1 \cdot 2a = -1$

$a = -\frac{1}{2}$

b $f(0) = 1$, dus $A(0, 1)$ en k: $y = x + 1$.

$g_a(0) = 4 + \dfrac{8}{(0 - 2)^2} = 4 + \dfrac{8}{4} = 4 + 2 = 6$, dus l: $y = 2ax + 6$.

k snijdt de x-as in $(-1, 0)$.

$(-1, 0)$ op l geeft $-2a + 6 = 0$

$-2a = -6$

$a = 3$

Bladzijde 177

21 **a** $f(p) = \frac{4}{p}$, dus $P\left(p, \frac{4}{p}\right)$.

$L(p) = d(O, P) = \sqrt{(p - 0)^2 + \left(\frac{4}{p} - 0\right)^2} = \sqrt{p^2 + \frac{16}{p^2}}$

b $L(p) = \sqrt{p^2 + \frac{16}{p^2}} = \sqrt{p^2 + 16p^{-2}}$ geeft

$L'(p) = \frac{1}{2\sqrt{p^2 + \frac{16}{p^2}}} \cdot (2p - 32p^{-3}) = \frac{p - 16p^{-3}}{\sqrt{p^2 + \frac{16}{p^2}}} = \frac{p^4 - 16}{p^3 \cdot \sqrt{p^2 + \frac{16}{p^2}}}$

$L'(p) = 0$ geeft $p^4 - 16 = 0$

$p^4 = 16$

$p = 2 \vee p = -2$

vold. vold. niet

$f(2) = \frac{4}{2} = 2$, dus $P(2, 2)$.

$f(x) = \frac{4}{x} = 4x^{-1}$ geeft $f'(x) = -4x^{-2} = -\frac{4}{x^2}$

$\left.\begin{array}{l} \text{rc}_k = f'(2) = -\frac{4}{2^2} = -1 \\ \text{rc}_{OP} = \frac{2-0}{2-0} = 1 \end{array}\right\} \text{rc}_k \cdot \text{rc}_{OP} = -1 \cdot 1 = -1$

Dus k en OP staan loodrecht op elkaar.

22 $f(x) = \frac{x^3 + 6}{x\sqrt{x}} = \frac{x^3 + 6}{x^{1\frac{1}{2}}} = x^{1\frac{1}{2}} + 6x^{-1\frac{1}{2}}$ geeft $f'(x) = 1\tfrac{1}{2}x^{\frac{1}{2}} - 9x^{-2\frac{1}{2}} = 1\tfrac{1}{2}\sqrt{x} - \frac{9}{x^2 \cdot \sqrt{x}} = \frac{3x^3 - 18}{2x^2 \cdot \sqrt{x}}$

$f'(x) = 0$ geeft $3x^3 - 18 = 0$

$3x^3 = 18$

$x^3 = 6$

$x = \sqrt[3]{6}$

$f(\sqrt[3]{6}) = \frac{6 + 6}{(\sqrt[3]{6})^{1\frac{1}{2}}} = \frac{12}{6^{\frac{1}{2}}} = \frac{12}{\sqrt{6}} = 2\sqrt{6}$

Dus $a = 6$ en $b = 2$.

23 $f_p(x) = \frac{10x + p}{x^2 + 1}$ geeft $f_p'(x) = \frac{(x^2 + 1) \cdot 10 - (10x + p) \cdot 2x}{(x^2 + 1)^2} = \frac{10x^2 + 10 - 20x^2 - 2px}{(x^2 + 1)^2} = \frac{-10x^2 - 2px + 10}{(x^2 + 1)^2}$

$f_p'(1) = \frac{-10 - 2p + 10}{(1 + 1)^2} = \frac{-2p}{4} = -\tfrac{1}{2}p$

Loodrecht snijden, dus $f_p'(1) \cdot \text{rc}_k = -1$

$-\tfrac{1}{2}p \cdot \tfrac{2}{3} = -1$

$-\tfrac{1}{3}p = -1$

$p = 3$

$f_3(x) = \frac{10x + 3}{x^2 + 1}$

$f_3(1) = \frac{10 + 3}{1 + 1} = \frac{13}{2} = 6\tfrac{1}{2}$, dus $A(1, 6\tfrac{1}{2})$.

$\left.\begin{array}{l} y = \tfrac{2}{3}x + q \\ \text{door } A(1, 6\tfrac{1}{2}) \end{array}\right\} \tfrac{2}{3} \cdot 1 + q = 6\tfrac{1}{2}$

$q = 5\tfrac{5}{6}$

Dus $p = 3$ en $q = 5\tfrac{5}{6}$.

7 Meetkunde met coördinaten

24 **a** $\left.\begin{array}{l} k\colon \dfrac{x}{2p+1}+\dfrac{y}{5}=1 \\ A(2, 3) \text{ op } k \end{array}\right\}$ $\dfrac{2}{2p+1}+\dfrac{3}{5}=1$

$\dfrac{2}{2p+1}=\dfrac{2}{5}$

$2p+1=5$

$2p=4$

$p=2$

vold.

b $\text{rc}_k=\dfrac{0-5}{2p+1-0}=\dfrac{-5}{2p+1}$

$k \,//\, m$, dus $\text{rc}_k=\text{rc}_m$

$\dfrac{-5}{2p+1}=-2$

$-2(2p+1)=-5$

$-4p-2=-5$

$-4p=-3$

$p=\frac{3}{4}$

$\text{rc}_l=\dfrac{q+2-0}{0-2}=\dfrac{q+2}{-2}$

$l \,//\, m$, dus $\text{rc}_l=\text{rc}_m$

$\dfrac{q+2}{-2}=-2$

$q+2=4$

$q=2$

Dus voor $p=\frac{3}{4} \wedge q=2$ zijn de lijnen k, l en m evenwijdig.

c De lijnen k en l vallen samen als k door het punt $(2, 0)$ en l door het punt $(0, 5)$ gaat.
Dus er moet gelden $2p+1=2$ en $q+2=5$. Hieruit volgt $p=\frac{1}{2}$ en $q=3$.
Dus voor $p=\frac{1}{2} \wedge q=3$ vallen de lijnen k en l samen.

25 **a** Stel $k\colon y=ax+b$.
$B(1, 6)$ op k geeft $a+b=6$, dus $b=-a+6$.
$k\colon y=ax-a+6$ oftewel $k\colon ax-y-a+6=0$

$d(A, k)=2$ geeft $\dfrac{|-3a-4-a+6|}{\sqrt{a^2+1}}=2$

$|-4a+2|=2\sqrt{a^2+1}$

$16a^2-16a+4=4a^2+4$

$12a^2-16a=0$

$4a(3a-4)=0$

$4a=0 \vee 3a=4$

$a=0 \vee a=1\frac{1}{3}$

$a=0$ geeft $b=6$, dus $k_1\colon y=6$.

$a=1\frac{1}{3}$ geeft $b=-1\frac{1}{3}+6=4\frac{2}{3}$, dus $k_2\colon y=1\frac{1}{3}x+4\frac{2}{3}$.

b Stel $AB\colon y=ax+b$ met $a=\dfrac{6-4}{1--3}=\frac{1}{2}$.

$\left.\begin{array}{l} y=\frac{1}{2}x+b \\ \text{door } B(1, 6) \end{array}\right\}$ $\frac{1}{2}\cdot 1+b=6$

$b=5\frac{1}{2}$

Dus $AB\colon y=\frac{1}{2}x+5\frac{1}{2}$ oftewel $AB\colon x-2y=-11$.

$l \,//\, AB$, dus $l\colon x-2y=c$.

$d(B, l)=2\sqrt{5}$ geeft $\dfrac{|1-2\cdot 6-c|}{\sqrt{1^2+(-2)^2}}=2\sqrt{5}$

$\dfrac{|-11-c|}{\sqrt{5}}=2\sqrt{5}$

$|-11-c|=10$

$-11-c=10 \vee -11-c=-10$

$c=-21 \vee c=-1$

Dus $l_1\colon x-2y=-21$ en $l_2\colon x-2y=-1$.

c Lijnen n evenwijdig m en op afstand 5 van m.
n: $3x + 4y = c$
$D(0, 2)$ is een punt op m.

$d(D, n) = 5$ geeft $\frac{|3 \cdot 0 + 4 \cdot 2 - c|}{\sqrt{3^2 + 4^2}} = 5$

$\frac{|8 - c|}{\sqrt{25}} = 5$

$\frac{|8 - c|}{5} = 5$

$|8 - c| = 25$

$8 - c = 25 \vee 8 - c = -25$

$c = -17 \vee c = 33$

n_1: $3x + 4y = -17$ snijden met AB: $y = \frac{1}{2}x + 5\frac{1}{2}$ geeft $3x + 4(\frac{1}{2}x + 5\frac{1}{2}) = -17$

$3x + 2x + 22 = -17$

$5x = -39$

$\left.\begin{array}{l} x = -7\frac{4}{5} \\ y = \frac{1}{2}x + 5\frac{1}{2} \end{array}\right\} y = \frac{1}{2} \cdot -7\frac{4}{5} + 5\frac{1}{2} = 1\frac{3}{5}$

n_2: $3x + 4y = 33$ snijden met AB: $y = \frac{1}{2}x + 5\frac{1}{2}$ geeft $3x + 4(\frac{1}{2}x + 5\frac{1}{2}) = 33$

$3x + 2x + 22 = 33$

$5x = 11$

$\left.\begin{array}{l} x = 2\frac{1}{5} \\ y = \frac{1}{2}x + 5\frac{1}{2} \end{array}\right\} y = \frac{1}{2} \cdot 2\frac{1}{5} + 5\frac{1}{2} = 6\frac{3}{5}$

De punten zijn $(-7\frac{4}{5}, 1\frac{3}{5})$ en $(2\frac{1}{5}, 6\frac{3}{5})$.

d $\text{rc}_{AC} = \frac{-1 - 4}{0 - -3} = -1\frac{2}{3}$

$\tan(\alpha) = \text{rc}_{AB} = \frac{1}{2}$ geeft $\alpha = 26{,}56...^\circ$

$\tan(\beta) = \text{rc}_{AC} = -1\frac{2}{3}$ geeft $\beta = -59{,}03...^\circ$

$\alpha - \beta = 26{,}56...^\circ - -59{,}03...^\circ \approx 85{,}6^\circ$

Dus $\angle(AB, AC) \approx 85{,}6^\circ$.

26 **a** $p = -3$ geeft k: $y = -3x + 1$

$\tan(\alpha) = \text{rc}_k = -3$ geeft $\alpha = -71{,}56...^\circ$

l: $2x + y = 3$ oftewel l: $y = -2x + 3$

$\tan(\beta) = \text{rc}_l = -2$ geeft $\beta = -63{,}43...^\circ$

$\beta - \alpha = -63{,}43...^\circ - -71{,}56...^\circ \approx 8{,}1^\circ$

Dus $\angle(k, l) \approx 8{,}1^\circ$.

b $-63{,}43...^\circ + 15^\circ = -48{,}43...^\circ$

$p = \text{rc}_k = \tan(-48{,}43...^\circ) \approx -1{,}128$

$-63{,}43...^\circ - 15^\circ = -78{,}43...^\circ$

$p = \text{rc}_k = \tan(-78{,}43...^\circ) \approx -4{,}887$

Bladzijde 178

27 **a** $d(A, B) = \sqrt{(5p - p)^2 + (2p + 3 - 5)^2} = \sqrt{(4p)^2 + (2p - 2)^2} = \sqrt{16p^2 + 4p^2 - 8p + 4} = \sqrt{20p^2 - 8p + 4}$

b $d(A, B) = 4\sqrt{29}$ geeft $\sqrt{20p^2 - 8p + 4} = 4\sqrt{29}$

$20p^2 - 8p + 4 = 464$

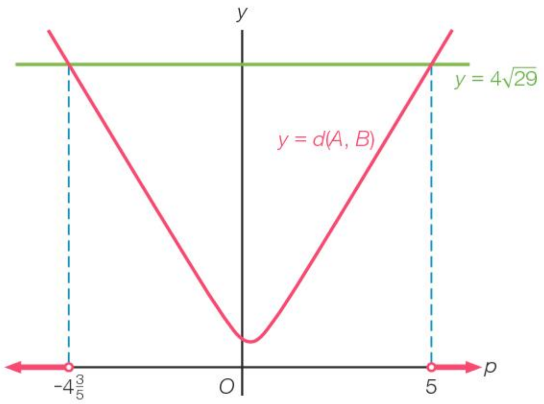
$20p^2 - 8p - 460 = 0$

$5p^2 - 2p - 115 = 0$

$D = (-2)^2 - 4 \cdot 5 \cdot -115 = 2304$

$p = \frac{2 + 48}{10} = 5 \vee p = \frac{2 - 48}{10} = -4\frac{3}{5}$

vold. vold.



$d(A, B) > 4\sqrt{29}$ geeft $p < -4\frac{3}{5} \vee p > 5$

c $M(\frac{1}{2}(p + 5p), \frac{1}{2}(5 + 2p + 3)) = M(3p, p + 4)$
Van c: $x^2 + y^2 = 16$ is het middelpunt $O(0, 0)$ en $r = 4$.
M op afstand 8 van c, dus M op de cirkel met middelpunt $O(0, 0)$ en straal 12.
M op $x^2 + y^2 = 144$ geeft $(3p)^2 + (p + 4)^2 = 144$
$9p^2 + p^2 + 8p + 16 = 144$
$10p^2 + 8p - 128 = 0$
$5p^2 + 4p - 64 = 0$
$D = 4^2 - 4 \cdot 5 \cdot -64 = 1296$
$p = \frac{-4 + 36}{10} \vee p = \frac{-4 - 36}{10}$
$p = 3\frac{1}{5} \vee p = -4$

d AB is een middellijn, dus van d is het middelpunt $M(3p, p + 4)$.
De straal is $d(A, M) = \sqrt{(3p - p)^2 + (p + 4 - 5)^2} = \sqrt{(2p)^2 + (p - 1)^2} = \sqrt{4p^2 + p^2 - 2p + 1} = \sqrt{5p^2 - 2p + 1}$.
Dus d: $(x - 3p)^2 + (y - p - 4)^2 = 5p^2 - 2p + 1$.
d door de oorsprong geeft $(-3p)^2 + (-p - 4)^2 = 5p^2 - 2p + 1$
$9p^2 + p^2 + 8p + 16 = 5p^2 - 2p + 1$
$5p^2 + 10p + 15 = 0$
$p^2 + 2p + 3 = 0$
$D = 2^2 - 4 \cdot 1 \cdot 3 = -8$
geen opl.
Dus er is geen waarde van p waarvoor d door de oorsprong gaat.

28 **a** $r = d(M, k) = \frac{|3 \cdot 8 - 4 \cdot 2 - 6|}{\sqrt{3^2 + (-4)^2}} = \frac{|10|}{\sqrt{25}} = \frac{10}{5} = 2$
Dus c: $(x - 8)^2 + (y - 2)^2 = 4$.

b Stel l: $3x - 4y = c$.
$P(2, 0)$ is een punt op k.
$d(P, l) = 3$ geeft $\frac{|3 \cdot 2 - 4 \cdot 0 - c|}{5} = 3$
$|6 - c| = 15$
$6 - c = 15 \vee 6 - c = -15$
$c = -9 \vee c = 21$
Dus l_1: $3x - 4y = -9$ en l_2: $3x - 4y = 21$.

c Stel een punt op m is $A(a, a)$.
$d(A, k) = 4$ geeft $\frac{|3a - 4a - 6|}{5} = 4$
$|-a - 6| = 20$
$-a - 6 = 20 \vee -a - 6 = -20$
$a = -26 \vee a = 14$
Dus $A(-26, -26)$ en $B(14, 14)$.

29 **a** Met dit werkschema wordt het middelpunt M van de cirkel gevonden door het snijpunt van twee middelloodlijnen van driehoek ABC te berekenen. Daarna geeft $d(M, A)$ de straal van de cirkel.

b $\text{rc}_{AB} = \frac{1-8}{7-6} = -7$, dus $\text{rc}_k = \frac{1}{7}$.

Het midden van AB is $(\frac{1}{2}(6+7), \frac{1}{2}(8+1)) = (6\frac{1}{2}, 4\frac{1}{2})$.

$\left.\begin{array}{l} k: y = \frac{1}{7}x + b \\ \text{door } (6\frac{1}{2}, 4\frac{1}{2}) \end{array}\right\} \frac{1}{7} \cdot 6\frac{1}{2} + b = 4\frac{1}{2}$

$\frac{13}{14} + b = 4\frac{1}{2}$

$b = 3\frac{4}{7}$

Dus $k: y = \frac{1}{7}x + 3\frac{4}{7}$.

$\text{rc}_{AC} = \frac{4-8}{-2-6} = \frac{1}{2}$, dus $\text{rc}_l = -2$.

Het midden van AC is $(\frac{1}{2}(6+-2), \frac{1}{2}(8+4)) = (2, 6)$.

$\left.\begin{array}{l} l: y = -2x + b \\ \text{door } (2, 6) \end{array}\right\} -2 \cdot 2 + b = 6$

$-4 + b = 6$

$b = 10$

Dus $l: y = -2x + 10$.

k en l snijden geeft

$\frac{1}{7}x + 3\frac{4}{7} = -2x + 10$

$2\frac{1}{7}x = 6\frac{3}{7}$

$\left.\begin{array}{l} x = 3 \\ y = -2x + 10 \end{array}\right\} y = -2 \cdot 3 + 10 = 4$

Dus $M(3, 4)$.

$d(M, A) = \sqrt{(6-3)^2 + (8-4)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Dus $c: (x-3)^2 + (y-4)^2 = 25$.

30 **a** $c: x^2 + y^2 - 2x - 6y + 5 = 0$

$x^2 - 2x + y^2 - 6y + 5 = 0$

$(x-1)^2 - 1 + (y-3)^2 - 9 + 5 = 0$

$(x-1)^2 + (y-3)^2 = 5$

Dus $M(1, 3)$ en $r = \sqrt{5}$.

Stel $y = ax + b$.

Door $A(5, 0)$ geeft $5a + b = 0$, dus $b = -5a$.

$y = ax - 5a$ oftewel $ax - y - 5a = 0$

$d(M, ax - y - 5a = 0) = r$ geeft $\frac{|a - 3 - 5a|}{\sqrt{a^2+1}} = \sqrt{5}$

$|-4a - 3| = \sqrt{5a^2 + 5}$

$16a^2 + 24a + 9 = 5a^2 + 5$

$11a^2 + 24a + 4 = 0$

$D = 24^2 - 4 \cdot 11 \cdot 4 = 400$

$a = \frac{-24 + 20}{22} = -\frac{2}{11} \vee a = \frac{-24 - 20}{22} = -2$

$a = -\frac{2}{11}$ geeft $b = -5 \cdot -\frac{2}{11} = \frac{10}{11}$, dus $l: y = -\frac{2}{11}x + \frac{10}{11}$.

$a = -2$ geeft $b = -5 \cdot -2 = 10$, dus $k: y = -2x + 10$.

b $\tan(\alpha) = \text{rc}_k = -2$ geeft $\alpha = -63{,}43...^\circ$

$\tan(\beta) = \text{rc}_l = -\frac{2}{11}$ geeft $\beta = -10{,}30...^\circ$

$\beta - \alpha = -10{,}30...^\circ - -63{,}43...^\circ \approx 53{,}1^\circ$

Dus $\angle(k, l) \approx 53{,}1^\circ$.

c d raakt de x-as in het punt $A(5, 0)$, dus het middelpunt van d is N met $x_N = 5$.
Stel $N(5, p)$. Dit geeft d: $(x-5)^2 + (y-p)^2 = p^2$.
d door $M(1, 3)$ geeft $(-4)^2 + (3-p)^2 = p^2$

$$16 + 9 - 6p + p^2 = p^2$$
$$-6p = -25$$
$$p = 4\tfrac{1}{6}$$

Dus d: $(x-5)^2 + (y - 4\tfrac{1}{6})^2 = (4\tfrac{1}{6})^2$ oftewel d: $(x-5)^2 + (y-4\tfrac{1}{6})^2 = 17\tfrac{13}{36}$.

Bladzijde 179

31 **a** c_1: $x^2 + y^2 - 8x - 4y + 15 = 0$

$$x^2 - 8x + y^2 - 4y + 15 = 0$$
$$(x-4)^2 - 16 + (y-2)^2 - 4 + 15 = 0$$
$$(x-4)^2 + (y-2)^2 = 5$$

Dus van c_1 is het middelpunt $M(4, 2)$ en de straal $r_1 = \sqrt{5}$.
$l \perp k$ geeft l: $x + 2y = c$.

$$d(M, l) = r \text{ geeft } \frac{|4 + 2 \cdot 2 - c|}{\sqrt{1^2 + 2^2}} = \sqrt{5}$$
$$\frac{|8 - c|}{\sqrt{5}} = \sqrt{5}$$
$$|8 - c| = 5$$
$$8 - c = 5 \vee 8 - c = -5$$
$$c = 3 \vee c = 13$$

Dus l_1: $x + 2y = 3$ en l_2: $x + 2y = 13$.

b $d(A, M) = \sqrt{(4-15)^2 + (2-9)^2} = \sqrt{121 + 49} = \sqrt{170}$

$d(A, c_1) = d(A, M) - r_1 = \sqrt{170} - \sqrt{5}$

$$d(M, k) = \frac{|2 \cdot 4 - 2 - 21|}{\sqrt{2^2 + (-1)^2}} = \frac{|-15|}{\sqrt{5}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}$$

$d(k, c_1) = d(M, k) - r_1 = 3\sqrt{5} - \sqrt{5} = 2\sqrt{5}$

$2\tfrac{1}{2} \cdot d(k, c_1) = 2\tfrac{1}{2} \cdot 2\sqrt{5} = 5\sqrt{5}$

Is $\sqrt{170} - \sqrt{5} > 5\sqrt{5}$ oftewel is $\sqrt{170} > 6\sqrt{5}$?

$6\sqrt{5} = \sqrt{36} \cdot \sqrt{5} = \sqrt{180}$

Er geldt dus niet dat $\sqrt{170} > 6\sqrt{5}$ en dus geldt niet dat $d(A, c_1) > 2\tfrac{1}{2} \cdot d(k, c_1)$.

c Van lijnstuk AB met $A(15, 9)$ en $B(1, -1)$ is het midden $N(8, 4)$.
N is tevens het middelpunt van c_2 en de straal van c_2 is
$r_2 = d(A, N) = \sqrt{(8-15)^2 + (4-9)^2} = \sqrt{49 + 25} = \sqrt{74}$.
$d(M, N) = \sqrt{(8-4)^2 + (4-2)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$
$d(c_1, c_2) = r_2 - d(M, N) - r_1 = \sqrt{74} - 2\sqrt{5} - \sqrt{5} = \sqrt{74} - 3\sqrt{5}$

32 **a** Stel k: $y = 3x + b$ oftewel k: $3x - y + b = 0$
Van c is het middelpunt $O(0, 0)$ en de straal $r = \sqrt{10}$.

$$d(O, k) = r \text{ geeft } \frac{|3 \cdot 0 - 0 + b|}{\sqrt{3^2 + (-1)^2}} = \sqrt{10}$$
$$\frac{|b|}{\sqrt{10}} = \sqrt{10}$$
$$|b| = 10$$
$$b = 10 \vee b = -10$$

Dus k_1: $y = 3x + 10$ en k_2: $y = 3x - 10$.

b Stel l: $y = ax + b$.

$A(4, 2)$ op l geeft $4a + b = 2$, dus $b = -4a + 2$.

l: $y = ax - 4a + 2$ oftewel $ax - y - 4a + 2 = 0$

$$d(O, l) = r \text{ geeft } \frac{|0 - 0 - 4a + 2|}{\sqrt{a^2 + 1}} = \sqrt{10}$$

$$|-4a + 2| = \sqrt{10a^2 + 10}$$

$$16a^2 - 16a + 4 = 10a^2 + 10$$

$$6a^2 - 16a - 6 = 0$$

$$3a^2 - 8a - 3 = 0$$

$$D = (-8)^2 - 4 \cdot 3 \cdot -3 = 100$$

$$a = \frac{8 + 10}{6} = 3 \vee a = \frac{8 - 10}{6} = -\tfrac{1}{3}$$

$a = 3$ geeft $b = -4 \cdot 3 + 2 = -10$, dus l_1: $y = 3x - 10$.

$a = -\frac{1}{3}$ geeft $b = -4 \cdot -\frac{1}{3} + 2 = 3\frac{1}{3}$, dus l_2: $y = -\frac{1}{3}x + 3\frac{1}{3}$.

c Noem de raakpunten P en Q.

$\sin(\angle OBQ) = \frac{r}{OB} = \frac{\sqrt{10}}{4}$ geeft $\angle OBQ = 52{,}23...°$

$\angle PBQ = 2 \cdot 52{,}23...° \approx 104{,}5°$

Dus $\angle(m_1, m_2) \approx 180° - 104{,}5° = 75{,}5°$.

33 **a** c: $x^2 + y^2 - 6x - 4y = 0$

$$x^2 - 6x + y^2 - 4y = 0$$

$$(x - 3)^2 - 9 + (y - 2)^2 - 4 = 0$$

$$(x - 3)^2 + (y - 2)^2 = 13$$

Dus $M(3, 2)$ en $r = \sqrt{13}$.

$x = 5$ geeft $(5 - 3)^2 + (y - 2)^2 = 13$

$$4 + (y - 2)^2 = 13$$

$$(y - 2)^2 = 9$$

$$y - 2 = 3 \vee y - 2 = -3$$

$$y = 5 \vee y = -1$$

Dus $A(5, 5)$ en $B(5, -1)$.

$\text{rc}_{AM} = \frac{5 - 2}{5 - 3} = 1\frac{1}{2}$, dus $\text{rc}_k = -\frac{2}{3}$.

$\left.\begin{array}{l} k: y = -\frac{2}{3}x + b \\ \text{door } A(5, 5) \end{array}\right\} -\frac{2}{3} \cdot 5 + b = 5$

$$-3\tfrac{1}{3} + b = 5$$

$$b = 8\tfrac{1}{3}$$

Dus k: $y = -\frac{2}{3}x + 8\frac{1}{3}$.

$\text{rc}_{BM} = \frac{-1 - 2}{5 - 3} = -1\frac{1}{2}$, dus $\text{rc}_l = \frac{2}{3}$.

$\left.\begin{array}{l} l: y = \frac{2}{3}x + b \\ \text{door } B(5, -1) \end{array}\right\} \frac{2}{3} \cdot 5 + b = -1$

$$3\tfrac{1}{3} + b = -1$$

$$b = -4\tfrac{1}{3}$$

Dus l: $y = \frac{2}{3}x - 4\frac{1}{3}$.

b m evenwijdig met n: $3x + 2y = 10$ geeft m: $3x + 2y = c$.

$$d(M, m) = r \text{ geeft } \frac{|3 \cdot 3 + 2 \cdot 2 - c|}{\sqrt{3^2 + 2^2}} = \sqrt{13}$$

$$\frac{|13 - c|}{\sqrt{13}} = \sqrt{13}$$

$$|13 - c| = 13$$

$$13 - c = 13 \vee 13 - c = -13$$

$$c = 0 \vee c = 26$$

Dus m_1: $3x + 2y = 0$ en m_2: $3x + 2y = 26$.

34 $c\colon x^2 + y^2 + 8x = 0$

$$x^2 + 8x + y^2 = 0$$
$$(x+4)^2 - 16 + y^2 = 0$$
$$(x+4)^2 + y^2 = 16$$

Dus middelpunt $M(-4, 0)$ en straal $r_c = 4$.
Lijn l gaat door M en staat loodrecht op k.

$$\left.\begin{array}{l} l\colon 3x - 4y = c \\ \text{door } M(-4, 0) \end{array}\right\} c = 3 \cdot -4 - 4 \cdot 0 = -12$$

Dus $l\colon 3x - 4y = -12$.
k en l snijden geeft het punt S, dat het middelpunt van cirkel d is.

$$\begin{cases} 4x + 3y = 9 \\ 3x - 4y = -12 \end{cases} \begin{vmatrix} 4 \\ 3 \end{vmatrix} \text{ geeft } \begin{cases} 16x + 12y = 36 \\ 9x - 12y = -36 \end{cases} +$$

$$25x = 0$$

$$\left.\begin{array}{l} x = 0 \\ 4x + 3y = 9 \end{array}\right\} \begin{array}{l} 4 \cdot 0 + 3y = 9 \\ y = 3 \end{array}$$

Dus $S(0, 3)$.
$d(M, S) = \sqrt{(0 - -4)^2 + (3 - 0)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$
De straal van d is $r_d = d(k, c) = d(M, S) - r_c = 5 - 4 = 1$.
Dus $d\colon x^2 + (y - 3)^2 = 1$.

35 $k\colon \dfrac{x}{2r} + \dfrac{y}{4} = 1$ oftewel $k\colon 2x + ry = 4r$

$d(O, k) = r$ geeft $\dfrac{|2 \cdot 0 + r \cdot 0 - 4r|}{\sqrt{4 + r^2}} = r$

$$|-4r| = r\sqrt{4 + r^2}$$
$$16r^2 = r^2(4 + r^2)$$
$$16 = 4 + r^2$$
$$r^2 = 12$$
$$r = \sqrt{12} = 2\sqrt{3}$$

Bladzijde 180

36 **a** De loodlijn uit N op de y-as snijdt de y-as in het punt $Q(0, 4)$.
In driehoek MNQ geeft de stelling van Pythagoras $QN^2 + QM^2 = MN^2$

$$QN^2 + 5^2 = 13^2$$
$$QN^2 = 169 - 25 = 144$$
$$QN = 12$$

Vierhoek $OPNM$ is een trapezium.
De oppervlakte van $OPNM$ is $\frac{1}{2}(9 + 4) \cdot 12 = 78$.

b De loodlijn uit N op de y-as snijdt de y-as in het punt $Q(0, s)$.
In driehoek MNQ geeft de stelling van Pythagoras $QN^2 + QM^2 = MN^2$

$$QN^2 + (r - s)^2 = (r + s)^2$$
$$QN^2 + r^2 - 2rs + s^2 = r^2 + 2rs + s^2$$
$$QN^2 = 4rs$$
$$QN = \sqrt{4rs} = 2\sqrt{rs}$$

De oppervlakte van $OPNM$ is $\frac{1}{2}(r + s) \cdot 2\sqrt{rs} = (r + s)\sqrt{rs}$.

c De stelling van Pythagoras in driehoek QNM geeft $QM^2 + QN^2 = MN^2$

$$(r - s)^2 + OP^2 = MN^2$$
$$(r - s)^2 + 15^2 = 25^2$$
$$(r - s)^2 = 625 - 225$$
$$(r - s)^2 = 400$$
$$r - s = 20$$

$MN = 25$ geeft $r + s = 25$

$$\begin{cases} r + s = 25 \\ r - s = 20 \end{cases}$$
$$2r = 45 \quad +$$

$$\left.\begin{array}{l} r = 22\frac{1}{2} \\ r + s = 25 \end{array}\right\} 22\frac{1}{2} + s = 25$$
$$s = 2\frac{1}{2}$$

Dus $r = 22\frac{1}{2}$ en $s = 2\frac{1}{2}$.

37 M op k en $x_M = -3$ geeft $y_M = -1\frac{1}{3} \cdot -3 + 4 = 8$.
Dus $M(-3, 8)$.
N op k en $y_N = -4$ geeft $-1\frac{1}{3}x_N + 4 = -4$

$$-1\frac{1}{3}x_N = -8$$
$$x_N = 6$$

Dus $N(6, -4)$.
$d(M, N) = \sqrt{(6 - -3)^2 + (-4 - 8)^2} = \sqrt{81 + 144} = \sqrt{225} = 15$, $r_1 = 3$ en $r_2 = 4$.
$d(c_1, c_2) = d(M, N) - r_1 - r_2 = 15 - 3 - 4 = 8$
$d(c_1, l) = d(c_2, l) = 4$
$d(M, l) = r_1 + d(c_1, l) = 3 + 4 = 7$
$d(N, l) = r_2 + d(c_2, l) = 4 + 4 = 8$
k: $y = -1\frac{1}{3}x + 4$ oftewel k: $1\frac{1}{3}x + y = 4$ oftewel k: $4x + 3y = 12$
$l \perp k$ geeft l: $3x - 4y = c$

$d(M, l) = 7$ geeft $\frac{|3 \cdot -3 - 4 \cdot 8 - c|}{\sqrt{3^2 + 4^2}} = 7$

$$\frac{|-41 - c|}{\sqrt{25}} = 7$$
$$\frac{|-41 - c|}{5} = 7$$
$$|-41 - c| = 35$$
$$-41 - c = 35 \vee -41 - c = -35$$
$$c = -76 \vee c = -6$$

Dus l_1: $3x - 4y = -76$ en l_2: $3x - 4y = -6$.

$d(N, l_1) = \frac{|3 \cdot 6 - 4 \cdot -4 + 76|}{5} = \frac{|110|}{5} = \frac{110}{5} = 22$, dus l_1 voldoet niet.

$d(N, l_2) = \frac{|3 \cdot 6 - 4 \cdot -4 + 6|}{5} = \frac{|40|}{5} = \frac{40}{5} = 8$, dus l_2 voldoet.

Een vergelijking van l is dus $3x - 4y = -6$.

8 Goniometrische functies

38 De draaiingshoek van A is $30° = \frac{1}{6}\pi$ rad.

De draaiingshoek van B is $\frac{1}{6}\pi + 2$ rad.

De draaiingshoek van C is $\frac{1}{6}\pi + 4$ rad.

$A(\cos(\frac{1}{6}\pi), \sin(\frac{1}{6}\pi)) \approx A(0{,}87; 0{,}5)$

$B(\cos(\frac{1}{6}\pi + 2), \sin(\frac{1}{6}\pi + 2)) \approx B(-0{,}82; 0{,}58)$

$C(\cos(\frac{1}{6}\pi + 4), \sin(\frac{1}{6}\pi + 4)) \approx C(-0{,}19; -0{,}98)$

Bladzijde 181

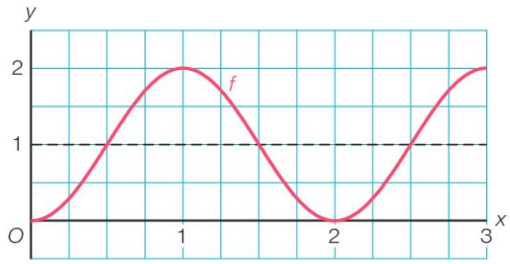
39 **a** $A(\cos(\frac{2}{3}\pi), \sin(\frac{2}{3}\pi)) = A(-\frac{1}{2}, \frac{1}{2}\sqrt{3})$

b $C(\cos(-\frac{1}{6}\pi), \sin(-\frac{1}{6}\pi)) = C(\frac{1}{2}\sqrt{3}, -\frac{1}{2})$

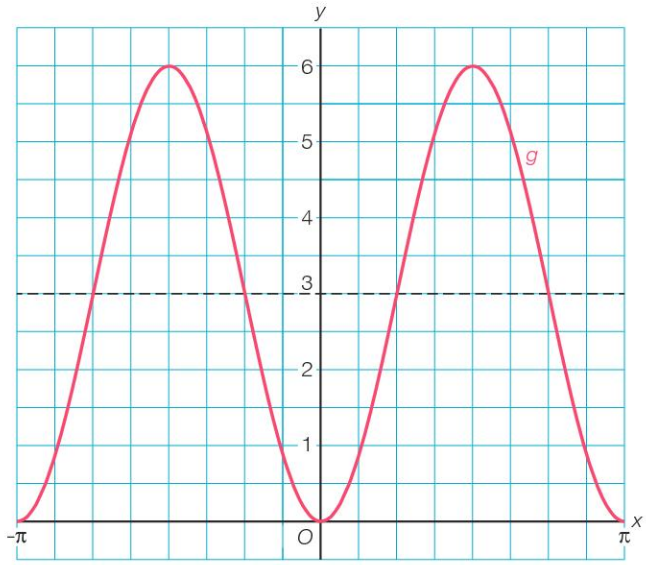
c $\cos(\beta) = -\frac{1}{2}\sqrt{2}$ geeft $\beta = -\frac{3}{4}\pi$

d De lengte van de langste cirkelboog BC is $1\frac{1}{4}\pi - -\frac{1}{6}\pi = 1\frac{5}{12}\pi$.

40 **a** $f(x) = 1 - \sin(\pi x + \frac{1}{2}\pi) = 1 - \sin(\pi(x + \frac{1}{2}))$
evenwichtsstand 1
amplitude 1
periode $\dfrac{2\pi}{\pi} = 2$
$-1 < 0$, dus grafiek dalend door het punt $(1\frac{1}{2}, 1)$.



b $g(x) = 3 + 3\cos(2x - \pi) = 3 + 3\cos(2(x - \frac{1}{2}\pi))$
evenwichtsstand 3
amplitude 3
periode $\dfrac{2\pi}{2} = \pi$
$3 > 0$, dus $(\frac{1}{2}\pi, 6)$ is een hoogste punt.



41 **a** $a = \dfrac{20 + -10}{2} = 5$
$b = 20 - 5 = 15$

Stijgend door de evenwichtsstand bij $t = 3$ en bij $t = 8$, dus periode $= 8 - 3 = 5$,
dus $c = \dfrac{2\pi}{5} = \frac{2}{5}\pi$.

Dalend door de evenwichtsstand bij $t = 3 - \frac{1}{2} \cdot$ periode $= 3 - \frac{1}{2} \cdot 5 = \frac{1}{2}$, dus $d = \frac{1}{2}$.

Dus $N = 5 - 15\sin(\frac{2}{5}\pi(t - \frac{1}{2}))$.

b Er is een hoogste punt bij $t = 3 + \frac{1}{4} \cdot$ periode $= 3 + \frac{1}{4} \cdot 5 = 4\frac{1}{4}$.

Dus $N = 5 + 15\cos(\frac{2}{5}\pi(t - 4\frac{1}{4}))$.

42 **a** Stel $y = a + b\sin(c(x - d))$.
$a = \dfrac{5 + -1}{2} = 2$
$b = 5 - 2 = 3$
periode $= 4\pi$, dus $c = \dfrac{2\pi}{4\pi} = \frac{1}{2}$.
Stijgend door de evenwichtsstand bij $x = \frac{1}{2}\pi$, dus $d = \frac{1}{2}\pi$.
Dus $y = 2 + 3\sin(\frac{1}{2}(x - \frac{1}{2}\pi))$.
Bijvoorbeeld:

$y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\sin(x) \xrightarrow{\text{verm. } y\text{-as, } 2} y = 3\sin(\frac{1}{2}x) \xrightarrow{\text{translatie } (\frac{1}{2}\pi,\ 2)} y = 2 + 3\sin(\frac{1}{2}(x - \frac{1}{2}\pi))$

b Een hoogste punt is $(1\frac{1}{2}\pi, 5)$, dus $y = 2 + 3\cos(\frac{1}{2}(x - 1\frac{1}{2}\pi))$.

Bijvoorbeeld:

$y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(x) \xrightarrow{\text{verm. } y\text{-as, } 2} y = 3\cos(\frac{1}{2}x) \xrightarrow{\text{translatie } (1\frac{1}{2}\pi,\ 2)} y = 2 + 3\cos(\frac{1}{2}(x - 1\frac{1}{2}\pi))$

43 **a** $\sin^2(x + \frac{1}{3}\pi) = 1$

$\sin(x + \frac{1}{3}\pi) = 1 \vee \sin(x + \frac{1}{3}\pi) = -1$

$x + \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot 2\pi \vee x + \frac{1}{3}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$

$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$

b $\cos(2x - \frac{1}{3}\pi) \cdot \sin(2x - \frac{1}{3}\pi) = 0$

$\cos(2x - \frac{1}{3}\pi) = 0 \vee \sin(2x - \frac{1}{3}\pi) = 0$

$2x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi \vee 2x - \frac{1}{3}\pi = k \cdot \pi$

$2x = \frac{5}{6}\pi + k \cdot \pi \vee 2x = \frac{1}{3}\pi + k \cdot \pi$

$x = \frac{5}{12}\pi + k \cdot \frac{1}{2}\pi \vee x = \frac{1}{6}\pi + k \cdot \frac{1}{2}\pi$

c $\tan(2x - \frac{1}{4}\pi) = \tan(\frac{1}{2}x + \frac{1}{3}\pi)$

$2x - \frac{1}{4}\pi = \frac{1}{2}x + \frac{1}{3}\pi + k \cdot \pi$

$1\frac{1}{2}x = \frac{7}{12}\pi + k \cdot \pi$

$x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$

d $\sin(\frac{1}{2}\pi x + \frac{1}{4}\pi) \cdot (\cos(\pi x) - 1) = 0$

$\sin(\frac{1}{2}\pi x + \frac{1}{4}\pi) = 0 \vee \cos(\pi x) - 1 = 0$

$\frac{1}{2}\pi x + \frac{1}{4}\pi = k \cdot \pi \vee \cos(\pi x) = 1$

$\frac{1}{2}\pi x = -\frac{1}{4}\pi + k \cdot \pi \vee \pi x = k \cdot 2\pi$

$x = -\frac{1}{2} + k \cdot 2 \vee x = k \cdot 2$

e $\tan^2(\pi x) = \tan(\pi x)$

$\tan(\pi x) = 0 \vee \tan(\pi x) = 1$

$\pi x = k \cdot \pi \vee \pi x = \frac{1}{4}\pi + k \cdot \pi$

$x = k \cdot 1 \vee x = \frac{1}{4} + k \cdot 1$

f $\sin(2x + \frac{1}{4}\pi) = \sin(x - \frac{1}{3}\pi)$

$2x + \frac{1}{4}\pi = x - \frac{1}{3}\pi + k \cdot 2\pi \vee 2x + \frac{1}{4}\pi = \pi - (x - \frac{1}{3}\pi) + k \cdot 2\pi$

$x = -\frac{7}{12}\pi + k \cdot 2\pi \vee 2x + \frac{1}{4}\pi = \pi - x + \frac{1}{3}\pi + k \cdot 2\pi$

$x = -\frac{7}{12}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{12}\pi + k \cdot 2\pi$

$x = -\frac{7}{12}\pi + k \cdot 2\pi \vee x = \frac{13}{36}\pi + k \cdot \frac{2}{3}\pi$

Bladzijde 182

44 **a** $\sin^2(\frac{1}{2}x + \frac{1}{4}\pi) = \frac{1}{2}$

$\sin(\frac{1}{2}x + \frac{1}{4}\pi) = \frac{1}{2}\sqrt{2} \vee \sin(\frac{1}{2}x + \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$

$\frac{1}{2}x + \frac{1}{4}\pi = \frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x + \frac{1}{4}\pi = \frac{3}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x + \frac{1}{4}\pi = 1\frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x + \frac{1}{4}\pi = -\frac{1}{4}\pi + k \cdot 2\pi$

$\frac{1}{2}x = k \cdot 2\pi \vee \frac{1}{2}x = \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = k \cdot 4\pi \vee x = \pi + k \cdot 4\pi \vee x = 2\pi + k \cdot 4\pi \vee x = -\pi + k \cdot 4\pi$

x in $[0, 2\pi]$ geeft $x = 0 \vee x = \pi \vee x = 2\pi$

b $\tan^2(x) = 3$

$\tan(x) = \sqrt{3} \vee \tan(x) = -\sqrt{3}$

$x = \frac{1}{3}\pi + k \cdot \pi \vee x = \frac{2}{3}\pi + k \cdot \pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = \frac{2}{3}\pi \vee x = 1\frac{2}{3}\pi$

c $\cos^2(x) - \frac{1}{2}\cos(x) = 0$

$\cos(x)(\cos(x) - \frac{1}{2}) = 0$

$\cos(x) = 0 \vee \cos(x) = \frac{1}{2}$

$x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi \vee x = \frac{1}{3}\pi \vee x = 1\frac{2}{3}\pi$

d $\cos(2x) - \cos(\frac{1}{2}x) = 0$

$\cos(2x) = \cos(\frac{1}{2}x)$

$2x = \frac{1}{2}x + k \cdot 2\pi \vee 2x = -\frac{1}{2}x + k \cdot 2\pi$

$1\frac{1}{2}x = k \cdot 2\pi \vee 2\frac{1}{2}x = k \cdot 2\pi$

$x = k \cdot 1\frac{1}{3}\pi \vee x = k \cdot \frac{4}{5}\pi$

x in $[0, 2\pi]$ geeft $x = 0 \vee x = 1\frac{1}{3}\pi \vee x = \frac{4}{5}\pi \vee x = 1\frac{3}{5}\pi$

45 **a** $\cos(\frac{2}{3}\pi(x-1)) = \frac{1}{2}\sqrt{3}$

$\frac{2}{3}\pi(x-1) = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{2}{3}\pi(x-1) = -\frac{1}{6}\pi + k \cdot 2\pi$

$\frac{2}{3}\pi x - \frac{2}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{2}{3}\pi x - \frac{2}{3}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$

$\frac{2}{3}\pi x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{2}{3}\pi x = \frac{1}{2}\pi + k \cdot 2\pi$

$x = 1\frac{1}{4} + k \cdot 3 \vee x = \frac{3}{4} + k \cdot 3$

x in $[0, 5]$ geeft $x = 1\frac{1}{4} \vee x = 4\frac{1}{4} \vee x = \frac{3}{4} \vee x = 3\frac{3}{4}$

b $\sin(\frac{1}{2}\pi x) = \sin(\frac{1}{2}\pi(x-1))$

$\frac{1}{2}\pi x = \frac{1}{2}\pi(x-1) + k \cdot 2\pi \vee \frac{1}{2}\pi x = \pi - \frac{1}{2}\pi(x-1) + k \cdot 2\pi$

$\frac{1}{2}\pi x = \frac{1}{2}\pi x - \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = \pi - \frac{1}{2}\pi x + \frac{1}{2}\pi + k \cdot 2\pi$

$0 = -\frac{1}{2}\pi + k \cdot 2\pi \vee \pi x = 1\frac{1}{2}\pi + k \cdot 2\pi$

geen opl. $\quad x = 1\frac{1}{2} + k \cdot 2$

x in $[0, 5]$ geeft $x = 1\frac{1}{2} \vee x = 3\frac{1}{2}$

c $\tan(\frac{1}{3}\pi x) + \sqrt{3} = 0$

$\tan(\frac{1}{3}\pi x) = -\sqrt{3}$

$\frac{1}{3}\pi x = \frac{2}{3}\pi + k \cdot \pi$

$x = 2 + k \cdot 3$

x in $[0, 5]$ geeft $x = 2 \vee x = 5$

d $\sin^2(\frac{1}{3}\pi x) = \frac{1}{2}\sqrt{3} \cdot \sin(\frac{1}{3}\pi x)$

$\sin(\frac{1}{3}\pi x) = 0 \vee \sin(\frac{1}{3}\pi x) = \frac{1}{2}\sqrt{3}$

$\frac{1}{3}\pi x = k \cdot \pi \vee \frac{1}{3}\pi x = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{1}{3}\pi x = \frac{2}{3}\pi + k \cdot 2\pi$

$x = k \cdot 3 \vee x = 1 + k \cdot 6 \vee x = 2 + k \cdot 6$

x in $[0, 5]$ geeft $x = 0 \vee x = 3 \vee x = 1 \vee x = 2$

46 **a** $f(x) = 2$ geeft $2 - 4\sin(2x - \frac{1}{3}\pi) = 2$

$4\sin(2x - \frac{1}{3}\pi) = 0$

$\sin(2x - \frac{1}{3}\pi) = 0$

$2x - \frac{1}{3}\pi = k \cdot \pi$

$2x = \frac{1}{3}\pi + k \cdot \pi$

$x = \frac{1}{6}\pi + k \cdot \frac{1}{2}\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{2}{3}\pi$

De grafiek van f snijdt de lijn van de evenwichtsstand in de punten $(\frac{1}{6}\pi, 2)$, $(\frac{2}{3}\pi, 2)$, $(1\frac{1}{6}\pi, 2)$ en $(1\frac{2}{3}\pi, 2)$.

b Toppen als $\sin(2x - \frac{1}{3}\pi) = 1 \vee \sin(2x - \frac{1}{3}\pi) = -1$

$2x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$

$2x = \frac{5}{6}\pi + k \cdot 2\pi \vee 2x = 1\frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{12}\pi + k \cdot \pi \vee x = \frac{11}{12}\pi + k \cdot \pi$

x in $[0, 2\pi]$ geeft $x = \frac{5}{12}\pi \vee x = 1\frac{5}{12}\pi \vee x = \frac{11}{12}\pi \vee x = 1\frac{11}{12}\pi$

De toppen van de grafiek zijn $(\frac{5}{12}\pi, -2)$, $(1\frac{5}{12}\pi, -2)$, $(\frac{11}{12}\pi, 6)$ en $(1\frac{11}{12}\pi, 6)$.

c $f(x) = 2 - 4\sin(x)$ geeft $2 - 4\sin(2x - \frac{1}{3}\pi) = 2 - 4\sin(x)$

$4\sin(2x - \frac{1}{3}\pi) = 4\sin(x)$

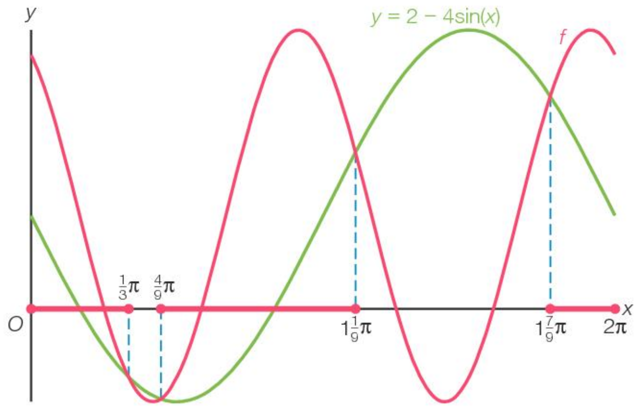
$\sin(2x - \frac{1}{3}\pi) = \sin(x)$

$2x - \frac{1}{3}\pi = x + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \pi - x + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = \frac{4}{9}\pi + k \cdot \frac{2}{3}\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = \frac{4}{9}\pi \vee x = 1\frac{1}{9}\pi \vee x = 1\frac{7}{9}\pi$



$f(x) \geq 2 - 4\sin(x)$ geeft $0 \leq x \leq \frac{1}{3}\pi \vee \frac{4}{9}\pi \leq x \leq 1\frac{1}{9}\pi \vee 1\frac{7}{9}\pi \leq x \leq 2\pi$

47 **a** $-\cos(2\pi x - \frac{1}{4}\pi) = \cos(2\pi x - \frac{1}{4}\pi + \pi) = \sin(2\pi x - \frac{1}{4}\pi + \pi + \frac{1}{2}\pi) = \sin(2\pi x + 1\frac{1}{4}\pi)$

b $-\sin(2(x + 1\frac{1}{4}\pi)) = -\sin(2x + 2\frac{1}{2}\pi) = \sin(2x + 2\frac{1}{2}\pi + \pi) = \cos(2x + 2\frac{1}{2}\pi + \pi - \frac{1}{2}\pi) =$
$\cos(2x + 3\pi) = \cos(2x + \pi)$

c $\sin(2x + \frac{2}{3}\pi) - 2\sin(2x - \frac{1}{3}\pi) = \cos(2x + \frac{2}{3}\pi - \frac{1}{2}\pi) - 2\cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) =$
$\cos(2x + \frac{1}{6}\pi) + 2\cos(2x - \frac{5}{6}\pi + \pi) = \cos(2x + \frac{1}{6}\pi) + 2\cos(2x + \frac{1}{6}\pi) = 3\cos(2x + \frac{1}{6}\pi)$

d $\dfrac{\cos(2x - \frac{1}{3}\pi)}{\sin(2x + \frac{2}{3}\pi)} = \dfrac{\sin(2x - \frac{1}{3}\pi + \frac{1}{2}\pi)}{\cos(2x + \frac{2}{3}\pi - \frac{1}{2}\pi)} = \dfrac{\sin(2x + \frac{1}{6}\pi)}{\cos(2x + \frac{1}{6}\pi)} = \tan(2x + \frac{1}{6}\pi)$

48 **a** $f(x) = x^2 \cdot \sin(3x - \frac{1}{2}\pi)$ geeft $f'(x) = 2x \cdot \sin(3x - \frac{1}{2}\pi) + x^2 \cdot \cos(3x - \frac{1}{2}\pi) \cdot 3$
$= 2x\sin(3x - \frac{1}{2}\pi) + 3x^2\cos(3x - \frac{1}{2}\pi)$

b $f(x) = \dfrac{x}{2 + \cos(x)}$ geeft $f'(x) = \dfrac{(2 + \cos(x)) \cdot 1 - x \cdot -\sin(x)}{(2 + \cos(x))^2} = \dfrac{2 + \cos(x) + x\sin(x)}{(2 + \cos(x))^2}$

c $f(x) = x^2 \cdot \cos^3(x)$ geeft $f'(x) = 2x \cdot \cos^3(x) + x^2 \cdot 3\cos^2(x) \cdot -\sin(x) = 2x\cos^3(x) - 3x^2\sin(x)\cos^2(x)$

d $f(x) = x^2 - \dfrac{1}{\tan^2(x)} = x^2 - \tan^{-2}(x)$ geeft $f'(x) = 2x + 2\tan^{-3}(x) \cdot (1 + \tan^2(x)) = 2x + \dfrac{2 + 2\tan^2(x)}{\tan^3(x)}$

e $f(x) = \sqrt{2\sin(x) + 3\cos(x)}$ geeft $f'(x) = \dfrac{1}{2\sqrt{2\sin(x) + 3\cos(x)}} \cdot (2\cos(x) - 3\sin(x)) = \dfrac{2\cos(x) - 3\sin(x)}{2\sqrt{2\sin(x) + 3\cos(x)}}$

f $f(x) = \dfrac{5}{4\sin(x) + 3\cos(x)}$ geeft $f'(x) = \dfrac{(4\sin(x) + 3\cos(x)) \cdot 0 - 5 \cdot (4\cos(x) - 3\sin(x))}{(4\sin(x) + 3\cos(x))^2} = \dfrac{15\sin(x) - 20\cos(x)}{(4\sin(x) + 3\cos(x))^2}$

49 **a** $f(x) = \dfrac{\cos(x)}{2 + \sin(x)}$ geeft $f'(x) = \dfrac{(2 + \sin(x)) \cdot -\sin(x) - \cos(x) \cdot \cos(x)}{(2 + \sin(x))^2}$

$= \dfrac{-2\sin(x) - \sin^2(x) - \cos^2(x)}{(2 + \sin(x))^2}$

$= \dfrac{-2\sin(x) - (\sin^2(x) + \cos^2(x))}{(2 + \sin(x))^2}$

$= \dfrac{-1 - 2\sin(x)}{(2 + \sin(x))^2}$

b Stel k: $y = ax + b$ met $a = f'(\frac{1}{2}\pi) = \dfrac{-1 - 2 \cdot 1}{(2 + 1)^2} = \dfrac{-3}{9} = -\frac{1}{3}$.

$\left.\begin{array}{l} y = -\frac{1}{3}x + b \\ f(\frac{1}{2}\pi) = 0, \text{ dus } A(\frac{1}{2}\pi, 0) \end{array}\right\} -\frac{1}{3} \cdot \frac{1}{2}\pi + b = 0$

$b = \frac{1}{6}\pi$

Dus k: $y = -\frac{1}{3}x + \frac{1}{6}\pi$.

c $f'(x) = 0$ geeft $\dfrac{-1 - 2\sin(x)}{(2 + \sin(x))^2} = 0$

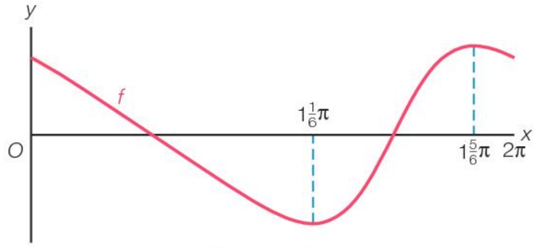
$-1 - 2\sin(x) = 0$

$2\sin(x) = -1$

$\sin(x) = -\frac{1}{2}$

$x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = 1\frac{1}{6}\pi$ (vold.) $\vee$ $x = 1\frac{5}{6}\pi$ (vold.)



min. is $f(1\frac{1}{6}\pi) = \dfrac{-\frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = \dfrac{-\frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = -\frac{1}{3}\sqrt{3}$.

max. is $f(1\frac{5}{6}\pi) = \dfrac{\frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = \dfrac{\frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = \frac{1}{3}\sqrt{3}$.

Dus $B_f = [-\frac{1}{3}\sqrt{3}, \frac{1}{3}\sqrt{3}]$.

Verantwoording

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Omslagontwerp: InOntwerp, Assen
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